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控 制 动 力 系 统 的 分 岔 现 象

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摘要：控制动力系统的分岔现象是一个比混沌控制历史要长的研究课题。随着混沌控制的发展，这一课题的研究工作又重新活跃起来。本文对分岔控制的现状和前景做了一个简单的综合介绍和评述，指出这一领域的重要性,以期引起动力系统和控制论研究人员的充分关注。

关键词：非线性动力系统，分岔，混沌，控制

1. 引言

动力系统的分岔现象指的是随着某些参数的变化,系统的动力性态发生质的改变,特别是系统的平衡状态发生稳定性改变或出现方程解的轨道分枝。各种典型的分岔及它们严格的数学定义可以在许多教科书里找到。动力系统的分岔现象不但在数学领域如动力系统理论,微分方程,拓扑,几何，而且在工程科学象神经网络,电网,电路,流体动力学,化学反应，等等都受到高度的重视并有相当长时间和非常大量的研究。值得注意的是,动力系统分岔的研究在近十多年来在工程系统控制理论中也进行得十分活跃。这主要发生在控制论的一个新的分枝 -- 分岔控制和混沌控制的领域内。

分岔控制指的是通过控制手段去改变动力系统分岔现象的各种特征。典型的分岔控制包括镇定不稳的分岔轨道[1-10]，延迟分岔的出现[5,11]，改变分岔点对应的系统参数值[12,13]，改变分岔轨道的形状或类型[6]，有目的地引进新的分岔[14-20]，控制极限环的个数，大小，周期，或重数[21-27]，优化系统在分岔点附近的动力行为[28]，通过控制分岔来控制混沌[3,4,12,15,29,30]，等等，有时甚至会是它们的某种组合。

从现有文献来看，人们对分岔动力学的认识显然比对混沌现象的认识清楚得多和深刻得多，其研究工作无论在质还是量++都显得相当完整。目前,人们对分岔控制好象不如对混沌控制更有兴趣。其实，控制动力系统的分岔现象是一个比混沌控制历史要长的研究课题。对文献稍作调查发现，就分岔控制而言，它的研究记录至少可以追溯到 A.A.Andronov 等人 60 年代的工作[31],其中对如何通过分岔控制来产生平面系统极限环的问题已有相当完整的论述。当然，在这项工作中，控制论的特征并不明显。但此后把控制论与动力系统理论结合起来的研究，以 E.H.Abed 等人的早期工作[1,2]为代表，尤其是近年来的长足发展[12,89,90]，已使该领域形成一种丰富多彩的局面。本文对目前国内外分岔控制的现状和前景提供一个简单的综合介绍和评述，并指出这一领域在工程应用上的重要性,以期引起国内同行的充分关注。

2. 分岔控制：一个实际例子

$$\begin{cases} \dot{\theta} &= \omega, \\ \dot{\omega} &= 16.6667\sin(\theta_L - \theta + 0.0873)V_L - 0.1667\omega + 1.8807, \\ \dot{\theta}_L &= 496.8718V_L^2 - 166.6667\cos(\theta_L - \theta - 0.0873)V_L \\ & \quad - 666.6667\cos(\theta_L - 0.2094)V_L - 93.3333V_L + 33.3333p + 43.333, \\ \dot{V}_L &= -78.7638V_L^2 + 26.2172\cos(\theta_L - \theta - 0.0124)V_L \\ & \quad + 104.8689\cos(\theta_L - 0.1346)V_L + 14.5229V_L - 5.2288p - 7.0327, \end{cases} \tag{1}$$

一个简单电网模型（图 1）可由下面的动力系统来描述[12]：

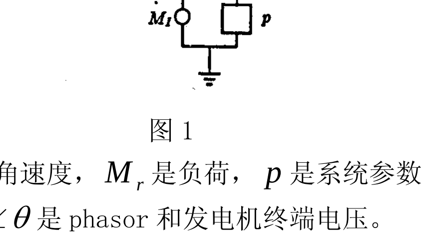

图 1

其中，θ 是发电机转角，$\omega = \dot{\theta}$ 是角速度，M_r 是负荷，p 是系统参数，$V_L \angle \theta_L$ 是负荷电压（V_L 是幅度，$\angle \theta_L$ 是相位），$E \angle 0°$ 和 $E_m \angle \theta$ 是 phasor 和发电机终端电压。

