Enhanced Blocking Probability Evaluation Method for Circuit-Switched Trunk Reservation Networks

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***Abstract*— We consider the problem of estimating steady-state blocking probability for circuit-switched networks that allow alternate routing with particular emphasis on networks protected by trunk reservation. We use a recently proposed blocking probability estimate, the Overflow Priority Classification Approximation (OPCA) as an alternative to the currently used Erlang's Fixed-Point Approximation (EFPA). We demonstrate empirically that OPCA provides a better blocking estimate than EFPA for circuit-switched networks with alternate routing, that are reasonably protected by trunk reservation.**

***Index Terms*— Erlang's Fixed Point Approximation, blocking probability, circuit switching, network design.**

I. Introduction

CIRCUIT-SWITCHED (CS) networks have been pervasive in telephony and it is envisaged that they will have a renewed role in future optical networks [1]. CS networks with *alternate routing* are examples of *overflow loss networks* which form an important class of network models. Overflow loss networks allow calls that have been blocked by one server group to overflow to a different server group under certain circumstances. The blocking probability is the probability that a call is blocked and not admitted to the network. Accurate techniques for blocking probability estimation are crucial in for performance evaluation and network design, and enable efficient network resource allocation.

The simplest and currently most used technique for estimating blocking in networks is Erlang's Fixed-Point Approximation (EFPA) [2], [3], [4]. EFPA makes reasonable sense intuitively, but yields a poor estimate in many overflow loss networks due to the crude assumptions it is based on, e.g. the assumption that the overflow streams behave as independent Poisson processes when in reality they are not. In fact, the variance is much greater than the mean and these streams are highly correlated [5]. In an attempt to improve the estimate, a novel method known as overflow priority classification approximation (OPCA) was proposed [5], [6].

Other extensions of EFPA for blocking probability estimation of CS networks have been proposed [7], [8] where [7] represents a higher order approximation while [8] seeks to capture correlation between links. These are the two distinct ways that the basic version of EFPA has been refined over the years [9]. The novelty of OPCA lies in the fact that a fictitious pre-emptive priority structure is imposed in order to improve the blocking estimate [5].

In [5], it was demonstrated in a hand-made example that OPCA yielded a superior estimate to EFPA and other methods based on EFPA. Here we demonstrate that OPCA yields a superior estimate to EFPA for the more practical case of CS networks with trunk reservation (TR).

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II. CS Networks with Alternate Routing

CS networks consist of switches (nodes) and server groups (links) between certain pairs of switches. Each server group consists of C independent cooperative servers (channels). Let $\mathcal{T}$ denote the set of all origin-destination (O-D) pairs and let $\mathcal{J}$ denote the set of all links. Call inter-arrival times at O-D pair $\tau \in \mathcal{T}$ are independent and exponentially distributed with parameter $a_\tau > 0$. Call holding times are exponentially distributed with unit mean.

The primary route for O-D pair τ is the path with the least number of links. Let $U_\tau(0)$ denote the set of links traversed by the primary route for O-D pair τ. The *alternate routes* for O-D pair τ are all the other paths from the origin node to the destination node with the condition that no two routes (primary or alternate) traverse a common link. Let $U_\tau(n)$, $n = 1, \dots, N_\tau$, denote the set of links traversed by the nth alternate route for O-D pair τ, where N_τ is the total number of alternate routes for O-D pair τ.

If a call arrives to O-D pair τ, it first seeks an idle channel in each of the links along the primary route $U_\tau(0)$. If unsuccessful, the call seeks an idle channel in each of the links along the first alternate route $U_\tau(1)$, etc. Denote a call that successfully engages its primary route a *primary call* and a call that does not, an *overflowed call*. The alternate routes are ordered based on least number of links, i.e., $|U_\tau(n_1)| \le |U_\tau(n_2)|$ for $n_1 < n_2$. If two routes contain the same number of links, the routes are ordered randomly. If a call is unsuccessful in seeking all routes, it is blocked and cleared from the network, signifying a blocking event.

