Overview of Signals and Systems

Chapter Intended Learning Outcomes:

(i) Understand basic concepts of signals and systems

(ii) Realize that signals and systems arise in our daily life

What is Signal?

- Anything that conveys information, e.g.,
  - Speech
  - Electrocardiogram (ECG)
  - Radar pulse
  - Traffic light
  - Medical image
  - Stock price
  - Orthogonal frequency division multiplexing waveform
  - Video
  - Smell

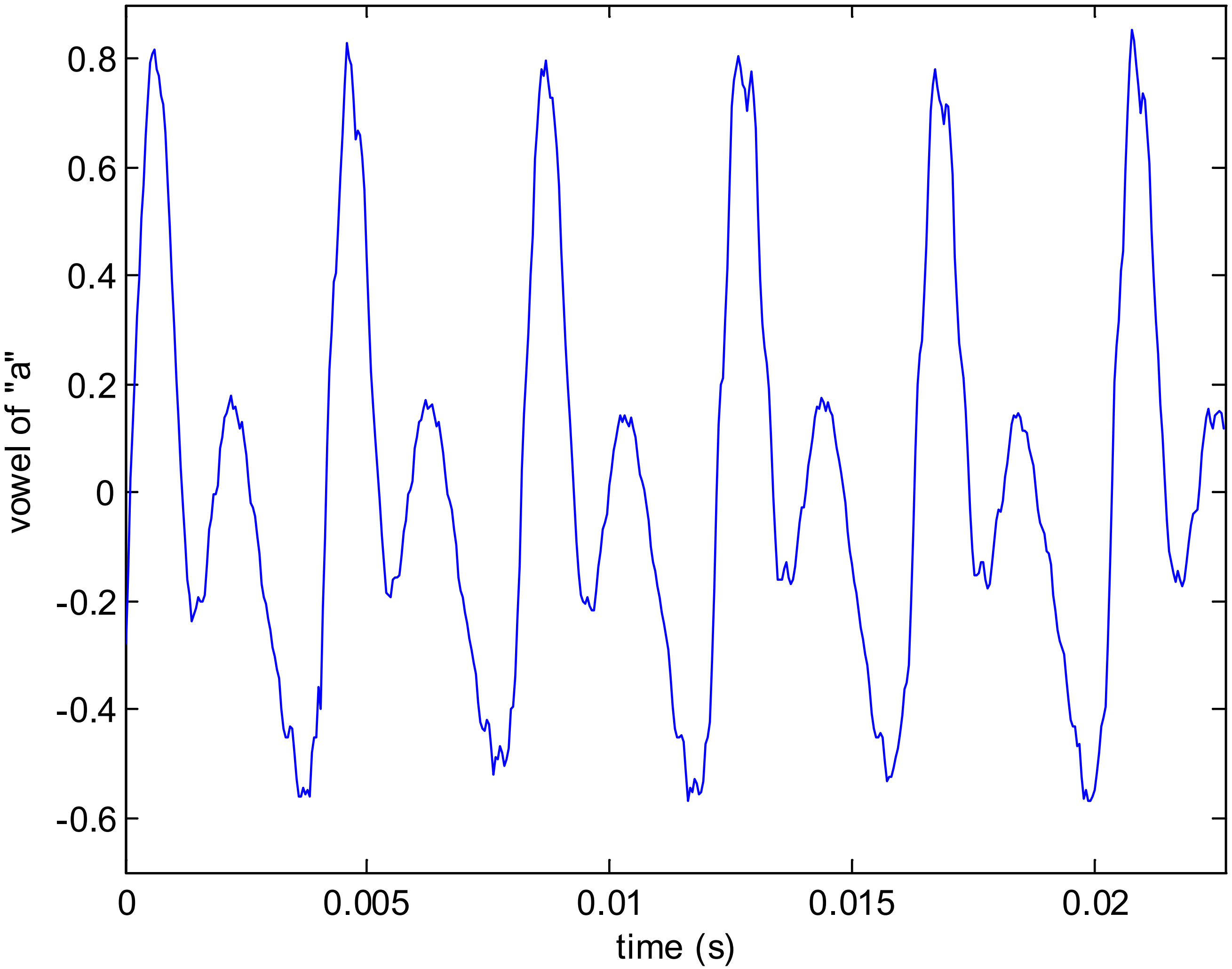


Fig.1.1: Speech

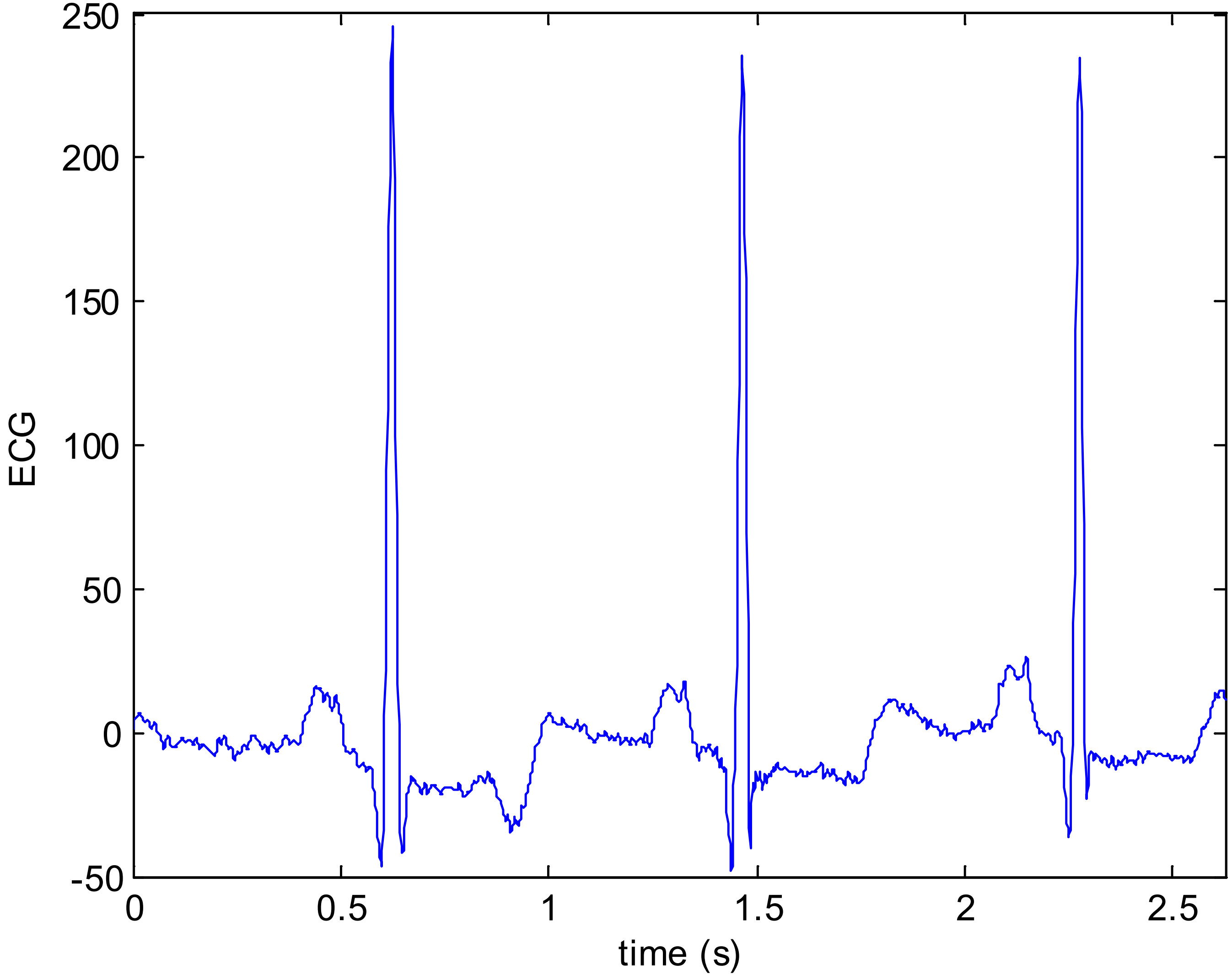


Fig.1.2: ECG

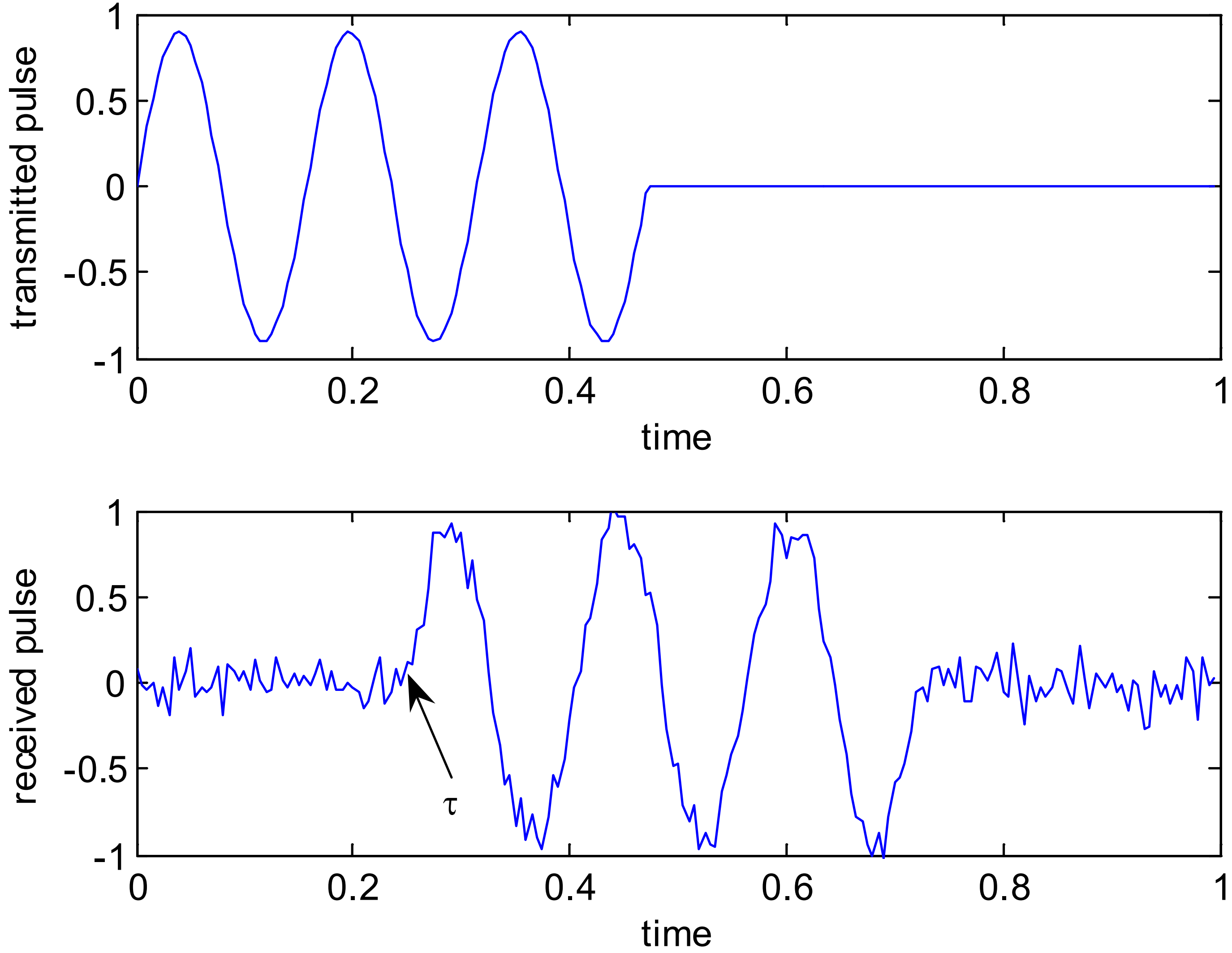
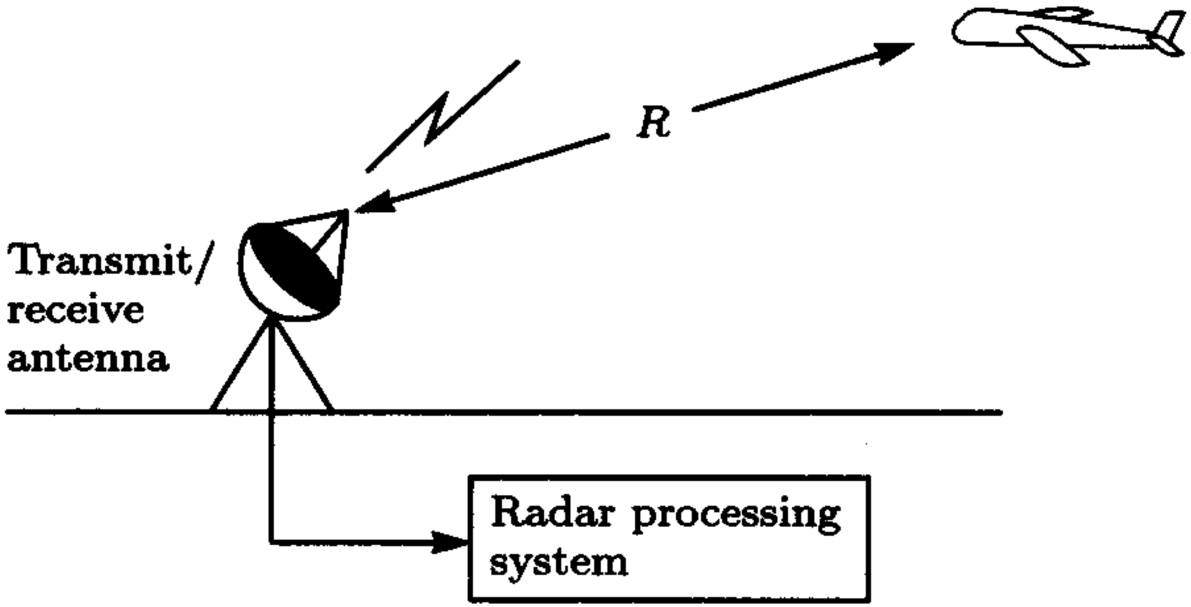


