Supplementary Information for paper "Communicating with sentences: A multi-word naming game model"

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1 Multi-word naming game scaling with population size

The convergence process of the multi-word naming game (MWNG) model, with population size 1000 and 1200, is presented here. Totally 5 conventional English language patterns are employed. The convergence processes are shown from 4 aspects, including the *number of total words*, *number of different words*, *number of total patterns* and *success rate*. Table S1 shows the network settings. Figures S1 to S4 show the convergence features for the case with 1000 agents, and Figures S5 to S8 show the convergence features for the case with 1200 agents. It shows that, when the population size is set to 500, 1000, and 1200, respectively, the convergence process is not influenced by the scaling.

Table S1 Network settings in simulations. The random-graph (RG), small-world (SW) and scale-free (SF) networks in a total of 24 networks are employed for further simulation. The networks are randomly generated and the properties including average node degree, average path length and average clustering coefficient are averaged over 30 independent runs.

Notation	Network type	Number of nodes	Average node degree	Average path length	Average clustering coefficient
RG/0.03	Random-graph network with $P = 0.03$	1000	29.9230	2.3660	0.0300
		1200	35.9133	2.3010	0.0300
RG/0.05	Random-graph network with $P = 0.05$	1000	50.0878	2.0269	0.05020
		1200	60.0281	1.9969	0.0501
RG/0.1	Random-graph network with $P = 0.1$	1000	99.8832	1.9001	0.1000
		1200	119.9957	1.8999	0.1001
SW/50/0.1	Small-world network with $K = 20$ and $RP = 0.1$	1000	100	1.9958	0.5523
		1200	100	2.0647	0.5508
SW/50/0.2	Small-world network with $K = 20$ and $RP = 0.2$	1000	100	1.9145	0.4067
		1200	100	1.9484	0.4025
SW/50/0.3	Small-world network with $K = 20$ and $RP = 0.3$	1000	100	1.9027	0.2977
		1200	100	1.9249	0.2901
SW/60/0.1	Small-world network with $K = 20$ and $RP = 0.1$	1000	120	1.9135	0.5563
		1200	120	1.9637	0.5540
SW/60/0.2	Small-world network with $K = 20$ and $RP = 0.2$	1000	120	1.8820	0.4130
		1200	120	1.9066	0.4084
SW/60/0.3	Small-world network with $K = 20$ and $RP = 0.3$	1000	120	1.8801	0.3076
		1200	120	1.9008	0.2985
SF/25	Scale-free with 26 initial nodes and 25 new edges added at each step	1000	49.31780	2.0563	0.1237
		1200	49.42950	2.0985	0.1091
SF/50	Scale-free with 51 initial nodes and 50 new edges added at each step	1000	97.39220	1.9044	0.1955
		1200	97.82530	1.9225	0.1729
SF/75	Scale-free with 76 initial nodes and 75 new edges added at each step	1000	144.2216	1.8557	0.2548
		1200	145.1818	1.8790	0.2259