当系统参数 p 逐渐增加或减小时，可观察到两系列的复杂动力现象[12]，如图 2 所示（图中，(1)表示稳定不动点，(2)表示稳定极限环，(3)和(4)表示不同的不稳定不动电，(5)和(6)表示不同的不稳定极限环）：

图的左面：

- p =10.818 出现周期解的转折点， p =10.873 出现第一个周期倍分，
- p =10.882 出现第二个周期倍分， p =10.946 出现不稳定的 Hopf 分岔。

图的右面：

- p =11.410 出现鞍型分岔， p =11.407 出现稳定的 Hopf 分岔，
- p =11.389 出现第一个周期倍分， p =10.384 出现第二个周期倍分。

如所熟知，周期倍分的结果是混沌现象，如图 3 所示。

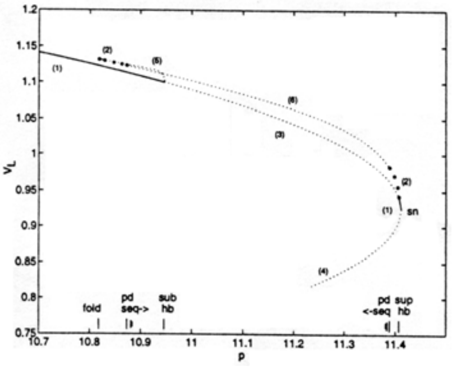

图 2

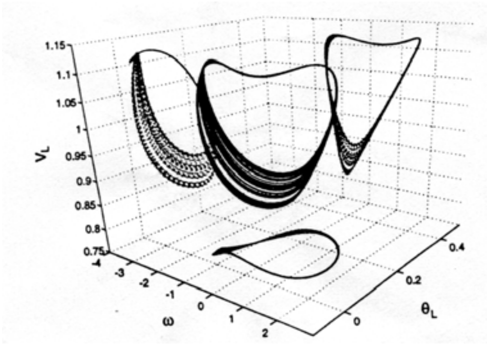

图 3

对于这个电力系统模型，分岔控制问题的提法包括：

1. 能否通过控制手段消除最终的系统混沌现象？
2. 能否通过控制手段延迟第一个周期倍分的发生？
3. 能否通过控制手段改变某个分岔的稳定性？
4. 能否通过分岔或混沌控制手段来避免电网灾变？

许多类似的控制问题都不是常规的和常见的，而且显然不容易，故此很有挑战性。

3. 分岔控制的一些方法

尽管分岔现象一般来说十分复杂，它并不象混沌现象那样放荡不羁而难于预测。因此，对于分岔能否控制的问题，人们一开始就没有疑问。然而，它又不象镇定混沌那样直接 (后来人们知道，大部分的常规控制方法都可以用来镇定混沌 -- 如果控制混沌的目的仅仅是把它消除掉并且允许使用强大输入即“brute force”的话 -- 当然，混沌控制领域包括很多非常规的目的和要求，因而需要发展很多新的理论和技术[12,29,30]。有兴趣的读者可参考本人在本刊的另一篇综述[94].) 分岔控制一开始就遇到如何应用常规技术的困难，因为这些控制方法通常都简单地把整个分岔现象清除掉，从而达不到镇定或改变其动力性态的目的。很快，人们就知道，常规技术需要改造，同时新颖技术需要发展。现在，具我们所知，分岔只可以用不多的几种方法来控制,尽管各种方法都有其理论分析和实验或模拟验证，并且在工程，生物，物理，化学，军事等领域都有一些非常规的应用。

目前，分岔控制的手段有几种线性和非线性反馈方法[1-3,17,18,21,29,30,36,38,42,57,63,64]，应用 wash-out-filter [5,15,16,20,74,75]， 频域分析和逼近方法[13,23-26,28-30,33,85-88]，以及利用标准型理论[6,24]，等等。较详尽的介绍可在文献[12,89,90] 中找到。一般而言，线性特别是非线性状态反馈是行之有效的分岔控制方法。例如，应用 wash-out-filter 的状态反馈分岔控制可以应用于高维系统和多种分岔现象的控制。如果控制问题牵涉到极限环，则频域分析和逼近方法十分有效，因为极限环通常都没有解析解，只能借助于逼近方法。而基于频域分析的逼近方法在这方面的研究和应用已有长时间的成功经验。此外，标准型理论是分岔分析中非常自然和精确的工具，因此在分岔控制方面看来很有前途。现就这几种方法作一简单介绍(当然, 有效方法绝不限于这些,见[12,89,90])。

