Pinning Control and Robust Controllability of Complex Networks: A Machine Learning Approach

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- Network Controllability
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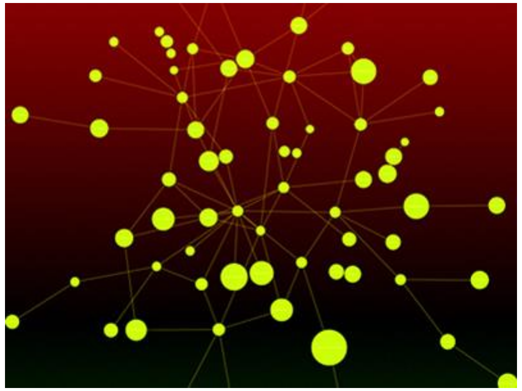
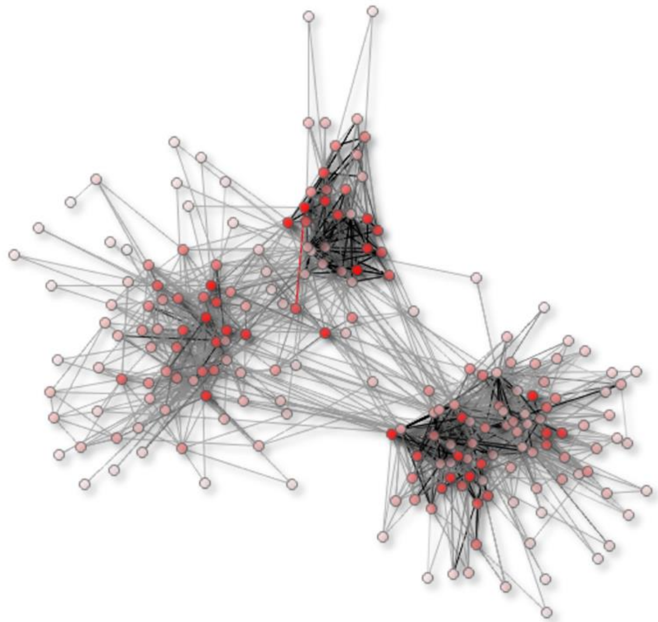
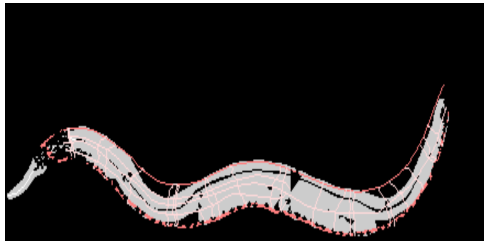
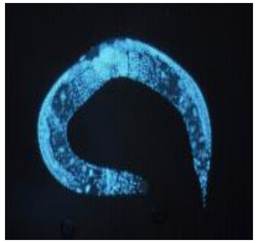


Motivational Examples



Example 1:





"The worm Caenorhabditis elegans has 297 nerve cells.
The neurons switch one another on or off, and, making
2345 connections among themselves. They form a network
that stretches through the nematode's millimeter-long body."

"How many neurons would you have to commandeer to control the network with complete precision?"

The answer is, on average: 49

-- Adrian Cho, ***Science***, 13 May **2011**, vol. 332, p 777

Here, control = stimuli

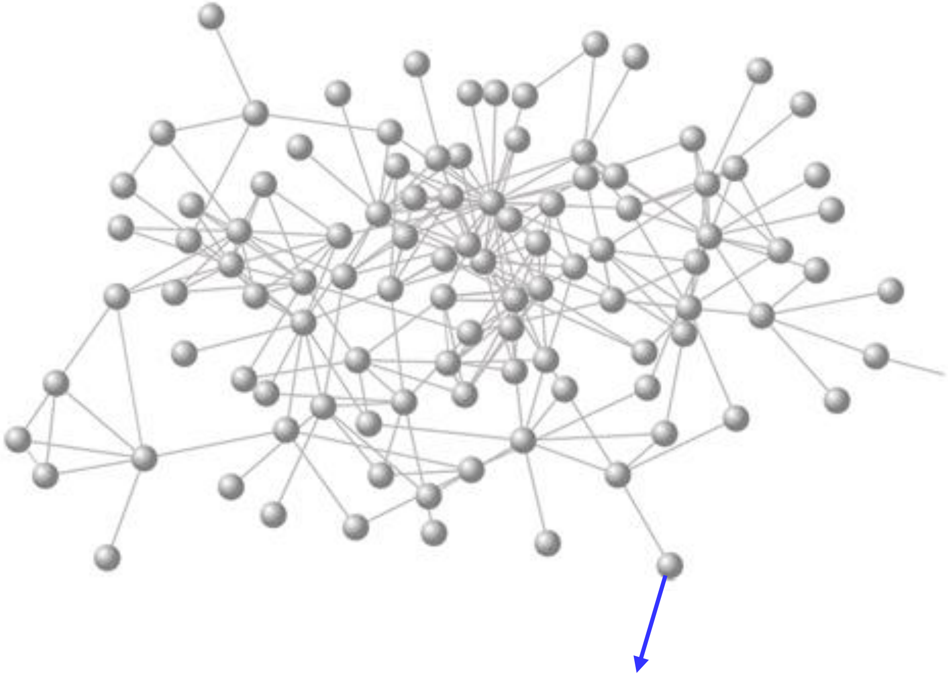
Example 2:

" … very few individuals (approximately **5**%) within honeybee swarms can guide the group to a new nest site."

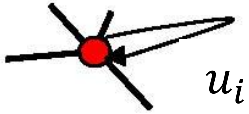
I.D. Couzin et al., ***Nature***, 3 Feb 2005, vol. 433, p 513

These **5%** of bees can be considered as "**controlling**" or "**controlled**" **agents**

Leader-Followers network



$$\frac{dx_i}{dt} = f(x_i), \qquad x_i \in R^n$$

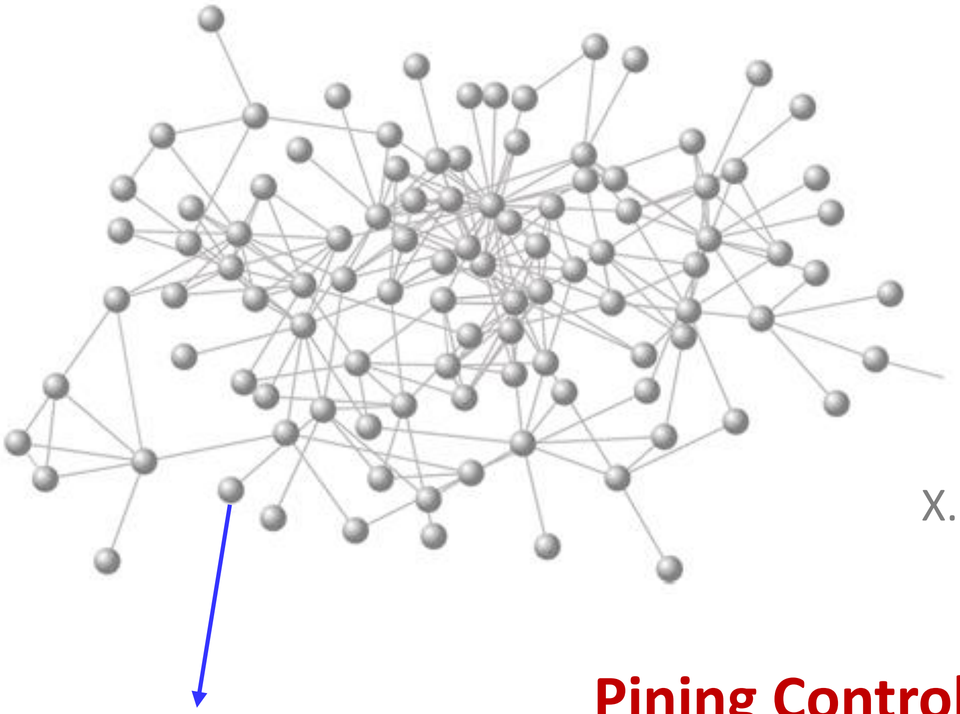


$$u_i = -\Gamma x_i$$

Mathematically:

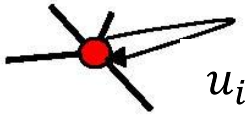
- Given a network of dynamical systems (e.g., ODEs)
- Given a specific control objective (e.g., synchronization)
- Assume: a certain class of controllers (e.g., local state-feedback controllers) are chosen to use

Control Problem



X. F. Wang and G. Chen, Physica A, 2002

$$\frac{dx_i}{dt} = f(x_i), \qquad x_i \in R^n$$



$$u_i = -\Gamma\, x_i$$

Pining Control:

- How many controllers to use?
- Where to “pin” them?

