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is beneficial. Chaos synchronization, on the other hand, refers to the task of enabling or disabling the dynamical synchrony of several connected chaotic systems by means of control tech-

## Synchronization and Control of Chaos: An Introduction for Scientists and Engineers

By J.M. GONZÁLEZ-MIRANDA

### CHAOS CONTROL

Chaos control refers to manipulating the dynamical behavior of a chaotic system, in which the goal is to suppress chaos when it is harmful or to enhance or create chaos when it

niques or through specially designed coupling configurations. These topics are addressed in the book *Synchronization and Control of Chaos: An Introduction for Scientists and Engineers* by J.M. González-Miranda.

Numerous research monographs, textbooks, and edited volumes are devoted to the subjects of synchronization and control of chaos. Examples include [1]–[10], although none of these works are included in the 217 citations of the book. Since the preface states that “The purpose of this book is to provide a systematic and broad account of that research for a wide audience,” I was interested in seeing how these topics are presented in a book intended as an introduction for scientists and engineers.

### CONTENTS

The book has eight chapters, none of which include any exercises.

Chapter 1 begins with an introduction to standard examples of free and damped second-order linear mechanical harmonic oscillators, evolving to one external-force driven and

two coupled oscillators, then to a cubic-nonlinear (Duffing) oscillator. This short introductory chapter is quite well written, preparing basic material required in the sequel.

Chapter 2 reviews basic concepts in chaos theory, including nonlinear continuous flows and discrete maps, using the Lorenz system and Hénon map as examples. The chapter also introduces Poincaré maps and bifurcation diagrams using Chua's circuit as the platform. Chaotic time series analysis, typically on Lyapunov exponents and power spectra, as well as a special feature of chaotic orbits (unstable periodic orbits) are discussed in detail. In this regard, however, the standard references [11], [12] on nonlinear and chaotic time-series analysis are regrettably not cited.

Chapter 3 introduces the concept of periodically driven chaotic oscillation along with the related topics of phase synchronization and chaos suppression through external-force input. Experimental and numerical observations of these phenomena are demonstrated using physical examples such as lasers, Josephson junctions, and single-neuron models. The author provides a good description of the basic phenomenon of chaotic phase synchronization. In referencing this topic, the overview article [13], which is not cited, can be beneficial for the reader's further edification on state-of-the-art developments in chaotic phase synchronization. As discussed below, several references cited in the book are either not the best selections from the available literature or are out of date despite the statement that "the book combines tutorial and review techniques . . . All the above is provided together with a bibliography aiming to allow the reader to probe deeper into the particular issues that especially interest him or her."

Chaos suppression by means of external weak resonant forces is discussed in the second half of Chapter 3, based on an open-loop control technique suggested in the early 1990s; see, for example, Chapter 4 of [5]. Brief comments on variants of this methodology are provided at the end of the chapter. I would refer the reader to the more updated and comprehensive reference [14].

Chapter 4 introduces the concept as well as elementary techniques for chaos synchronization, mainly based on the drive-response (also called masterslave) configuration. Complete and generalized synchronizations, for both identical and nonidentical systems, are discussed. The reader can learn the essentials of chaos synchronization by reading through this well-written chapter. In particular, "marginal synchronization of chaos" in the phase space, attributed to the author of the book, is studied in detail. The chapter closes with observations on generalized chaos synchronization. It is important to note that there exist alternative synchronization settings, such as chaotic switching and chaotic modulation, that are not introduced in the chapter. For additional chaos synchronization configurations, see Chapter 11 of [5]. As an introductory treatise on synchronization and control of chaos, this omission leaves the book incomplete.