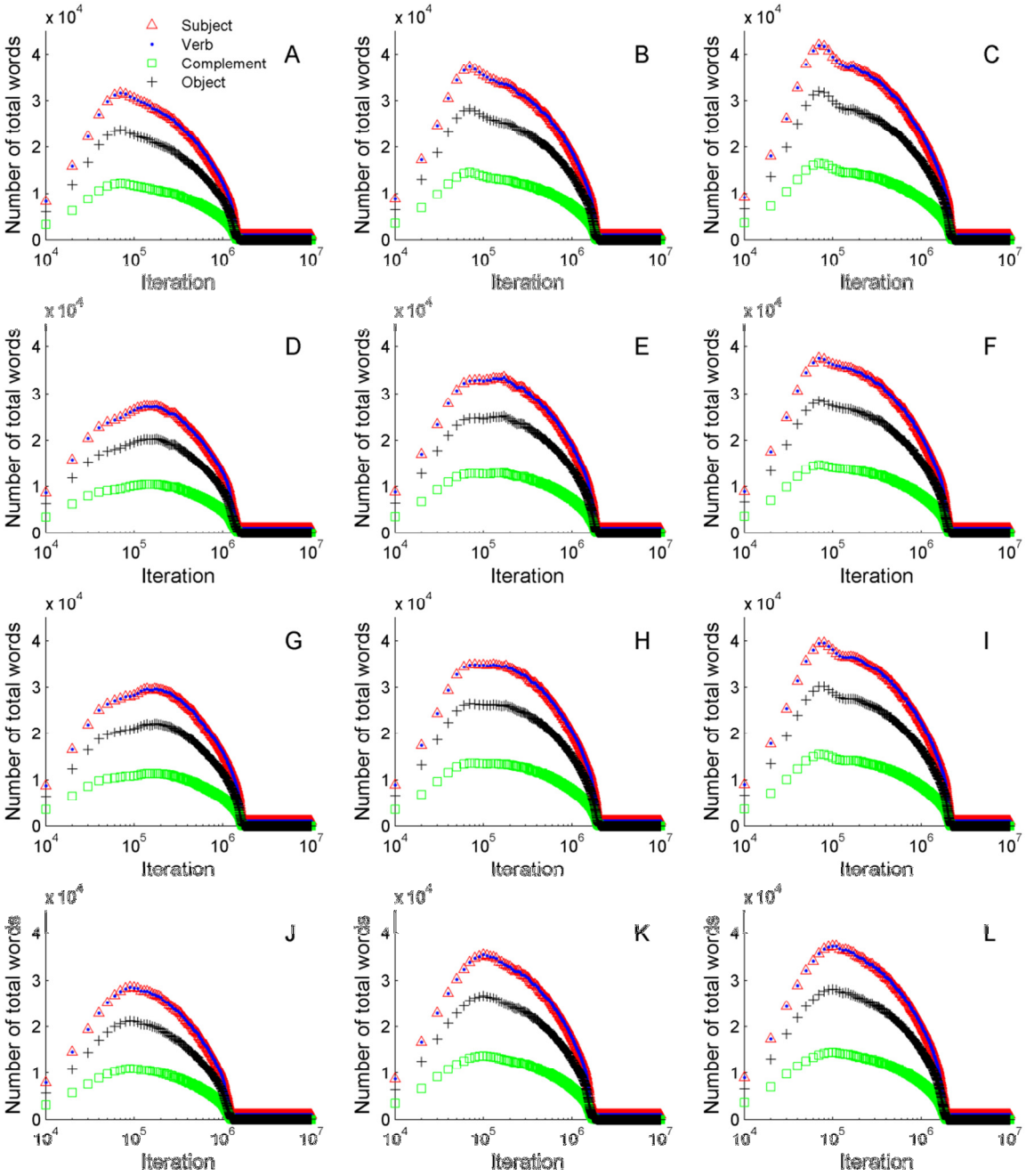

Figure S1 Convergence curves in terms of the number of total words vs. iterations: (A) RG/0.03; (B) RG/0.05; (C) RG/0.1; (D) SW/50/0.1; (E) SW/50/0.2; (F) SW/50/0.3; (G) SW/60/0.1; (H) SW/60/0.2; (I) SW/60/0.3; (J) SF/25; (K) SF/50; (L) SF/75. In each subfigure, the converging process is plotted as 4 curves, representing 4 categories, '*subject*', '*verb*', '*complement*' and '*object*'. Note that the numbers of complements and objects reach zero when the population converges, while the numbers of subjects and verbs reach the population size, 1000. The shapes and features of the convergence curves in terms of the number of total words are similar to those with population sizes 500 and 1200, respectively.