Fig.1.3: Transmitted & received radar waveforms: $s(t)$ & $r(t)$



$$r(t) = s(t - \tau) + w(t)$$

Fig.1.4: Radar ranging

Given the signal propagation speed, denoted by c, the time delay τ is related to R as:

$$\tau = \frac{2R}{c} \tag{1.1}$$

Hence radar pulse contains the object range information.

- Can be a function of one, two or three independent variables, e.g., speech is one-dimensional (1-D) signal, function of time; image is 2-D, function of space; wind is 3-D, function of latitude, longitude and elevation.

- 3 types of signals that are functions of time:
  - Continuous-time (analog) $x(t)$: defined on a continuous range of time t, amplitude can be any value.
  - Discrete-time $x(nT)$ (sampled): defined only at discrete instants of time $t = \cdots -T, 0, T, 2T, \cdots$, amplitude can be any value.
  - Digital (quantized) $x_Q(nT)$: both time and amplitude are discrete, i.e., it is defined only at $t = \cdots -T, 0, T, 2T, \cdots$ and amplitude is confined to a finite set of numbers.

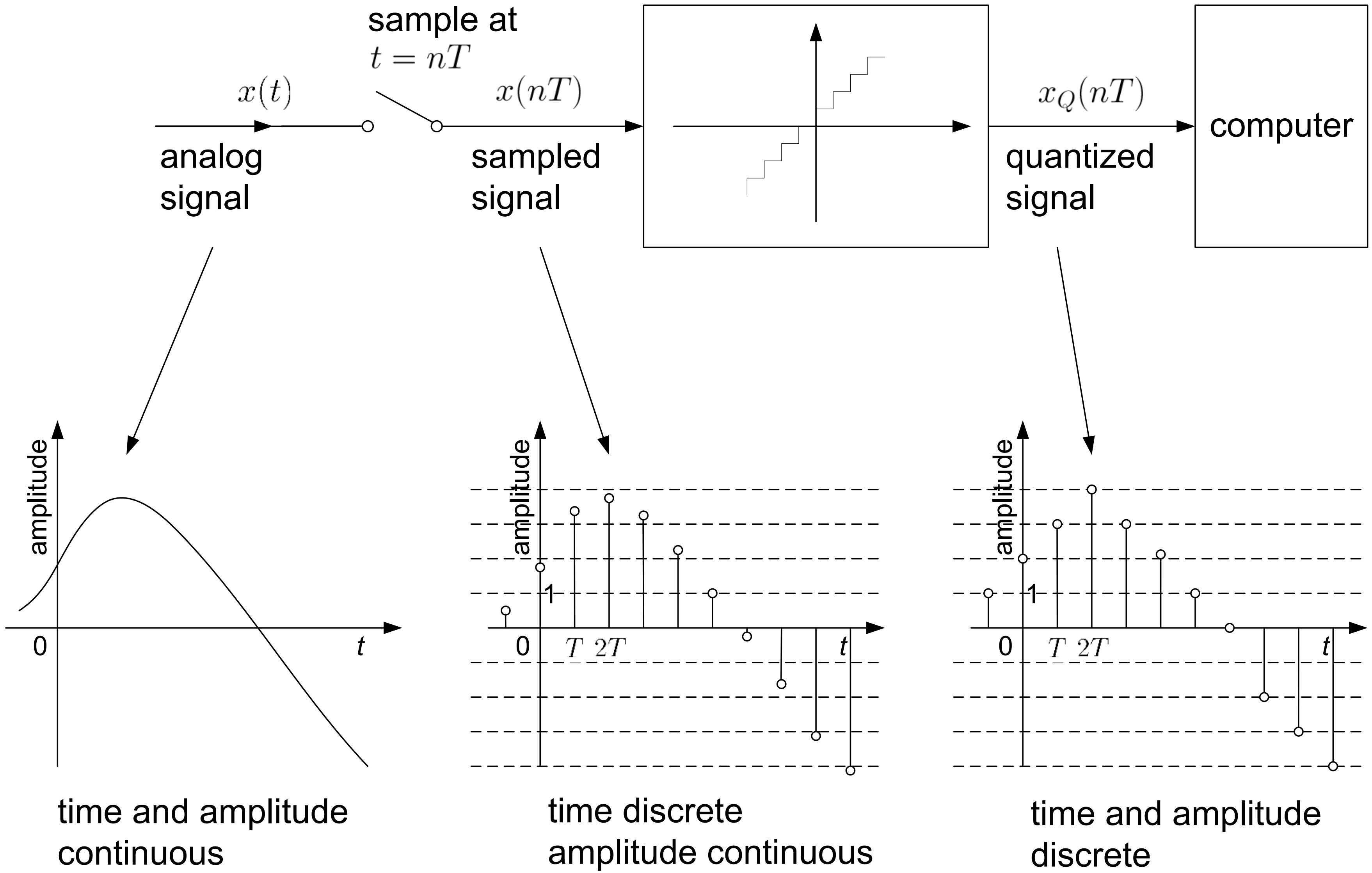


Fig. 1.5: Relationships between $x(t)$, $x(nT)$ and $x_Q(nT)$

$x(nT)$ at $n=0$ is close to 2 and $x_Q(0)=2$.

$x(nT) \in (3,4)$ at $n=1$ and $x_Q(T)=3$.