III. Trunk Reservation

Implementing alternate routing can cause instability for a highly congested network due to overflowed calls generally requiring more links (and hence more channels) than primary calls [10]. Trunk reservation alleviates this issue by reserving a certain number of channels for primary calls, i.e., there exists some threshold $M < C$ such that if there are no more than M

busy channels in a trunk group, an overflowed call is rejected. The trunk threshold will be referred to as a percentage. For instance, $C = 10$ and $M = 9$ means a threshold of 10%.

IV. OPCA

OPCA imposes a pre-emptive priority structure on the original system and then applies Erlang's Fixed-Point Approximation to the new system. Henceforth, we refer to the original CS model as the *true model* and the new system with priority structure applied as the *fictitious model*. The idea is to apply EFPA to the fictitious model to achieve better blocking probability estimate rather than applying EFPA directly to the true model.

The pre-emptive priority structure gives priority to calls that overflowed fewer times. Namely, if a call that has overflowed n times (denoted as an n-*call*) arrives at a fully engaged trunk group where m_i-calls, $i = 1, 2 \dots K$, are being served where $n < m_1 < m_2 < \dots < m_K$, the n-call replaces an m_K-call, forcing the m_K-call to find another route, or if all alternate routes have been tried, the call is blocked.

The key reason for the success of OPCA is that it increases the proportion of 0-calls, i.e., primary calls, by giving them highest priority. This reduces the error associated with assuming that all streams follow a Poisson process since 0-calls do not violate the Poisson assumption. Further, having the fictitious model discriminates between different streams of calls helps reduce the error caused by the independence assumption [5], [6]. The success of OPCA relies on the fact the fictitious model and true model have similar overall blocking probabilities, allowing the advantages described above to take effect.

To derive equations required to apply OPCA to arbitrary CS networks with TR, we define an (n, τ)-*call* as an n-call pertaining to O-D pair τ and assume that the inter-arrival time of an (n, τ)-call process at link $j \in U_\tau(n)$ is independent and exponentially distributed with parameter $a(n, \tau, j)$. (This is analogous to the Poisson assumption inherent in EFPA.) Also let $b_j(n)$ be the blocking probability perceived by any n-call offered to link $j \in \mathcal{J}$.

The inter-arrival time of the (n, τ)-call process is independent and exponentially distributed with parameter $a(n, \tau)$. Noting that (n, τ)-calls occur only when $(n-1, \tau)$-calls are blocked for O-D pair τ and $n = 1, 2, \dots, N_\tau + 1$, we have

$$a(n,\tau) = a(n-1,\tau)\left(1 - \prod_{j \in U_\tau(n-1)} \big(1 - b_j(n-1)\big)\right), \quad (1)$$

and $a(0, \tau) = a_\tau$. We determine $a(n\tau, j)$ in terms of $a(n, \tau)$ by using the reduced load EFPA [2], [4]. Further, calls are carried only if they are accepted by all links along the route. In particular, for O-D pair τ and link $j \in U_\tau(n)$, we have

$$a(n,\tau,j) = a(n,\tau)\frac{\prod_{i \in U_\tau(n)} \big(1 - b_i(n)\big)}{1 - b_j(n)}, \quad (2)$$

for $n = 0, 1, \dots, N_\tau$. While for $n > N_\tau$ or $j \notin U_\tau(n)$, we have $a(n, \tau, j) = 0$.

Since we assume that inter-arrival times of the (n, τ)-calls process at link $j \in U_\tau(n)$ are is independent and exponentially distributed with parameter $a(n, \tau, j)$, we can superpose each of the $a(n, \tau, j)$ streams originating from different O-D pairs $\tau \in \mathcal{T}$. In particular, these inter-arrival times (originating from any O-D pair) are independent and exponentially distributed with parameter denoted as $a_j(n) = \sum_{\tau \in \mathcal{T}} a(n, \tau, j)$.