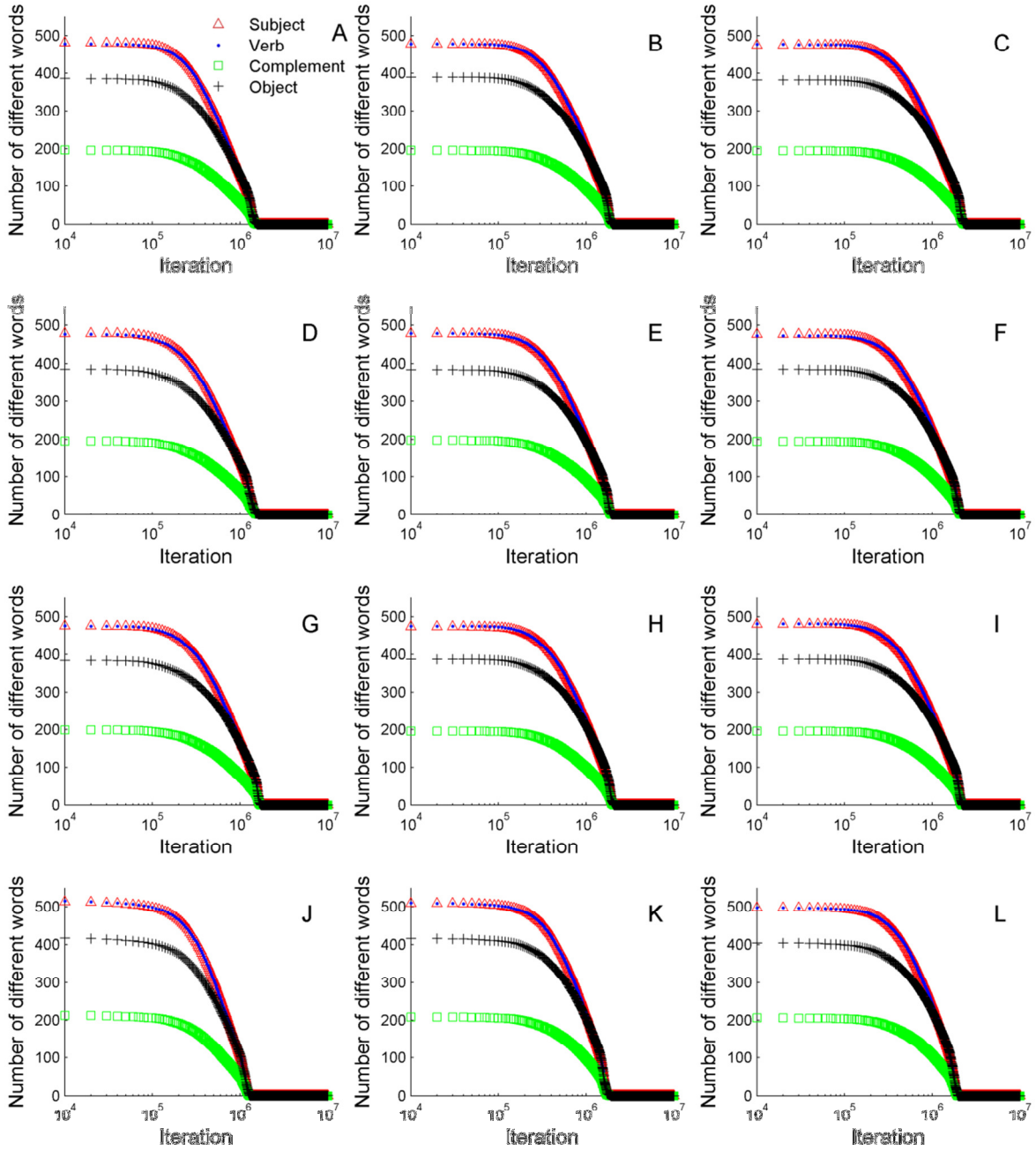

Figure S2 Convergence curves in terms of the number of different words vs. iterations: (A) RG/0.03; (B) RG/0.05; (C) RG/0.1; (D) SW/50/0.1; (E) SW/50/0.2; (F) SW/50/0.3; (G) SW/60/0.1; (H) SW/60/0.2; (I) SW/60/0.3; (J) SF/25; (K) SF/50; (L) SF/75. Differing from the curves of the number of total words, no matter horizontally or vertically, the shapes of the curves are nearly unchanged, but only slightly shifted. The population size is 1000.