3.1. 线性和非线性反馈方法

作为例子，考虑一个简单的二维系统：

$$\begin{cases} \dot{x} = f(x, y; \mu), \\ \dot{y} = g(x, y; \mu). \end{cases} \tag{2}$$

其中 μ 是实参数， f 和 g 为一次连续可微函数，满足 $f(x^*, y^*; \mu) = g(x^*, y^*, \mu) = 0, (x^*, y^*)$ 为系统的不动点， $u = u(x, y; \mu)$ 是待设计的状态反馈控制器。

假定原系统并没有 Hopf 分岔，但我们希望设计出一个线性状态反馈控制器使得被控系统以一个指定的相平面点$(x^0, y^0; \mu^0)$为其新的不动点并在其上出现 Hopf 分岔。

根据不动点的要求，可知点(x^0, y^0)需满足 1

$$\begin{cases} f(x^0, y^0; \mu) = 0, \\ g(x^0, y^0; \mu) + u(x^0, y^0; \mu) = 0, \end{cases} \quad (3)$$

再根据分岔的基本条件，可知系统在点(x^0, y^0)上的特征值

$$\lambda_{1,2}^c(\mu) = \frac{1}{2}(f_x + g_y + u_y) \pm \frac{1}{2}\sqrt{(f_x + g_y + u_y)^2 - 4[f_x(g_y + u_y) - f_y(g_x + u)]} \quad (4)$$

应能满足（充分条件）

$$\begin{aligned} &(f_x + g_y + u_y)\Big|_{\mu=\mu^0} = 0, \\ &f_x(g_y + u_y) - f_y(g_x + u_x)\Big|_{\mu=\mu^0} > 0, \\ &(f_x + g_y + u_y)^2 - 4\big[f_x(g_y + u_y) - f_y(g_x + u_x)\big]\Big|_{\mu\neq\mu^0} < 0, \\ &\frac{\partial \operatorname{Re}\{\lambda_1^c(\mu)\}}{\partial\mu}\Big|_{\mu=\mu^0} = \frac{\partial(f_x + g_y + u_y)}{\partial\mu}\Big|_{\mu=\mu^0} > 0. \end{aligned} \quad (5)$$

上述条件给出了控制器设计的一些基本准则。以熟悉的 van der Pol 振子

$$\begin{cases} \dot{x} = y, \\ \dot{y} = \mu(1 - x^2)y - x \end{cases} \quad (6)$$

为例，它在不动点(0,0)上并无 Hopf 分岔。假定我们希望在原来的不动点上产生 Hopf 分岔，则上述条件给出线性状态反馈控制器

$$u = -\mu^0(1 - (x^0)^2)y, \quad (7)$$

把它加入到 van der Pol 系统的第二个方程的右端，可以得到期望的 Hopf 分岔。值得指出的是，所设计的控制器满足

$$u(x^*, y^*; \mu) = -\mu^0(1 - (x^0)^2)y^* = 0, \quad (8)$$

故此不改变原来的系统的不动点。既要不改变原来的系统的不动点，又要改变系统的其他动力行为，一般的常规线性控制器是不容易设计成功的。

3.2 应用 wash-out-filter 方法

作为例子，考虑 Lorenz 系统

$$\begin{cases} \dot{x} = -p(x - y), \\ \dot{y} = -xz - y, \\ \dot{z} = xy - z - r, \end{cases} \quad (9)$$