Using 4-bit representation, $x_Q(0)=0010$ and $x_Q(T)=0011$, and in general, the value of $x_Q(nT)$ is restricted to be an integer between -8 and 7 according to the two's complement representation.

We focus on continuous-time and discrete-time signals. Discrete-time signal is also commonly represented by $x[n]$ with $n=\cdots-1,0,1,\cdots$ being the time index.

The digital signal can be considered as discrete-time if the quantizer has very (infinite) high resolution.

What is System?

- Mathematical model or abstraction of a physical process that relates input to output:

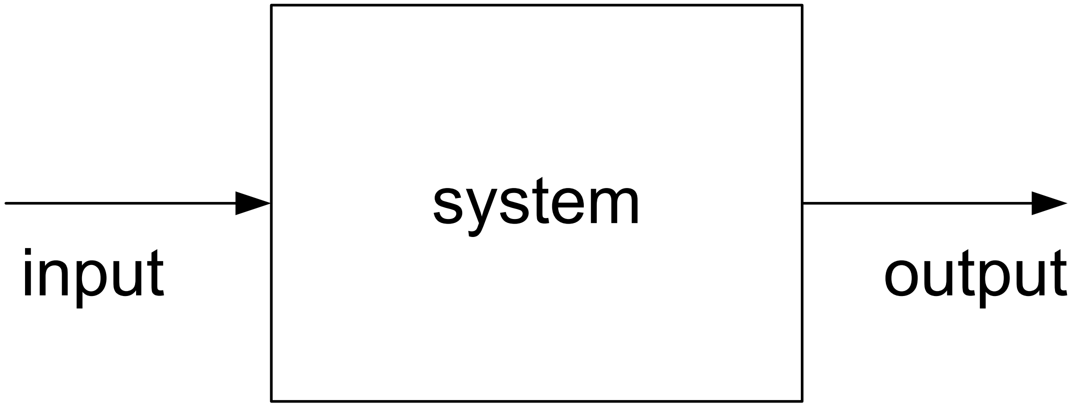


Fig.1.6: System with input and output

- It operates on an input to produce an output, e.g.:
  - Grading system: inputs are coursework and examination marks, output is grade.

- Squaring system: input is 5, then the output is 25.
- Amplifier: input is $\cos(\omega t)$, then output is $10\cos(\omega t)$.
- Communication system: input to mobile phone is voice, output from mobile phone is 5G waveform.
- Noise reduction system: input is a noisy speech, output is a noise-reduced speech.
- Feature extraction system: input is $\cos(\omega t)$, output is ω.