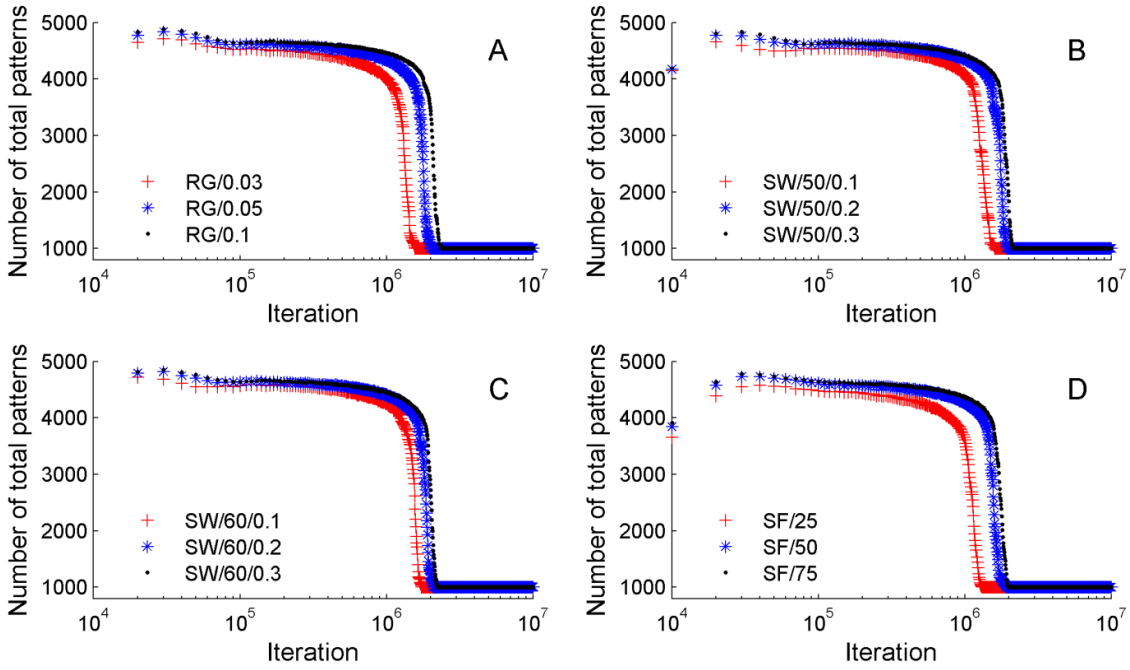

Figure S3 Convergence curves in terms of the number of total patterns vs. iterations: (A) Random-graph networks; (B) and (C) Small-world networks; (D) Scale-free networks. The shapes of curves are similar, but slightly shifted to the upper-right, when the (re-)connection probability (as well as the average node degree) increases. Totally 5 patterns are employed and the population size is 1000. The peaks of other curves are higher than 4500, but (slightly) lower than 5000, which means that there is one period that, on the average, the agents have learned more than 4 patterns and many of them even have learned all 5 patterns.