Pinning Control: Our Research Progress

Wang XF, Chen G, Pinning control of scale-free dynamical networks, Physica A, 310: 521-531, 2002.

Li X, Wang XF, Chen G, Pinning a complex dynamical network to its equilibrium, IEEE Trans. Circ. Syst. –I, 51: 2074-2087, 2004.

Sorrentino F, di Bernardo M, Garofalo F, Chen G, Controllability of complex networks via pinning, Phys. Rev. E, 75: 046103, 2007.

… …

Yu WW, Chen G, Lu JH, Kurths J, Synchronization via pinning control on general complex networks, SIAM J. Contr. Optim., 51: 1395-1416, 2013.

Chen G, Pinning control and synchronization on complex dynamical networks, Int. J. Contr., Auto. Syst., 12: 221-230, 2014.

… …

Liu H, Xu X, Lu J-A, Chen, G, Zeng Z G, “Optimizing pinning control of complex dynamical networks based on spectral properties of grounded Laplacian matrices,” IEEE Trans. Syst. Man Cybern., 51: 786-796, 2021.

Sanchez EN, Vega CJ, Suarez OJ, Chen, G. Nonlinear Pinning Control of Complex Dynamical Networks, Springer 2021.

Network Model

Linearly coupled network:

$$\dot{x}_i = f(x_i) + c\sum_{j=1}^{N}\beta_{ij}Hx_j \qquad x_i \in R^n \quad i = 1,2,\ldots,N$$

- General assumption: $f(.)$ is Lipschitz. Here, it is linear (or linearized):

$$\dot{x}_i = Ax_i + c\sum_{j=1}^{N}\beta_{ij}Hx_j \qquad x_i \in R^n \quad i = 1,2,\ldots,N$$

- Coupling strength $c > 0$ and H – input coupling matrix

- Adjacency matrix: $\left[\beta_{ij}\right]_{N\times N}$

If node i points to node j $(j \neq i)$, then $\beta_{ij} = 1$; otherwise $\beta_{ij} = 0$; and $\beta_{ii} = 0$

For undirected networks, $\left[\beta_{ij}\right]_{N\times N}$ is symmetrical; for directed networks, may not be so

How many? Where to pin?

$$\dot{x}_i = Ax_i + c\sum_{j=1}^{N}\beta_{ij}Hx_j \leftarrow + Bu_i \qquad (e.g., \quad u_i = -\Gamma x_i)$$

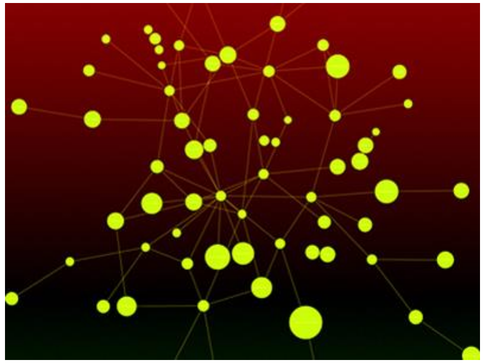
$\Longrightarrow$

$$\dot{x}_i = Ax_i + c\sum_{j=1}^{N}\beta_{ij}Hx_j + \delta_i Bu_i$$

$$\delta_i = \begin{cases} 1 & if \;\; to-control \\ 0 & if \;\; not-control \end{cases}$$

Q: How many $\delta_i = 1$? Which i ? $\quad (i = 1,2,\ldots,N)$ → **Pinning Control**

Controllability Theory



In retrospect, …

J.S.I.A.M. Control
Ser. A, Vol. 1, No. 2
Printed in U.S.A., 1963

(1930-2016)

MATHEMATICAL DESCRIPTION OF LINEAR DYNAMICAL SYSTEMS*

R. E. KALMAN†

Abstract. There are two different ways of describing dynamical systems: (i) by means of state variables and (ii) by input/output relations. The first method may be regarded as an axiomatization of Newton's laws of mechanics and is taken to be the basic definition of a system.

It is then shown (in the linear case) that the input/output relations determine only one part of a system, that which is completely observable and completely controllable. Using the theory of controllability and observability, methods are given for calculating irreducible realizations of a given impulse-response matrix. In particular, an explicit procedure is given to determine the minimal number of state variables necessary to realize a given transfer-function matrix. Difficulties arising from the use of reducible realizations are discussed briefly.

System Controllability

Linear Time-Invariant (LTI) system

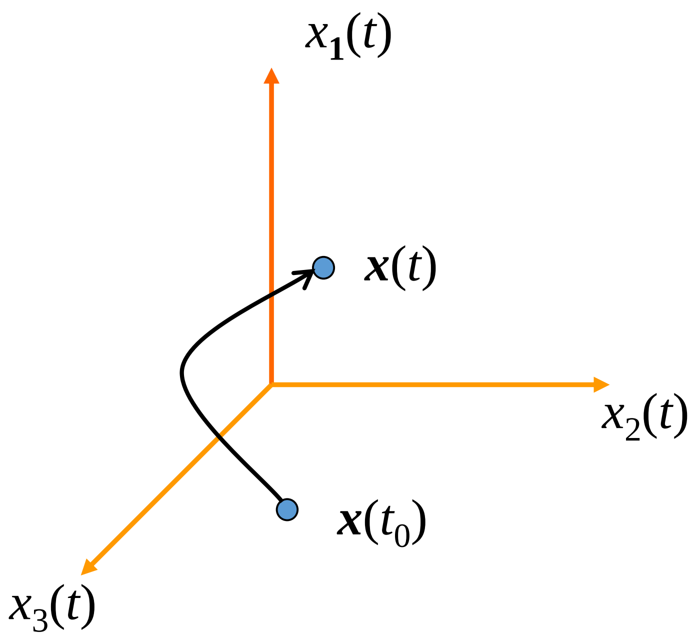
$$\dot{x}(t) = Ax(t) + Bu(t)$$

$x \in R^n$: state vector

$u \in R^p$: control input

$A \in R^{n \times n}$: system matrix

$B \in R^{n \times p}$: control matrix



Controllable: The system orbit can be driven by an input from any initial state to any target state in finite time.

C. K. Chui and G. Chen, **Linear Systems and Optimal Control**, Springer, 1989

State Controllability Theorems

(i) Kalman Rank Criterion $\dot{x}(t) = Ax(t) + Bu(t)$

The controllability matrix Q **has full row rank:**

$$Q = [B \;\; AB \;\; \cdots \;\; A^{n-1}B]$$

(ii) Popov-Belevitch-Hautus (PBH) Test

The following hold:

$$v^T A = \lambda v^T, \quad v^T B \neq 0$$

λ : eigenvalue of A

v : nonzero left eigenvactor with λ

System Observability

Linear Time-Invariant (LTI) system

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t)$$

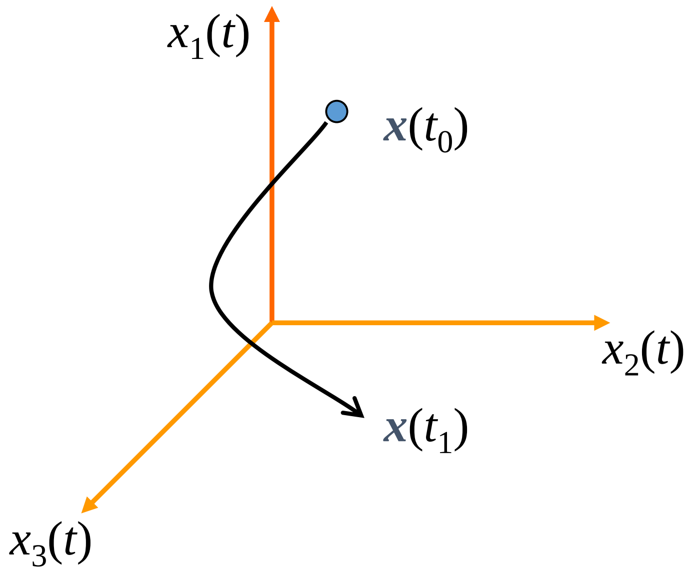
$x \in R^n$: state vector

$u \in R^p$: control input

$A \in R^{n \times n}$: system matrix

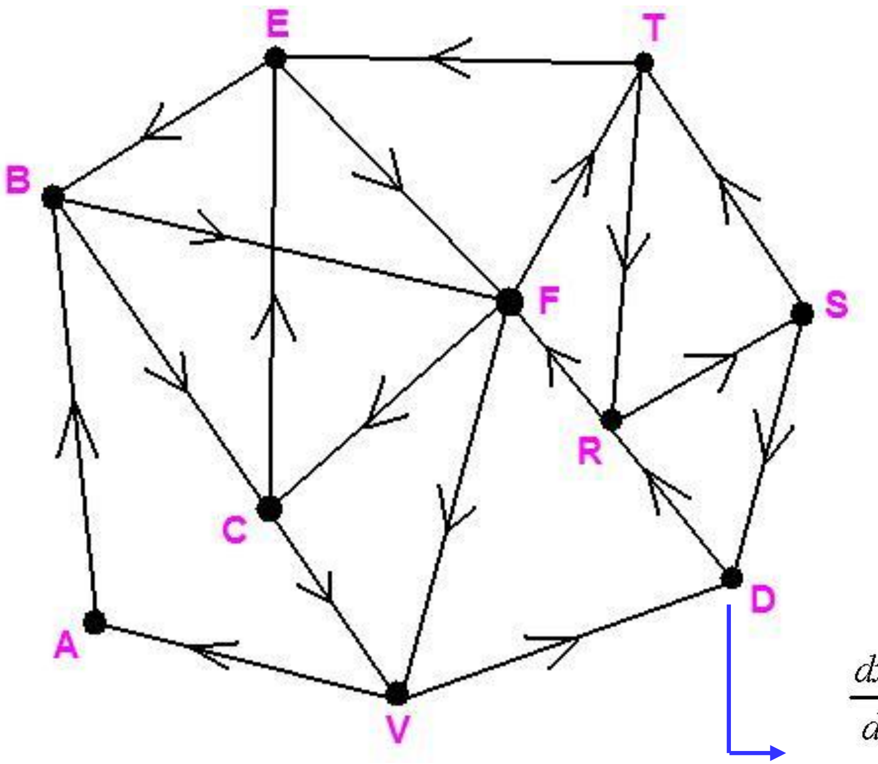
$B \in R^{n \times p}$: control matrix

$$x(t) = x(t_0)e^{(t-t_0)A} + \int_{t_0}^{t} e^{(t-\tau)A} Bu(\tau)d\tau$$

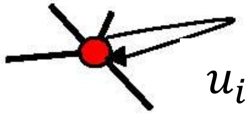


Observability: Input-output pair $(u(t), y(t))$ on $[t_1, t_2]$ uniquely determines the initial state $x(t_0)$

What About Directed Networks?



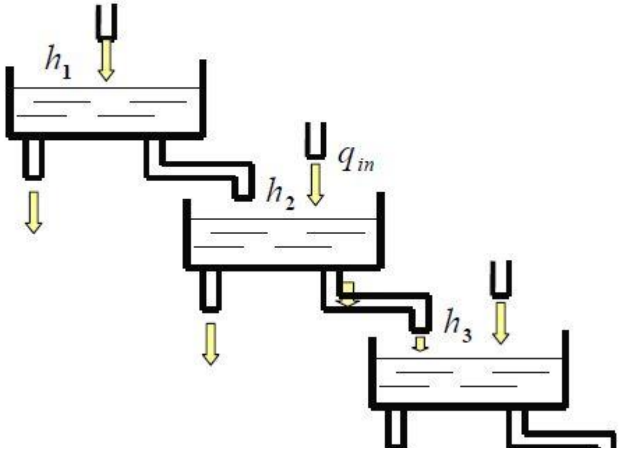
$$\frac{dx_i}{dt} = f(x_i), \qquad x_i \in R^n$$



$$u_i = -\Gamma \, x_i$$

In retrospect: large-scale systems theory

Structural Analysis of Dynamical Systems



A directed network

Q:

Is this kind of structure controllable?

Structural Controllability

Corresponding linearized system has the following general form:

$$\begin{aligned}\dot{x}_1 &= a_{11}x_1 \\ \dot{x}_2 &= a_{21}x_1 + a_{22}x_2 \qquad + bu, \\ \dot{x}_3 &= \qquad a_{32}x_2 + a_{33}x_3\end{aligned}$$

$$\mathbf{A} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ 0 & a_{32} & a_{33} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}$$

$$\textbf{rank}\left[\mathbf{B}, \mathbf{AB}, \mathbf{A}^2\mathbf{B}\right] = \begin{bmatrix} 0 & 0 & 0 \\ b & a_{22}b & a_{22}^2 b \\ 0 & a_{32}b & a_{32}(a_{22}+a_{33})b \end{bmatrix} \leq 2$$

→ Uncontrollable

Structural Controllability

In the controllability matrix: $Q = [B \;\; AB \;\; \cdots \;\; A^{n-1}B]$

All 0 are fixed

There is a realization of independent nonzero parameters such that Q has full row rank

Example 1:

$$Q = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$$

Realization: All admissible parameters

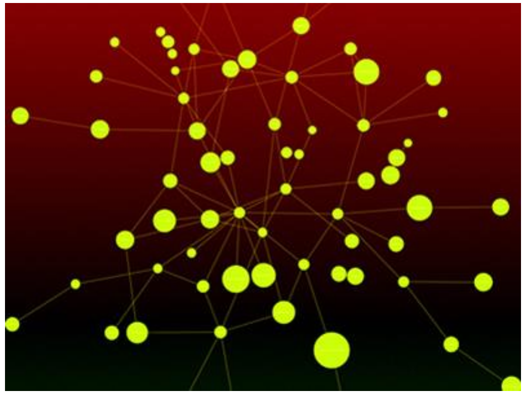
$$a \neq 0, \;\; d \neq 0$$

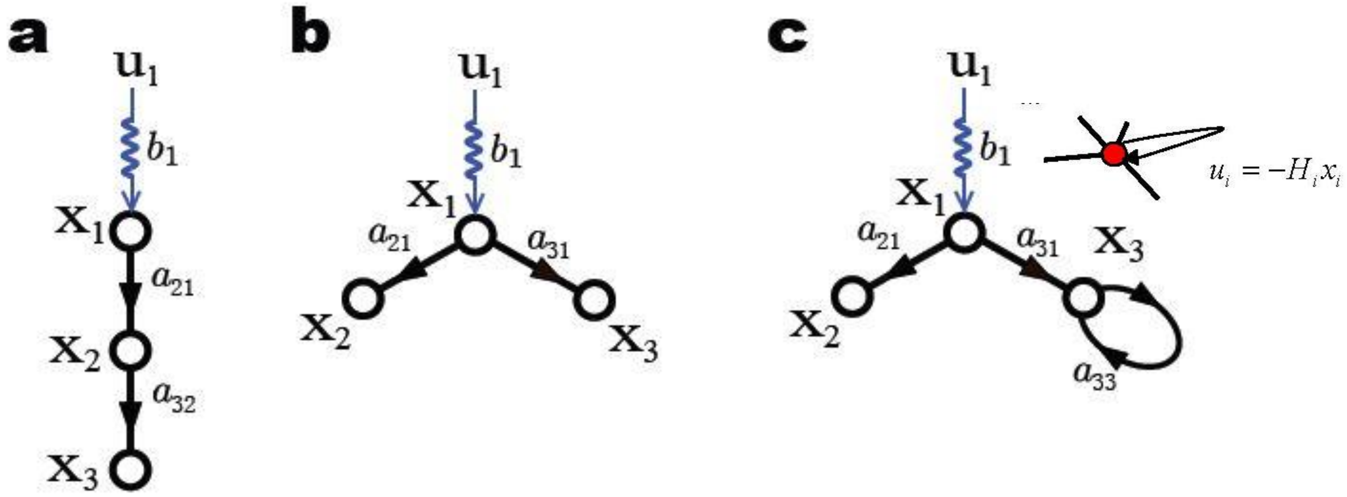
Example 2: Frobinius Canonical Form

$$Q = \begin{bmatrix} -a_1 & -a_2 & -a_3 & \cdots & -a_{n-1} & -a_n \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