Finally, we determine $b_j(n)$ in terms of $a(n, \tau, j)$ by taking into account that an (n, τ_1)-call can preempt an (m, τ_2)-call, where $n < m$ and τ_1 may be equal to τ_2. We have for all $j \in \mathcal{J}$ and $n > 0$,

$$b_j(n) = \frac{a_j(0)Q_j(n) + R_j(n)\sum_{i=1}^{n} a_j(i) - \nu_j(n)}{a_j(n)}, \quad (3)$$

where $\nu_j(n) = \sum_{i=0}^{n-1} a_j(i) b_j(i)$ and $b_j(0)$ is given by the Erlang B formula with offered load $a_j(0)$ and C servers. Eq. (3) follows since the probability an n-call is blocked at link j is the ratio of the intensity of blocked n-calls to total n-calls along link j. $Q_j(n)$ and $R_j(n)$ are functions of the steady-state probabilities of a one-dimensional birth-and-death process characterizing each link $j \in \mathcal{J}$. For $j \in \mathcal{J}$, we have $Q_j(n) = \pi_j^C(n)$ and $R_j(n) = \pi_j^M(n) + \pi_j^{M+1}(n) + \dots + \pi_j^C(n)$. For a given n, we numerically compute the steady-state probabilities $\{\pi_j^i(n)\}_{i=0}^C$ via the following recursion: for $i = 1, \dots, M$, we have

$$\pi_j^i(n) = \frac{[a_j(0) + a_j(1) + \dots + a_j(n)]^i \pi_j^0(n)}{i!},$$

while for $i = M+1, \dots, C$, we have

$$\pi_j^i(n) = \frac{a_j(0)^{i-M}[a_j(0) + a_j(1) + \dots + a_j(n)]^M \pi_j^0(n)}{i!}.$$

The normalization constant $\pi_j^0(n)$ is determined by solving $\sum_{i=0}^C \pi_j^i(n) = 1$. The case of routing without TR occurs when $M = C$ resulting in $Q_j(n) = R_j(n) = \pi_j^C(n)$.

Equations (1), (2) and (3) give rise to a coupled set of nonlinear equations, which may be solved using successive substitution. The probability that a call originating from O-D pair τ is blocked is given by $a(N_\tau + 1, \tau)/a_\tau$.

V. Empirical Results and Discussion

The performance of OPCA compared to EFPA is tested for a broad range of networks with different topologies, number of trunks and trunk thresholds. Since exact blocking probabilities are not tractable except for very small networks, simulation results are used as a benchmark. The results are subtly different depending on the various network parameters, but the general trends are consistent. Other results are available in [11], [12].

In Fig. 1, results for the NSF network with topology shown in Fig. 10(c) of [5], 20 channels per link and a threshold of 10% are presented. Fig. 2 displays results for a fully-meshed 6-node network with 10 channels and threshold of 20%. Blocking in the range of $[10^{-5}, 10^{-1}]$ is considered. The results demonstrate that OPCA yields a superior estimate to EFPA for these two networks. In general, OPCA yields an accurate estimate for networks with trunk threshold over 10%. OPCA is not as accurate when the threshold is less than 10%. Fig. 3 presents blocking probabilities obtained by OPCA, EFPA and simulation for an 6-node fully-meshed network with 10 channels without TR (0% threshold). Although the

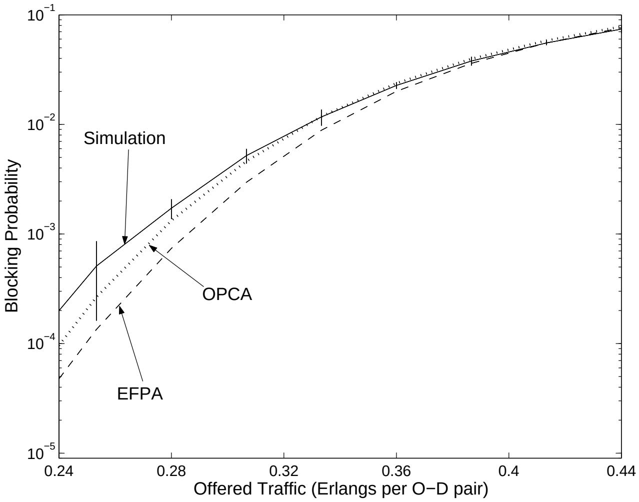

Fig. 1. Blocking probability comparison of OPCA vs EFPA for an NSF network with 20 channels and 10% threshold.