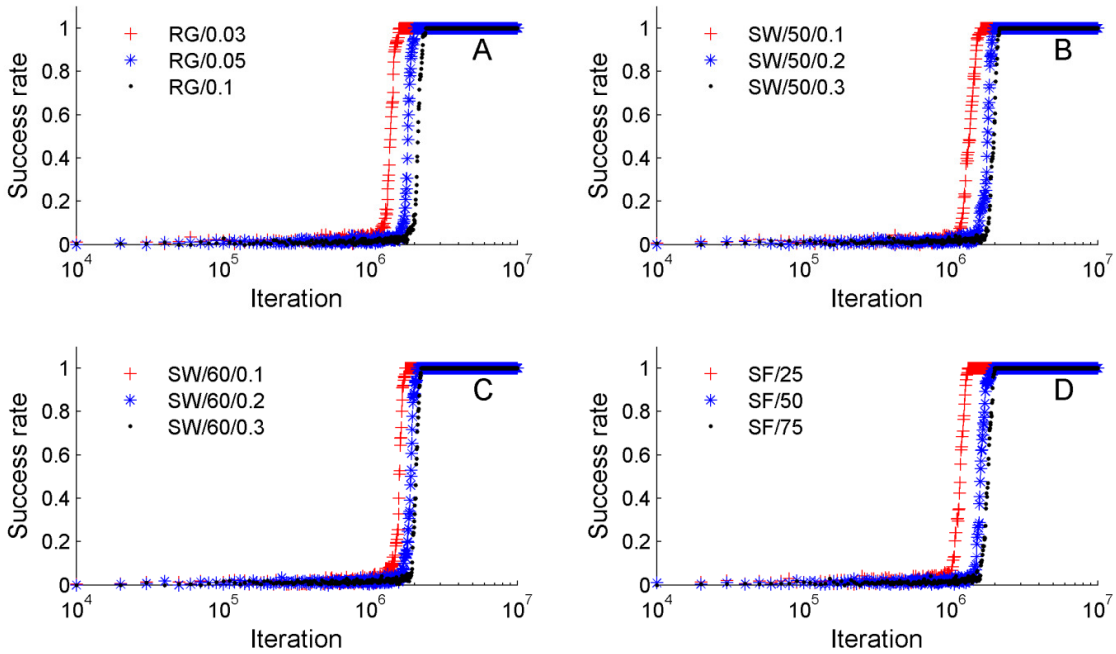

Figure S4 Curves of the success rate: (A) Random-graph networks; (B) and (C) Small-world networks; (D) Scale-free networks. The success rate curves of MWNG are simple as compared with the oscillatory success rate curves of small-world networks in atomic NG. Before the population converge takes place, the success rate stays below 0.1, then in the converging phase, the success rate increases dramatically, and finally reaches 1.0.