Structural Controllability

A network of single-input/single-output (SISO) node systems





$$\mathbf{C} = [\mathbf{B}, \mathbf{A} \cdot \mathbf{B}, \mathbf{A}^2 \cdot \mathbf{B}]$$

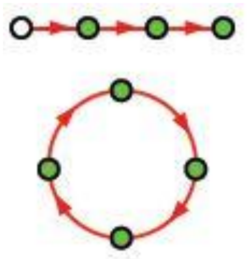
$$Q = b_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & a_{21} & 0 \\ 0 & 0 & a_{32}a_{21} \end{bmatrix}, \quad b_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & a_{21} & 0 \\ 0 & a_{31} & 0 \end{bmatrix}, \quad b_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & a_{21} & 0 \\ 0 & a_{31} & a_{33}a_{31} \end{bmatrix},$$

rank C $= 3 = n$	**rank C** $= 2 < n = 3$	**rank C** $= 3 = n$
controllable	**uncontrollable**	**controllable**

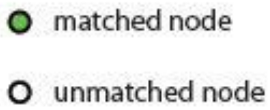
Y.Y. Liu, J.J. Slotine, and A.L. Barabási, *Nature* (2011)

Matching in Directed Networks

- **Matching**: a set of directed edges without common heads and tails
- **Unmatched node**: the tail node of a matching edge

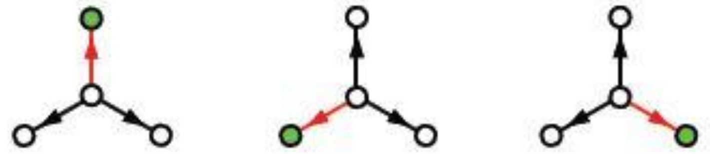


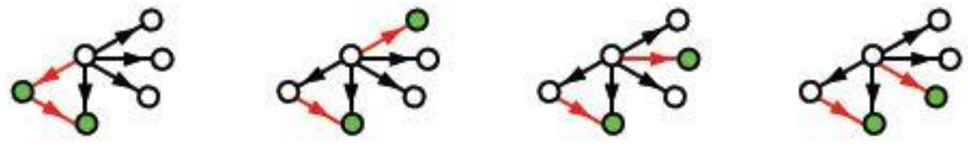
← Perfect matching



Maximum matching:
Cannot be extended

Perfect matching:
All nodes are matched nodes





← Maximum but not perfect matching

Solution to Pinning Control:

Minimum Inputs Theorem

Q: How many?

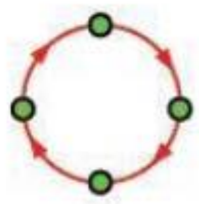
A: The minimum number of inputs N_D needed is:

Case 1: If there is a perfect matching, then

$$N_D = 1$$

Case 2: If there is no perfect matching, then

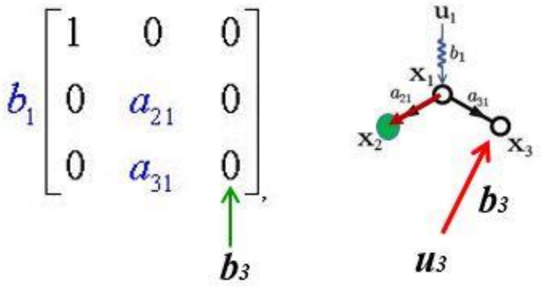
N_D = number of unmatched nodes



Q: Where to put them?

A: Case 1: Anywhere

Case 2: At unmatched nodes



This completely answer the pinning control question for SISO networks

Y. Y. Liu, J. J. Slotine, and A. L. Barabási, *Nature* (2011)

Characterization of General Topology with SISO Nodes

$$\dot{x}_i = Ax_i + \sum_{j=1}^{N} \beta_{ij} HCx_j + \delta_i Bu_i, \quad i = 1, 2, \cdots, N \qquad x_i \in R^n \quad y_i \in R^m \quad u_i \in R^p$$

$$L = [\beta_{ij}] \in R^{N \times N} \quad \Delta = diag(\delta_1, \cdots, \delta_N)$$

A network with SISO nodes is **controllable if and only if**

(A, H) is controllable

(A, C) is observable

For any eigenvalue s of A and $\alpha = Re(s)$, $\alpha L \neq 0$ for $\alpha \neq 0$

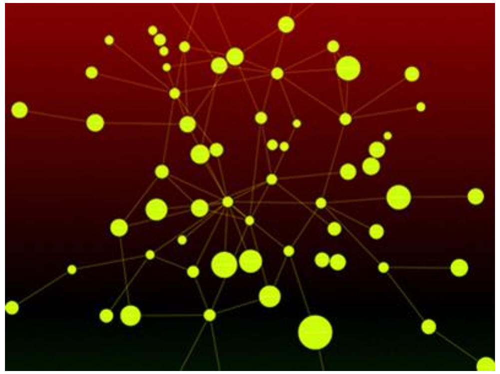
For any eigenvalue s of A, $rank(I - L\Gamma_1, \ \Delta\Gamma_2) = N$,

with $\Gamma_1 = C[sI - A]^{-1}H, \Gamma_2 = C[sI - A]^{-1}B$

L. Wang, X.F. Wang and G. Chen, Royal Phil Trans A (2016)

State Controllability

A network of multi-input/multi-output (MIMO) node systems, where the node systems are of higher-dimensional



A Network of Multi-Input/Multi-Output LTI Systems

Node system $\dot{x}_i = Ax_i + Bu_i \qquad y_i = Cx_i \qquad x_i \in R^n \quad y_i \in R^m \quad u_i \in R^p$

Networked system $\dot{x}_i = Ax_i + \sum_{j=1}^{N} \beta_{ij} H y_j, \quad y_i = Cx_i, \qquad i = 1, 2, \cdots, N$

Networked system with external control $\dot{x}_i = Ax_i + \sum_{j=1}^{N} \beta_{ij} H C x_j + \delta_i B u_i, \quad i = 1, 2, \cdots, N$

$\delta_i = 1$: **with** external control $\qquad \delta_i = 0$: **without** external control

Some notations

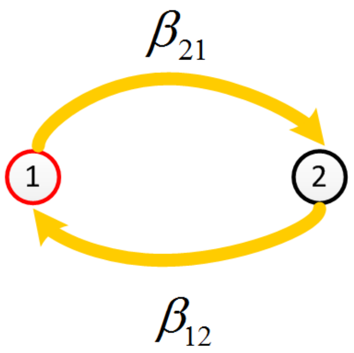
Node system (A,B,C) $\qquad$ Network structure $L = [\beta_{ij}] \in R^{N \times N}$

Coupling matrix H $\qquad$ External control inputs $\Delta = diag(\delta_1, \cdots, \delta_N)$

L. Wang, X.F. Wang, G. Chen and W.K.S. Tang, *Automatica* (2016)

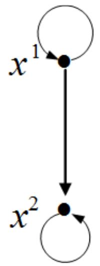
Counter-intuitive example 1

Network structure



$$L = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Node system



$$H = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

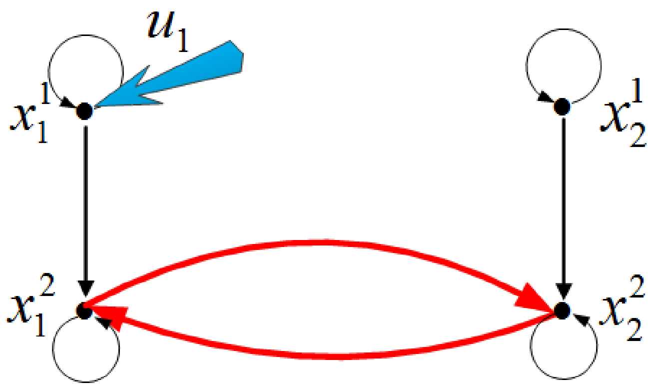
$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