Chapter 5 is devoted to the study of chaos control by means of perturbations, mainly the Ott-Grebogi-Yorke

(OGY) method and delayed feedback control. This 21-page chapter (pp. 111–131), as well as the eight-page "chaos suppression" portion in Chapter 3 (pp. 70–77) reviewed above, constitute the main subject of chaos control as covered by the book. Compared to the materials on chaos synchronization, this emphasis is unbalanced. As a result, all other approaches to chaos control available in the literature, which includes more than 20 well-known methods and techniques (see, for example, [1]–[9] and particularly Chapters 5–10, pp. 173–534, of [5]), are not presented here. Control engineers are familiar with linear and nonlinear feedback control, adaptive control, optimal control, sliding-mode control, self-tuning control, robust control, digital control, stochastic control, intelligent control, distributed control, and many other system regulation and stabilization methodologies. Seeing the eye-catching title of the book, an engineer who is interested in delving into this new research field may wonder whether these conventional control techniques work equally well for chaotic systems and, if so, whether there are any special features to be anticipated by taking advantage of chaos theory. Although intended as an introduction for scientists and engineers, presumably targeting control engineers, this omission fails to provide the whole picture of chaos control but, rather, shows only a very small piece of it. Setting aside this important issue, the key topics of OGY control and delayed feedback control are not introduced with updated information. Although these approaches are described quite well in the text, the OGY method is further developed in subsections 3.2.6–3.2.7 of [5] and Chapter 12 of [8]. Also, the delayed feedback control method has an inherent defect of the odd number limitation [15], [16], which should be noted in an introductory text.

Chapters 6 and 7 extend the chaos synchronization framework from a pair of coupled oscillators to a network of multi-coupled oscillators. The case of identical chaotic oscillators is studied in Chapter 6, while the nonidentical setting is discussed in Chapter 7. In these chapters, network synchronization is presented by numerical simulations, with physical experiments for lower dimensional cases. Phase synchronization is presented with detailed discussions. The recent trend of chaotic network synchronization [17], [18], however, is not reflected in these chapters or elsewhere in the book.

Chapter 8, in the author's own words, "is devoted to a summary of the main results presented in the book, to discuss their possible technical and scientific applications, and to speculate on the perspectives of the field both from the fundamental and the applied points of view." This well-written summary includes three short subsections on applications, namely, synchronization in neurobiology, synchronization in the earth sciences, and chaotic communications. Chaos control and synchronization have many more potential applications; see, for example, chapters 11 and 12 of [5].

The subject of chaos synchronization in the human brain is a good selection for study. As noted in [19], "The brain

transforms sensory messages into conscious perceptions almost instantly. Chaotic, collective activity involving millions of neurons seems essential for such rapid recognition." Within this context, the chaotic and collective activity often refers to chaos synchronization [19]. As the author points out on page 193, "Modelling of climatic and meteorological scenarios is another application of interest." The topic of synchronized motions in patterns of atmospheric circulation is a natural choice for discussion in this section, based on the familiar Lorenz model. Generally speaking, these topics are well presented.

Readers of *IEEE Control Systems Magazine* may be interested in the third subject, chaotic communications. This topic, unfortunately, is not well presented in the book, and only some initial work from the early 1990s is introduced. If one is interested in the novelty and merits of chaotic communications versus conventional communications, then the best research includes [20] and [21]. If the reader is interested in the security aspect of chaotic communications, then the setting based on chaotic signal masking and discovery introduced in this chapter is known to be insecure, as cryptanalyzed by several papers (see, for example, [22] and the references in [23]). In general, chaotic communication and encryption technologies are not well reflected in this part of the book. The goal of the author "to exemplify what might be a major line of development of this field in the next few years: the application of the new ideas and techniques in all the sciences, medicine and engineering" (p. 196) is far from being reached.

## CONCLUSIONS

As noted in the preface, the book's author has "been working on coupled and driven chaotic oscillators for the last ten years." Indeed, the corresponding part of the contents is clearly written and well presented. If the reader seeks to learn or teach chaos synchronization, this book is useful. In particular, the topic of chaotic phase synchronization is well presented, and the discussion is thorough; thus, this part of the text is highly recommended.

However, I do not recommend this text as an introduction for scientists and engineers. I believe that the goal of this book, as stated in the preface, "to provide a systematic and broad account of that research," has not been achieved. For the reader's information and further reading, given the vast literature on the rapidly evolving research subjects of chaos control and chaos synchronization, the bibliographies [24] and [25] in conjunction with [1]–[10] are more informative. Nevertheless, in the information-explosive era today, writing a good research monograph or textbook is never an easy task. From this point of view, I commend the author for the courage, time, and effort devoted to this book.

—Guanrong Chen

## REVIEWER INFORMATION

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