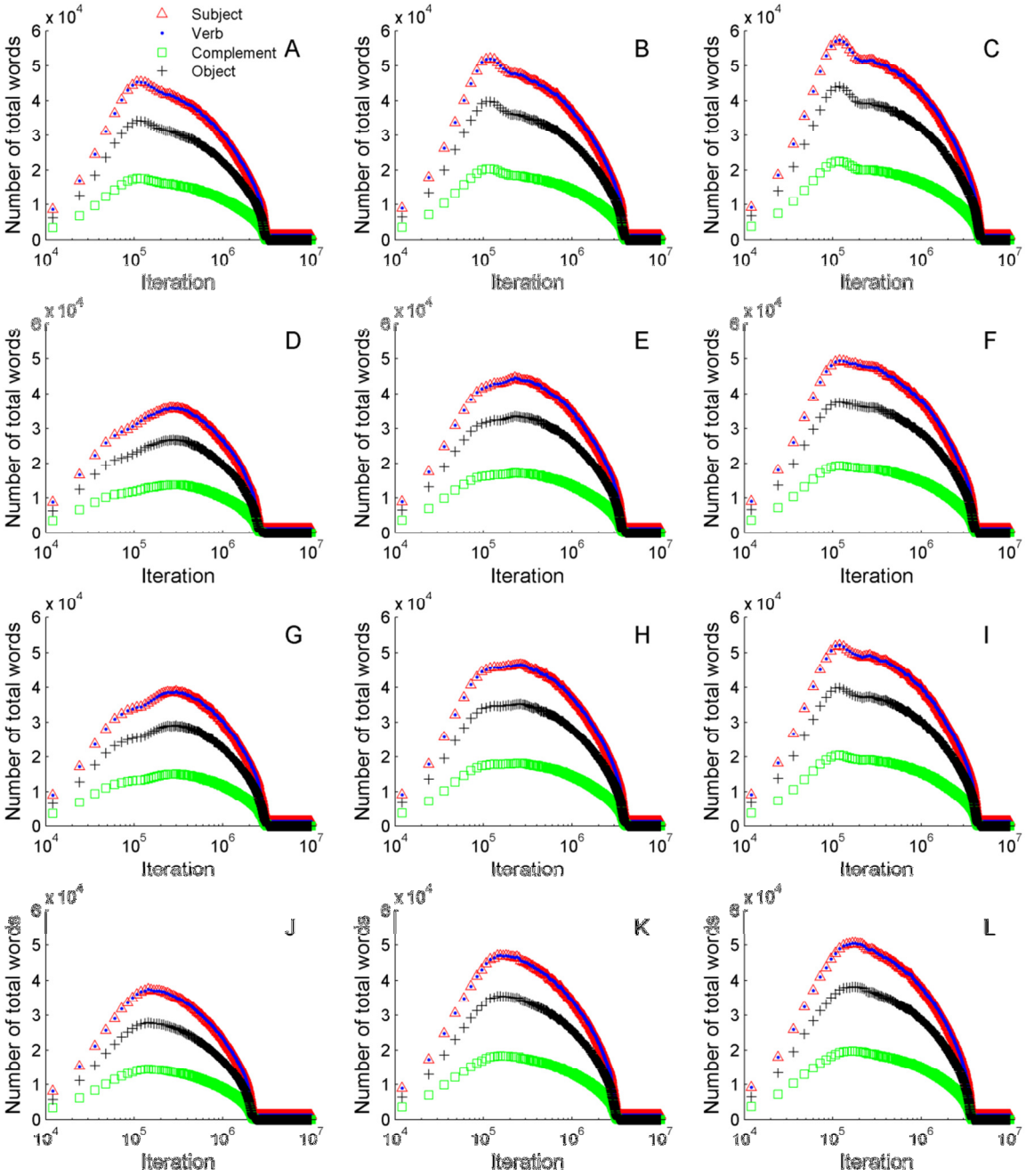

Figure S5 Convergence curves in terms of the number of total words vs. iterations: (A) RG/0.03; (B) RG/0.05; (C) RG/0.1; (D) SW/50/0.1; (E) SW/50/0.2; (F) SW/50/0.3; (G) SW/60/0.1; (H) SW/60/0.2; (I) SW/60/0.3; (J) SF/25; (K) SF/50; (L) SF/75. In each subfigure, the converging process is plotted as 4 curves, representing 4 categories, '*subject*', '*verb*', '*complement*' and '*object*'. Note that the numbers of complements and objects reach zero when the population converges, while the numbers of subjects and verbs reach the population size, 1200. The shapes and features of the convergence curves in terms of the number of total words are similar to those with population size 500 and 1000, respectively.