(A,B) is controllable

(A,C) is observable

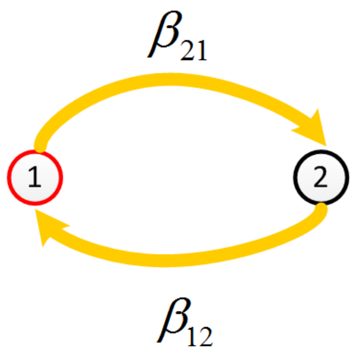
Networked MIMO system



state uncontrollable

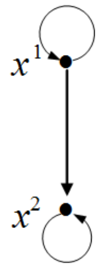
Counter-intuitive example 2

Network structure



$$L = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Node system



$$H = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

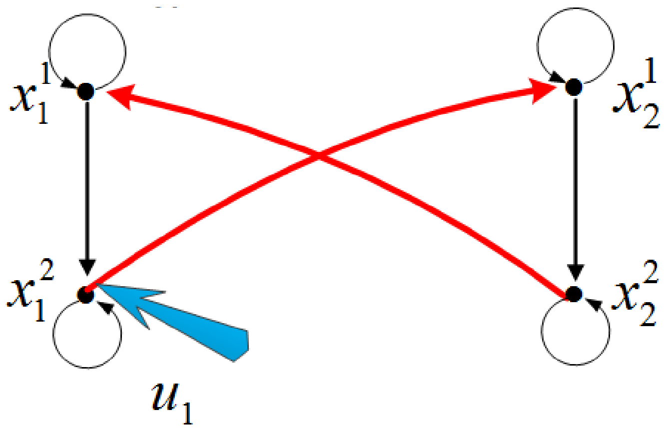
$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

(A,B) is uncontrollable

(A,C) is observable

Networked MIMO system



state controllable

A Network of Multi-Input/Multi-Output LTI Systems

A necessary and sufficient condition

$$\dot{x}_i = Ax_i + \sum_{j=1}^{N} \beta_{ij} HCx_j + \sum_{k=1}^{s} \delta_{ik} Bu_k, \qquad x_i \in R^n, \quad i = 1, \cdots N$$

$$u_k \in R^p, \quad k = 1, \cdots s$$

$$y_l = \sum_{j=1}^{N} m_{lj} Dx_j \qquad y_l \in R^q, \quad l = 1, \cdots r$$

$$L = [\beta_{ij}] \in R^{N \times N} \qquad \Delta = [\delta_{ij}] \in R^{N \times s}$$

State Controllable — **If and only if** — **Matrix equations**

$\Delta^T XB = 0, L^T XHC = X(\lambda \mathrm{I} - \mathrm{A})$ $\forall \lambda \in$ ¢

have a unique solution $X = 0$

L. Wang, X.F. Wang, G. Chen and W.K.S. Tang, *Automatica* (2016)

Pinning Control of MIMO Networks

State Controllable	If and only if ⟷	Matrix equations $\Delta^T XB = 0, L^T XHC = X(\lambda I - A)\ \forall \lambda \in ¢$ have a unique solution $X = 0$

Solution to Pinning Control: How many? Where to pin?

→ Select $\Delta = diag[\delta_i]$ such that the above algebraic matrix equations has a unique zero solution X

→ How many $\delta_i = 1$ and which $\delta_i = 1$

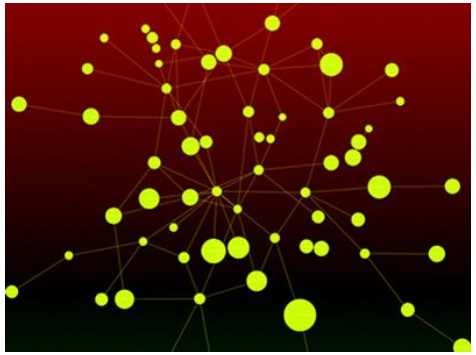
This completely answer the pinning control question for MIMO networks

Robustness of Network Controllability

Robustness of Controllability

Against Destructive Attacks

(Node-Removals / Edge-Removals)



Measure for Controllability Robustness

Let N_D be the minimum number of external control input needed to maintain the network controllability

Define

Controllability index:

$$n_D = N_D/N$$

Controllability Robustness:

The smaller the value of n_D, the better the robustness against (node-removal or edge-removal) attacks

Complex Network Models

- Random-Graph (RG) Network
- Scale-Free (SF) Network
- Multiplex Congruence Network (MCN)
- *q*-Snapback Network (QSN)
- Random Triangle Network (RTN)
- Random Rectangle Network (RRN)

Comparison of Controllability Robustness

- Attack Methods
- Simulation Results
- Comparisons

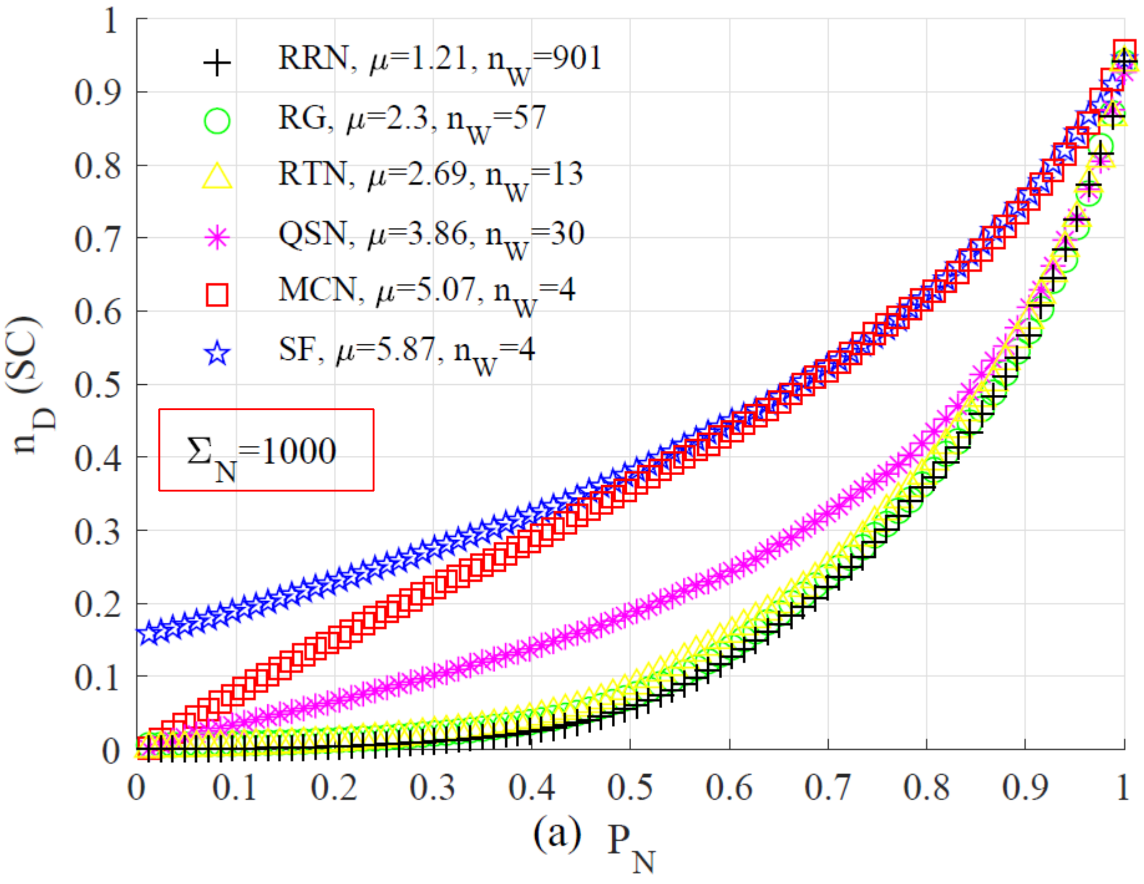
Y. Lou, L. Wang, G. Chen, “Enhancing controllability robustness of q-snapback networks through re-directing edges,” *Research* (2019)

Attack Methods

		Node-removal	Edge-removal
Targeted	Betweenness	Remove the node with the largest betweenness	Remove the edge with the largest betweenness
	Degree	Remove the node with the largest out-degree	Remove the edge with the largest edge degree
Random		Remove a node randomly	Remove an edge randomly

Edge degree for an edge A_{ij} is $\sqrt{k_i \times k_j}$, where k_i and k_j are the out-degrees of nodes i and j, respectively.