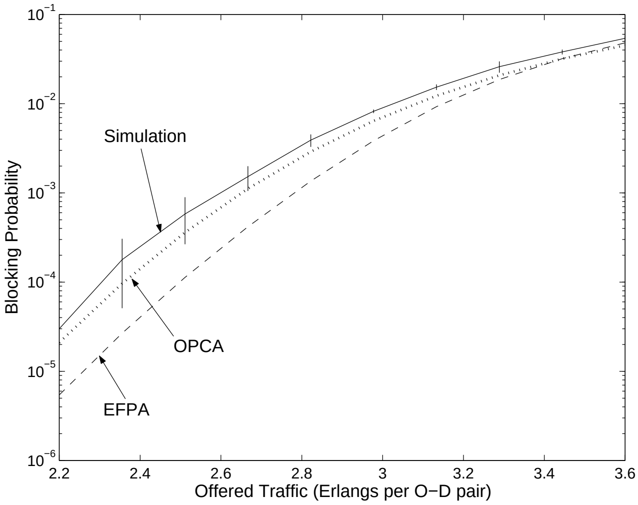

Fig. 2. Blocking probability comparison of OPCA vs EFPA for an 6-node fully meshed network with 10 channels and 20% threshold.

performance of OPCA may still be perceived as superior to EFPA for low traffic, the OPCA curve does not follow the simulation curve as closely as when TR is employed. In particular, OPCA underestimates blocking for high traffic.

To explain the poor performance of OPCA for networks without TR at high traffic, we contrast the behavior of the true and the fictitious models. At high traffic, since the network becomes congested and the true model imposes no priority, the chance of a call being serviced along its primary route is small. Therefore, since most if not all calls overflow and overflowed calls use more resources (channels), fewer calls are serviced than if alternate routing were not permitted. The fictitious model places priority on calls that have not overflowed, so a primary call will never be blocked by an overflowed call. Therefore, at high traffic the fictitious model carrying more primary calls is much less congested than the true model since primary calls consume fewer network resources. As a result, the fictitious structure significantly changes the overall blocking behavior of the original system, causing the true and fictitious models to have significantly different blocking

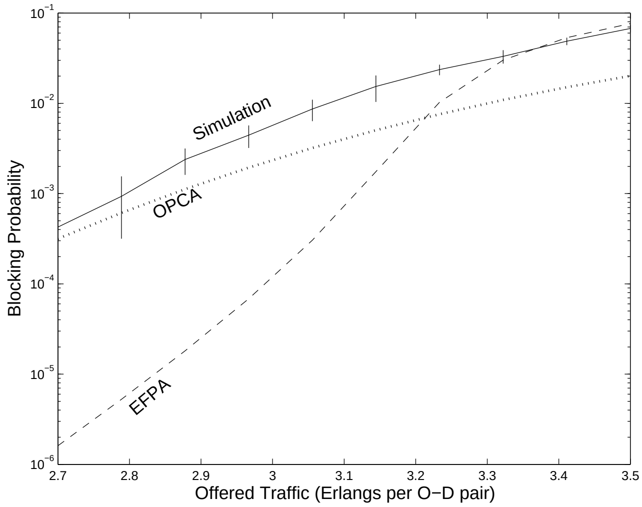

Fig. 3. Blocking probability comparison of OPCA vs EFPA for an 6-node fully meshed network with 10 channels and alternate routing without TR.

probabilities. This offsets the advantages described in section IV causing OPCA to be less accurate than EFPA.

TR closes the gap between the true model and the fictitious model. Under TR, the true model gives priority to primary calls over overflowed calls which more closely mirrors the fictitious model than in the case of routing without TR. Hence, OPCA blocking probability prediction improves with TR.

VI. Conclusion

For circuit-switched networks with reasonable trunk reservation threshold of over 10% or more, OPCA yields a better blocking probability estimate than EFPA while if the threshold is less than 10% it is not always the case.

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