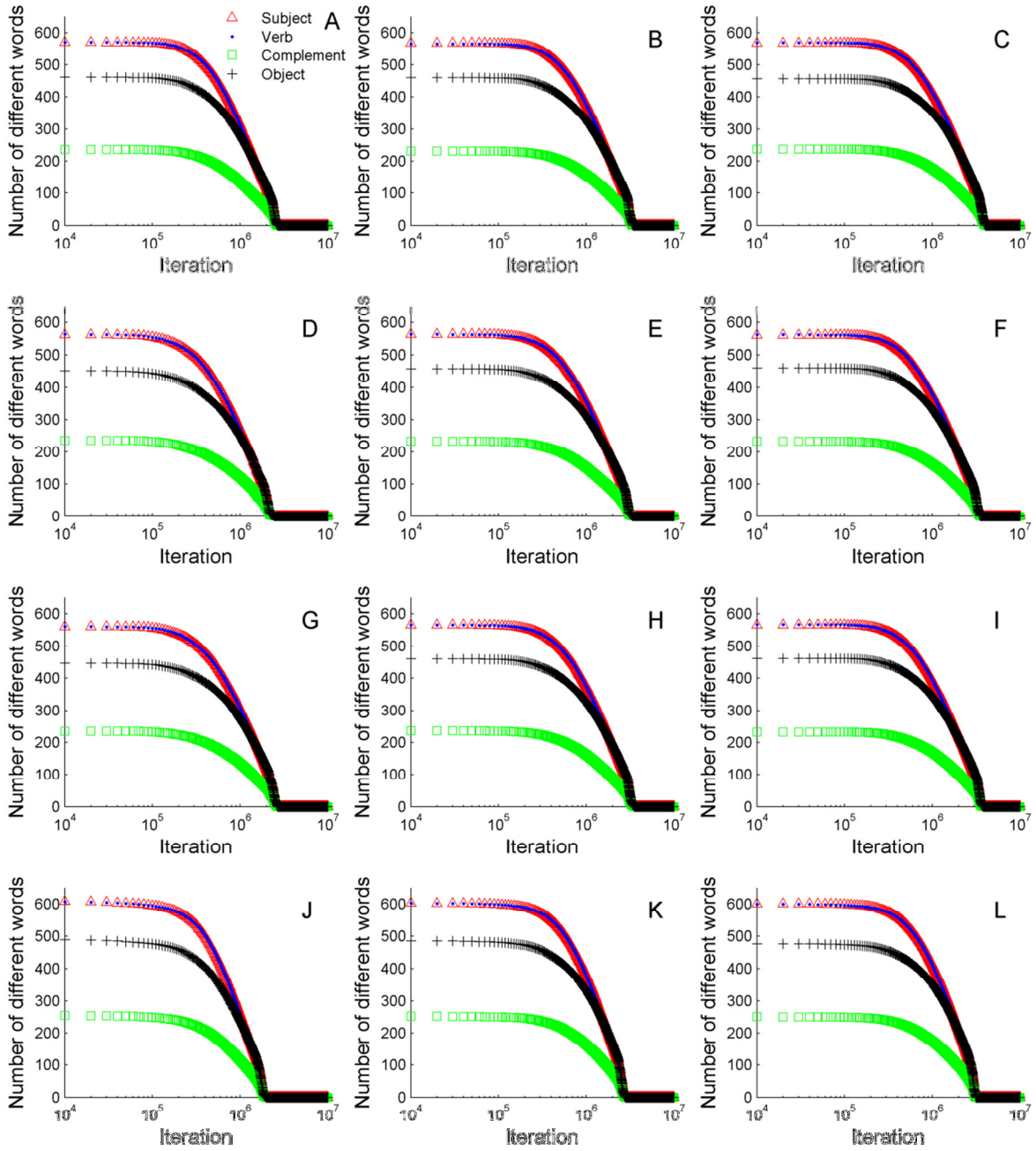

Figure S6 Convergence curves in terms of the number of different words vs. iterations: (A) RG/0.03; (B) RG/0.05; (C) RG/0.1; (D) SW/50/0.1; (E) SW/50/0.2; (F) SW/50/0.3; (G) SW/60/0.1; (H) SW/60/0.2; (I) SW/60/0.3; (J) SF/25; (K) SF/50; (L) SF/75. Differing from the curves of the number of total words, no matter horizontally or vertically, the shapes of the curves are nearly unchanged, but slightly shifted. The population size is 1200.