Simulation Results (Comparison)



Average over 100 trials

Random Node-Removal

RRN outperforms the other networks.

RRN, RG, and RTN performs similarly.

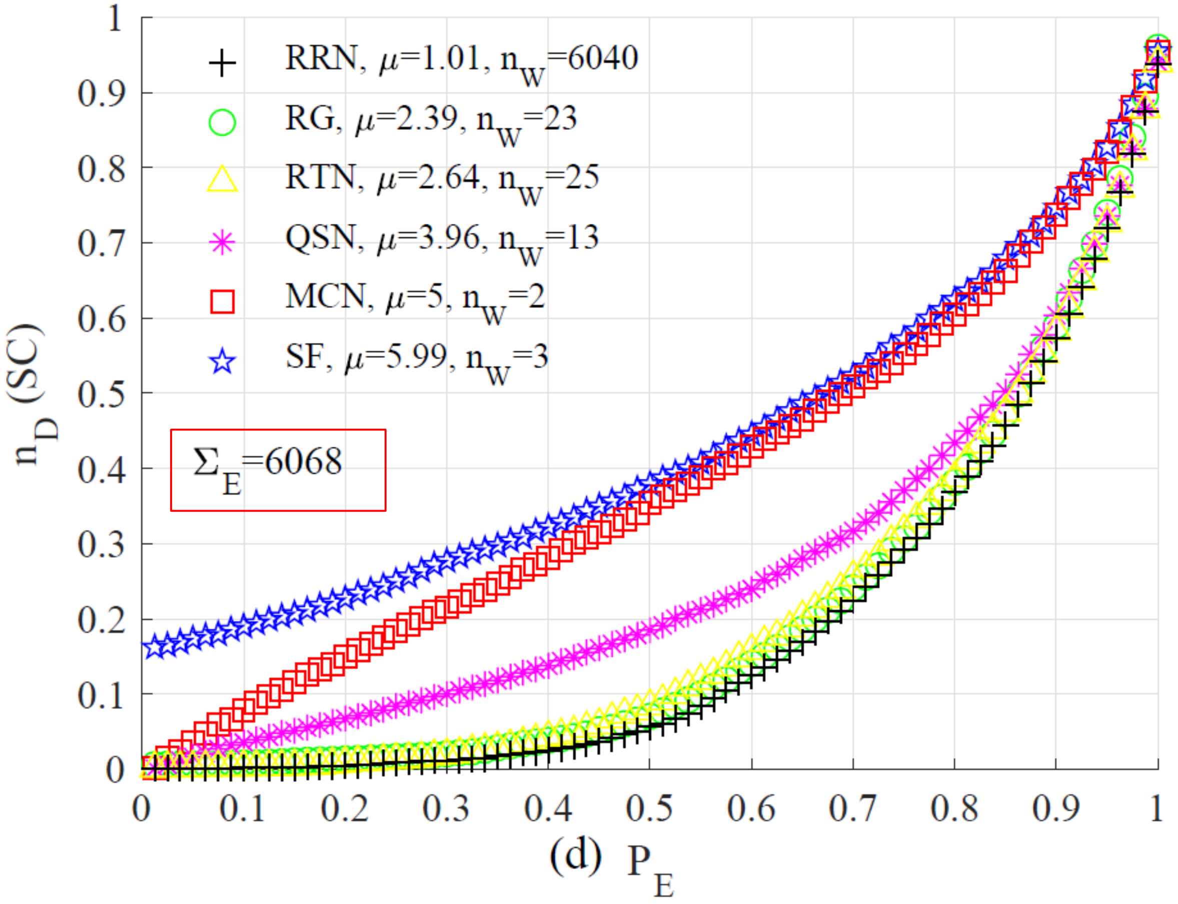
SF performs the worst.

Observation:

RRN, RTN have many loops

RG is homogeneous

Simulation Results (Comparison)



Average over 100 trials

Random Edge-Removal

RRN outperforms the other networks.

RRN, RG, and RTN performs similarly.

SF performs the worst.

Observation:

RRN, RTN have many loops

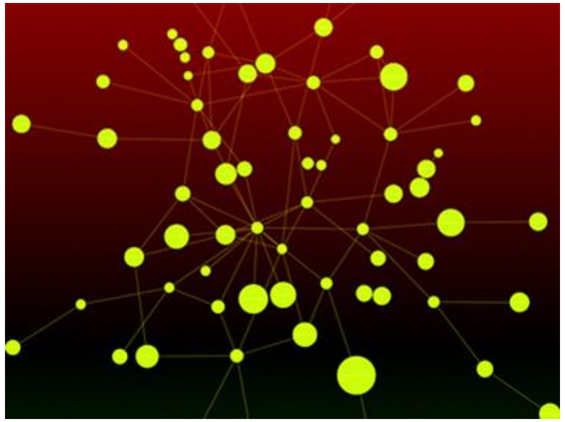
RG is homogeneous

Machine Learning

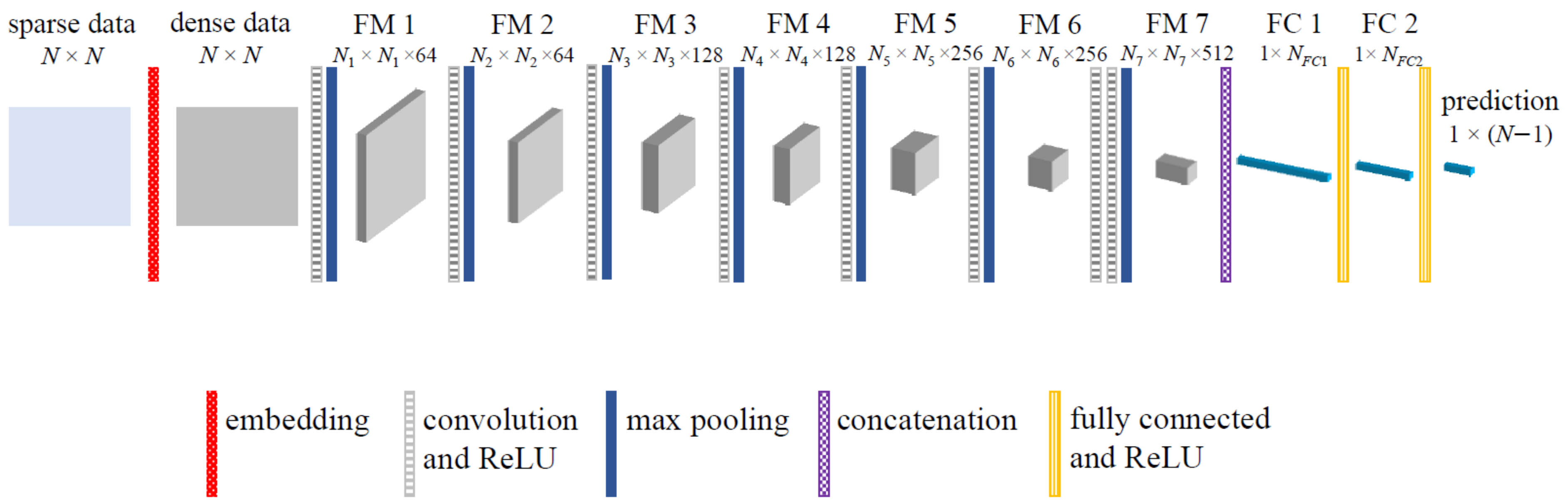
Motivation of applying **Machine Learning:**

There is no clear correlation between the topological features and the controllability robustness of a general (directed or undirected) network

Y. Lou, Y. He, L. Wang, and G. Chen, "Predicting Network Controllability Robustness: A Convolutional Neural Network Approach", *IEEE Transactions on Cybernetics*, 2020 (online)