当r变化时，这系统有熟知的分岔现象如图 4 所示。

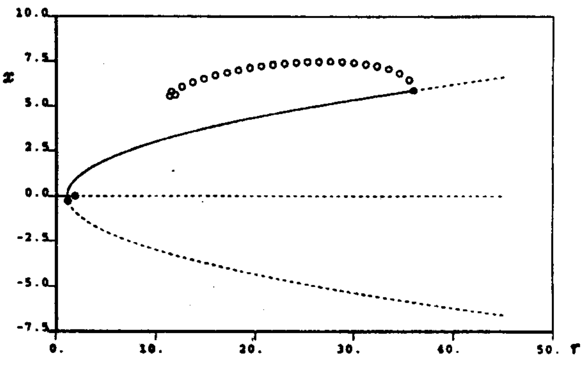

图 4

所谓的 wash-out-filter 可以理解为一种扩充的线性或非线性状态反馈控制器。把它加到 Lorenz 系统后具有形式

$$\begin{cases} \dot{x} = -p(x - y), \\ \dot{y} = -xz - y, \\ \dot{z} = xy - z - r + u, \\ \dot{v} = y - cv, \end{cases} \quad (10)$$

其中 v 是新引进的状态变量，

$$u = -k_c(y - cv) - k_n(y - cv)^3 \qquad (11)$$

是控制器，其余的都是常数或代定参数。当 c=0.5, k_c=2.5 和 k_n=0.009 时，原有的一不稳定的周期轨道分枝被控制变成一稳定点[5]，如图 5 所示。

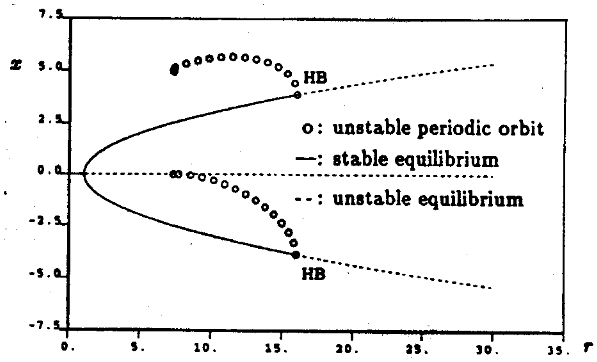

图 5

3.3 频域分析和逼近方法

作为例子，考虑典型的 Lur′e 系统（图 6）

$$f * (g \circ y + k_c \circ y) + y = 0, \qquad (12)$$

其中 ∗ 和 ∘ 表示卷积和复合运算，f 和 g 为给定系统及内反馈，k_c 为代设计的控制器。假定系统具有一族可预测的一阶极限环，并在 p_h〈 p〈 p_c 参数范围内稳定。