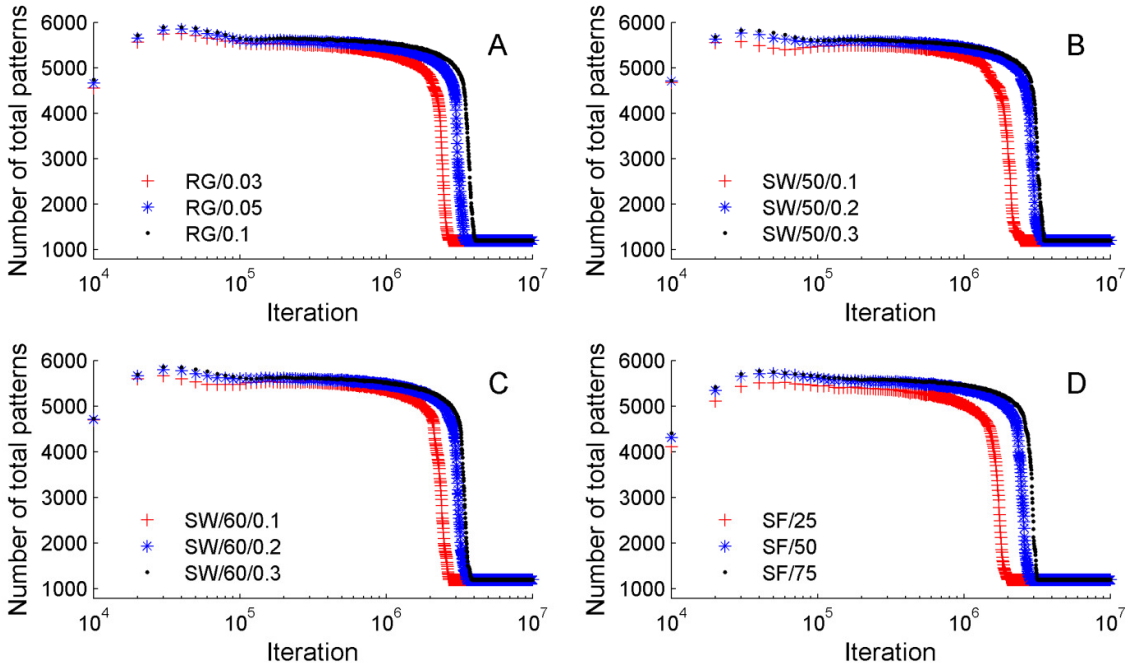

Figure S7 Convergence curves in terms of the number of total patterns vs. iterations: (A) Random-graph networks; (B) and (C) Small-world networks; (D) Scale-free networks. The shapes of curves are similar, but slightly shifted to the upper-right, when the (re-)connection probability (as well as the average node degree) increases. Totally 5 patterns are employed and the population size is 1200. The peaks of other curves are higher than 5500, but (slightly) lower than 6000, which means that there is one period that, on the average, the agents have learned more than 4 patterns and many of them even have learned 5 patterns.

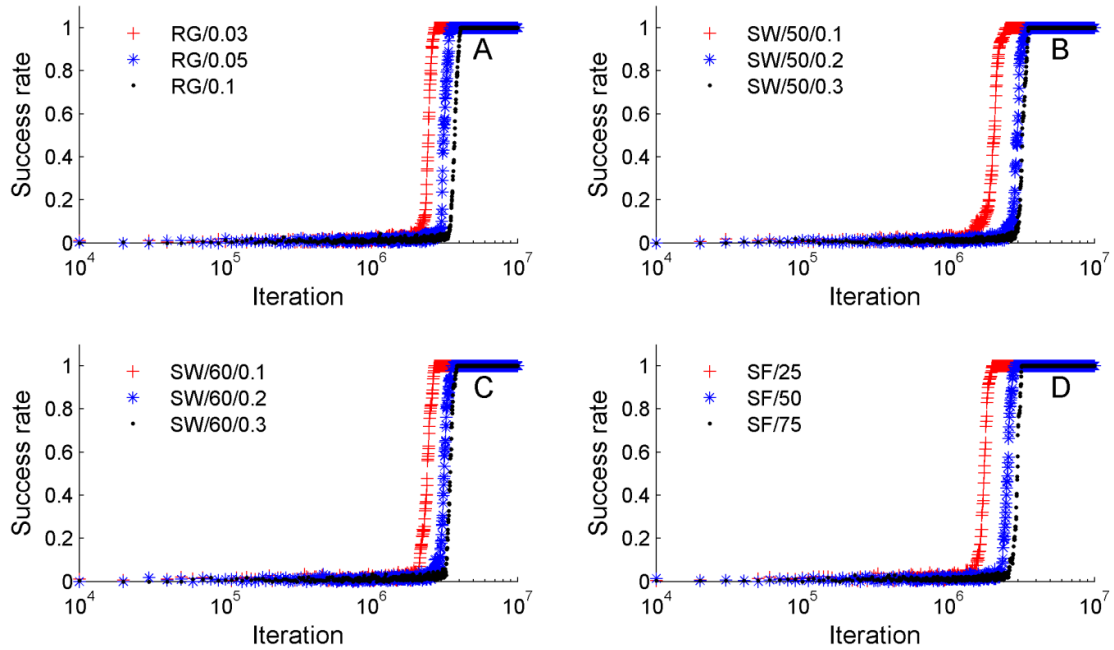

Figure S8 Curves of the success rate: (A) Random-graph networks; (B) and (C) Small-world networks; (D) Scale-free networks. The success rate curves of MWNG are simple as compared with the oscillatory success rate curves of small-world networks in atomic NG. Before the population converge takes place, the success rate stays below 0.1, then in the converging phase, the success rate increases dramatically, and finally reaches 1.0.