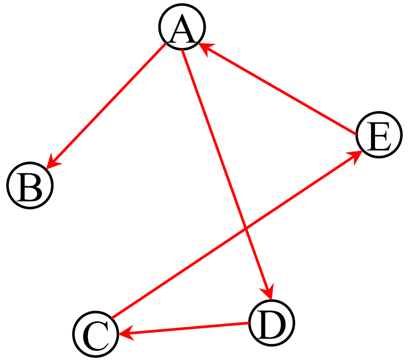


Machine Learning
using **Convolutionary Neural Network (CNN)**



CNN architecture used for controllability robustness prediction
FM – feature map
FC – fully connected
data size $N_i = \lceil N/(i+1) \rceil$, for $i = 1,2,\ldots,7$.
$N_{FC1} = N_7 \times N_7 \times 512$, $N_{FC2} \in (N_{FC1}, N-1)$ is a hyperparameter
$N_{FC2} = 4096$ for $N = \{800,1000,1200\}$

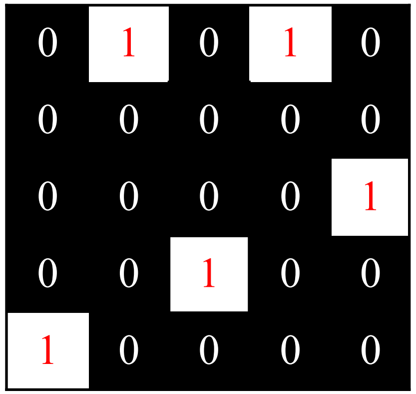
Networks and Image Representation

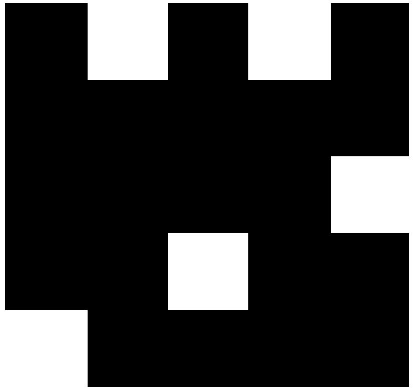


topology

	A	B	C	D	E
A	0	1	0	1	0
B	0	0	0	0	0
C	0	0	0	0	1
D	0	0	1	0	0
E	1	0	0	0	0

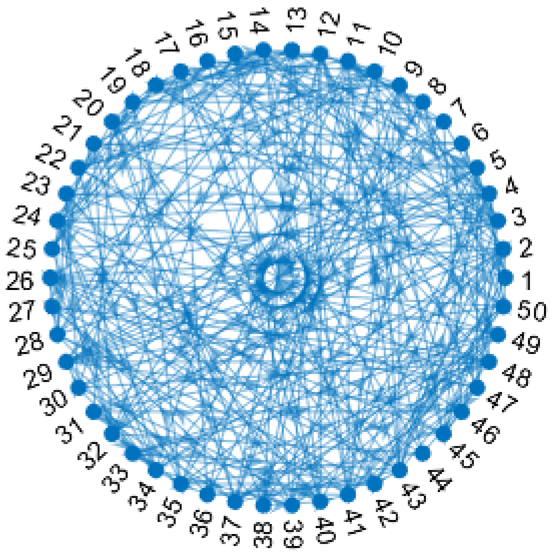
adjacency matrix

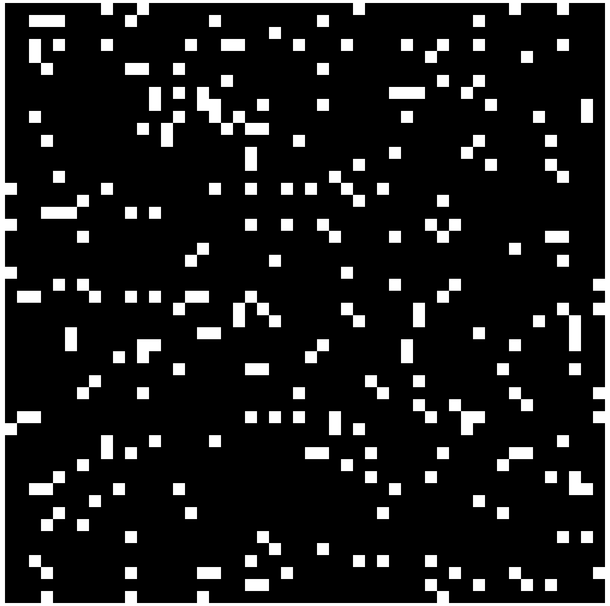




image

Erdos-Renyi Random Graph (ER)

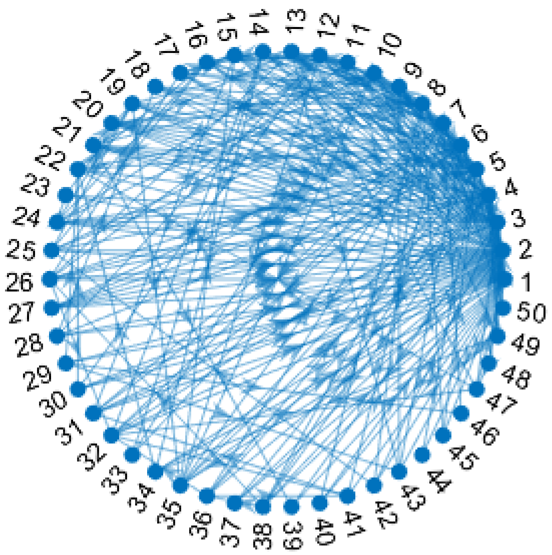


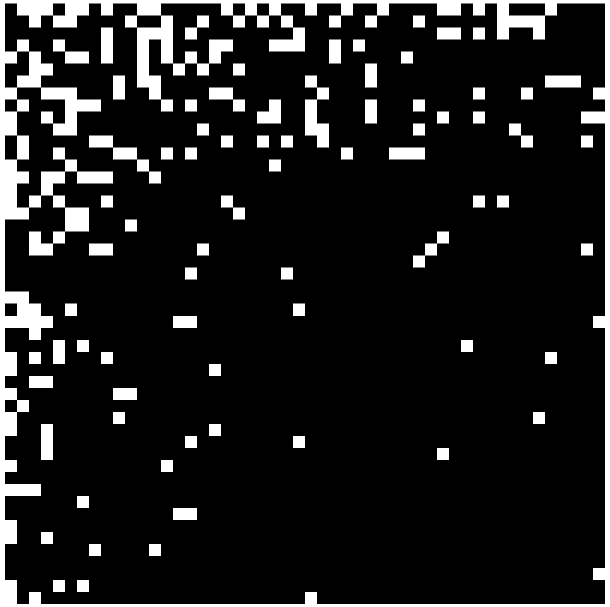


ER: uniformly randomly connect any two nodes by M edges; the directions are evenly-randomly assigned

ER-image: uniformly randomly distribute the M light pixels into an $N \times N$ matrix

Barabasi-Albert Scale-Free Network (SF)



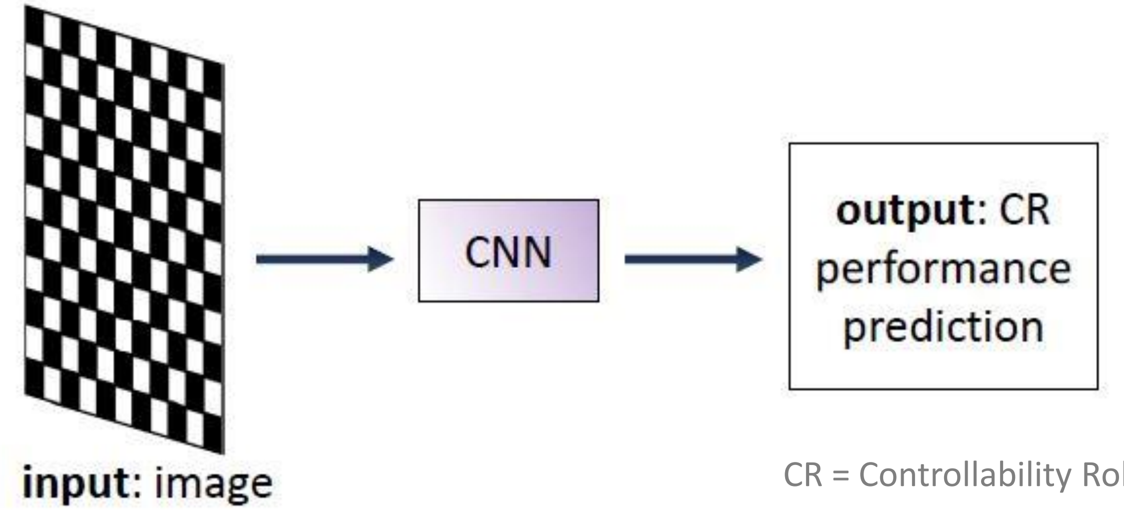


SF: nodes i and j $(i \neq j, i, j = 1,2, \dots, N)$ are randomly picked with a probability proportional to their weights w_i and w_j, respectively. Then, an edge A_{ij} from i to j is added only if they are not connected