$$y^{<1>}(t) = y_0 + y_1 \sin(\omega\ t) \qquad (13)$$

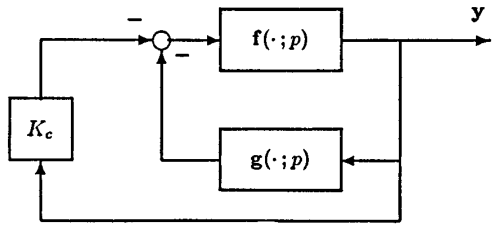

图 6

为其预测一阶极限环的近似。Harmonic balance 分析表明[11]，一个适当设计的控制器

$$k_c(s) = k_c \frac{s(s^2 + w^2(p_h))}{(s+a)^3}, \qquad (14)$$

可以达到延迟原来内在周期倍分的发生，如图 7 所示。

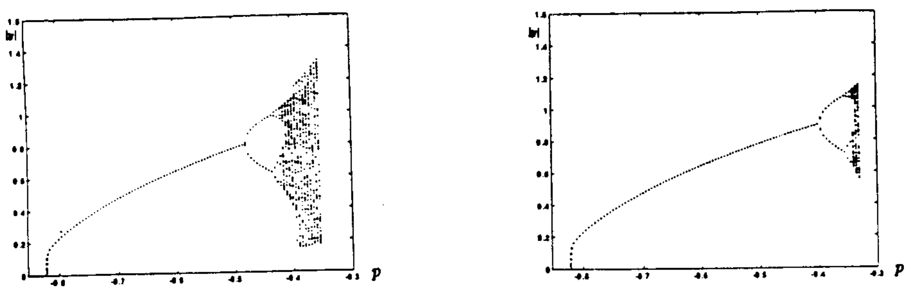
(a) before control (b) after control

图 7

4. 分岔控制的若干工程应用

就目前所知，分岔控制的研究能够在化学工程[32,33]，机械工程[33-42]，电子工程[10,43-52]，航天和航空工程[8,53-70]，生物医学[71-77]，物理和化学[78-83]，以及气象[84]等方面找到分析和应用，显示出其可观的前景和巨大的潜力。就分岔控制本身而言，人体心脏脉搏以及两心室交替跳动的控制提供了一个很好的例子。心脏非正常脉搏跳动呈现复杂的分岔和混沌动力性态，分岔控制很可能提供一种新的有效的调整手段[71-77]。又如，大型电网临近严重超负荷导致灾变时，非线性动力分岔现象非常显著。通过分岔控制的办法能够有效地延迟分岔的发生，或暂时镇定危险的分岔过程，使有足够的时来采取保护措施，以及避免这种危险的灾变[44-47]。此外，在诸如热交换过程控制 [5]，卫星姿态控制[55,65,67,92]，以及电路，激光，机器人，神经网络，机床切削等控制问题中都存在大量的分岔现象，因而都是分岔控制的应用领域。特别是在航天和航空应用方面，分岔控制在发动机控制中有着特殊的功用。近年来，中轴流体压缩 (axial flow compressor) 的研究工作极其活跃，其中一个主要方面是提高汽轮喷气式发动机 (gas turbine jet engine) 在接近最高压力时的临界工作效率。这项研究在空军战斗机高府冲角临界状态时的稳定性分析中特别重要，因为这时机体旋转引起的失速和高频振动与动力分岔现象直接相联。常规的设计是躲开这种危险，其代价是牺牲战机的战斗能力。分岔控制的思想是通过新的控制器来延迟分岔的发生，或者把不稳定的分岔变为稳定的分岔 (所谓“软化”分岔控制)，导致扩大战机的安全运行范围，以此来充分利用和提高战斗机的作战能力 [34-38]。当应用到宇航，分岔控制很有可能用来充分利用和发挥飞船喷气机的最大潜力，以实现少燃料远距离太空航行 [91-93]。

5. 结束语

正如前面谈到的那样，分岔控制的一个难点是要求所用的控制器不能改变分岔的存在性，而这一要求通常是通过首先满足不改变不动点的存在性而达到的。很多常规的控制器都不满足这个要求，因为它

们大都是按稳定性原则来设计的，而这一原则又大都依赖于被控系统的渐近性态。然而，分岔，不管是静态分岔还是动态分岔，大都不是随时间推演的渐近行为。更不必说，在这要求的基础上还有许多别的约束，诸如镇定原来不稳的分岔轨道，延迟原有分岔的出现，改变本来分岔点对应的系统参数值，改变先前分岔轨道的形状或类型，有目的地引进新的分岔，控制极限环（个数，大小，周期，或重数)，优化系统在分岔点附近的动力行为, 通过控制分岔来实现混沌控制, 等等，有时还会是它们的某种组合。

当然，如果控制器允许设计得很复杂，这些也许都不难。不过，控制论，特别是工程控制论，有一条基本原则就是控制器不能过于复杂，特别是不能比给定的系统还要复杂，否则就失去实用价值(不能想象一架飞机的控制器比飞机本身还要复杂庞大)。因此，综合起来考虑，分岔控制器的设计确实是很有挑战性的。不过，正因为这样，这个新研究领域才更有吸引力，更值得引起数学，物理，工程，化学，生物，特别是控制论的专门家们的注意。

分岔控制就如混沌控制一样, 蓬勃发展，方兴未艾。可以预言，分岔控制的理论和实践在今后相当长的一段时间内将会有重大的飞跃和突破，并与混沌控制一起, 成为控制论的一个富有特色的重要分支。

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Controlling Bifurcations in Dynamical Systems

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Abstract： Bifurcation control deals with modification of bifurcation characteristics of a parameterized nonlinear system by a designed control input. Typical bifurcation control objectives include delaying the onset of an inherent bifurcation, stabilizing a bifurcated solution or branch, changing the parameter value of an existing bifurcation point, modifying the shape or type of a bifurcation chain,introducing a new bifurcation at a preferable parameter value, monitoring the multiplicity, amplitude, and/or frequency of some limit cycles emerging from bifurcation, optimizing the system performance near a bifurcation point, or a combination of some of these objectives. This article offers an overview of this emerging, challenging, stimulating, and yet promising field of research,putting the main subject of bifurcation control into perspective.

Key word: nonlinear dynamic control; bifurcation; chaos; control

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