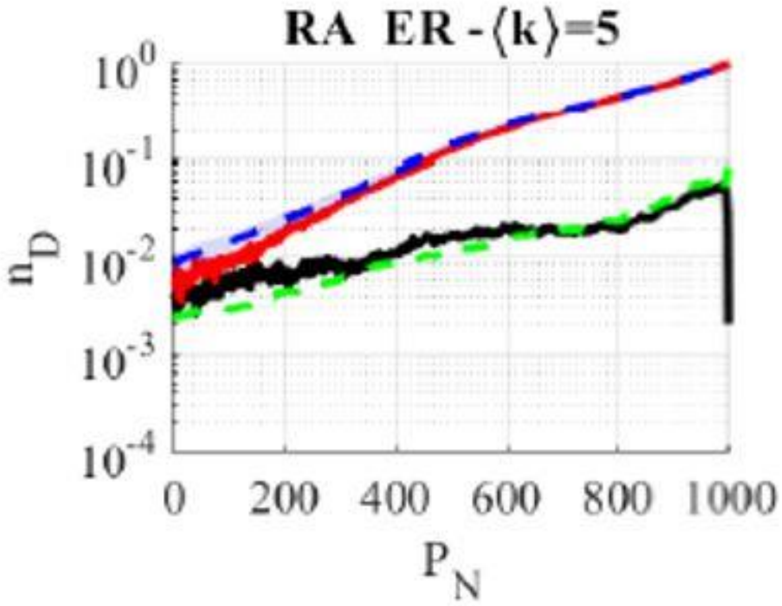
SF-image: a heterogeneous network and thus a heterogeneous image; very strong structural characteristics

Simulations

(There are many simulation results, but only one is shown for illustration)



CR = Controllability Robustness

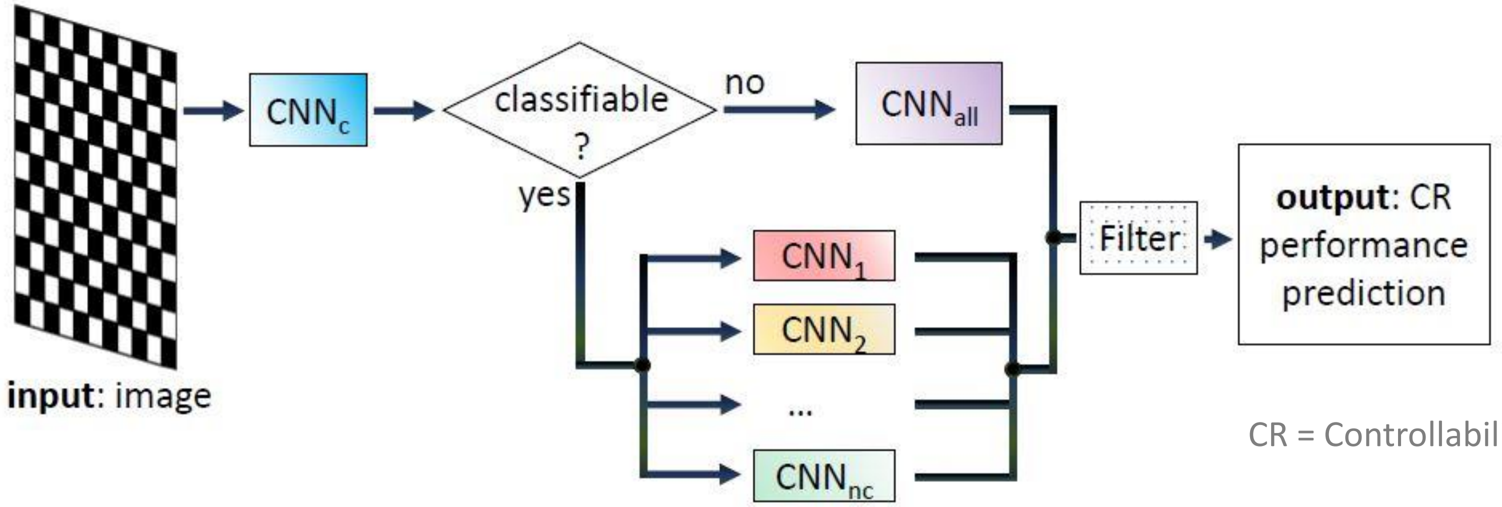


RA – Random Attack
ER – Erdos-Renyi Random Network

Blue/Red – True/Prediction
Black/Green – Errors/Deviations

Knowledge-Based Learning

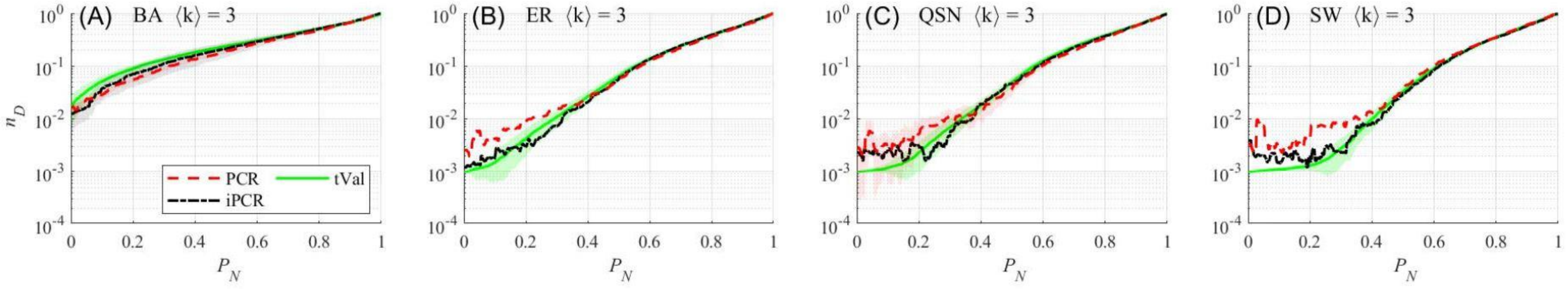
Sufficiently utilize the prior knowledge (network types) in pre-processing for improving predictions



CR = Controllability Robustness

Y Lou, Y D He, L Wang, K F Tsang, G Chen. "Knowledge-based prediction of network controllability robustness," *IEEE Trans. Neur. Nets. Learn. Sys*., accepted, 2021

Simulation

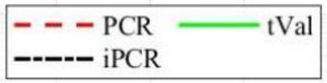


BA = BA scale-free network

ER = ER random-graph network

QSN = q-snapback network

SW = Small-world network



PCR = Predicted controllability robustness

tVal = True value

iPCR = improved Predicted controllability robustness

Training size = 4000

Testing size = 1000

Network size = 200

P_N = Attack probability

Significant Finding:

Cycles and **Homogeneity** are **good** for

both **Controllability** and **Robustness**

An empirical necessary (homogeneity) condition:

$$\lfloor M/N \rfloor \leq k_i^{in,out} \leq \lceil M/N \rceil \quad (i = 1,2,\ldots,N)$$

M- number of edges, N – number of nodes, k- degree

Y. Lou, L. Wang, K. F. Tsang and G. Chen, IEEE Trans. Circ. Syst.-I (2020)

Research Outlook

General Theory ?

Higher-order Topology ?

Cycle, Clique, Cavity

Betti Number, Euler Characteristic Number

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Thank You

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