- An analog or continuous-time system has continuous-time input and output while a discrete-time system deals with discrete-time input and output.

- A system can be realized in hardware or software via an algorithm.

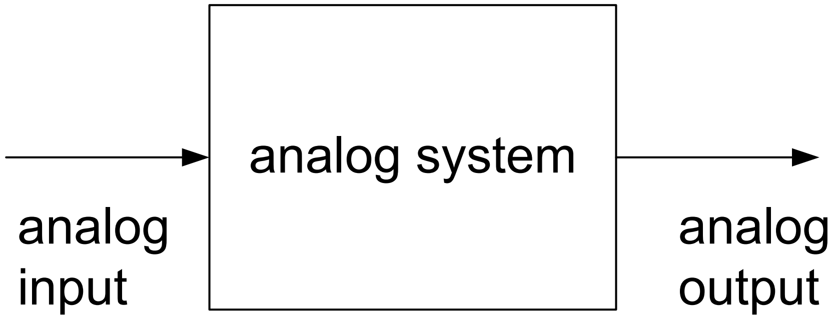


Fig.1.7: Continuous-time system

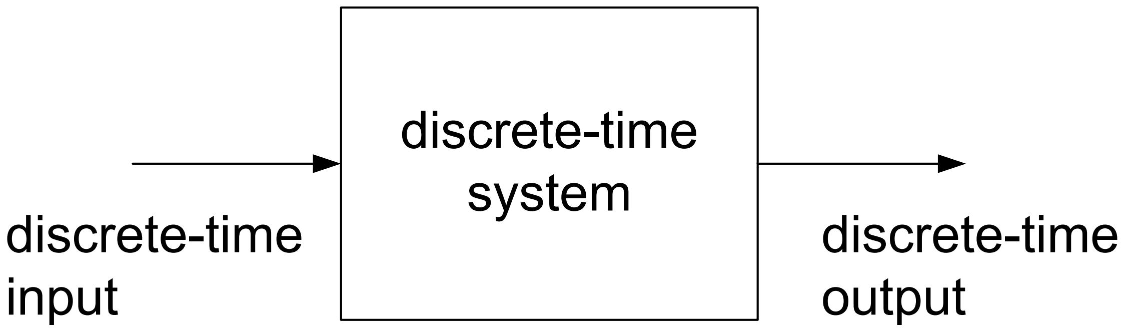


Fig.1.8: Discrete-time system

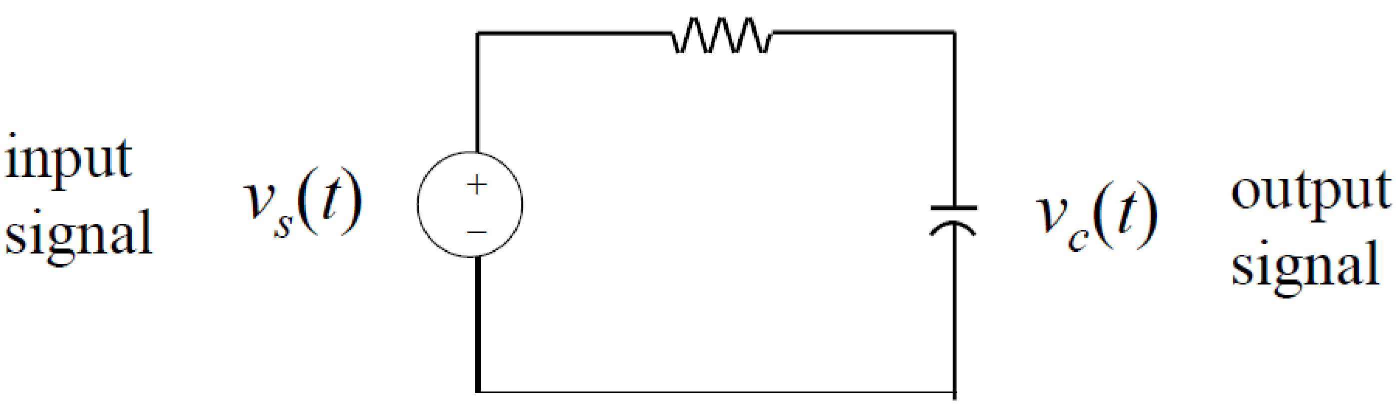


Fig.1.9: Hardware system of resistor-capacitor circuit

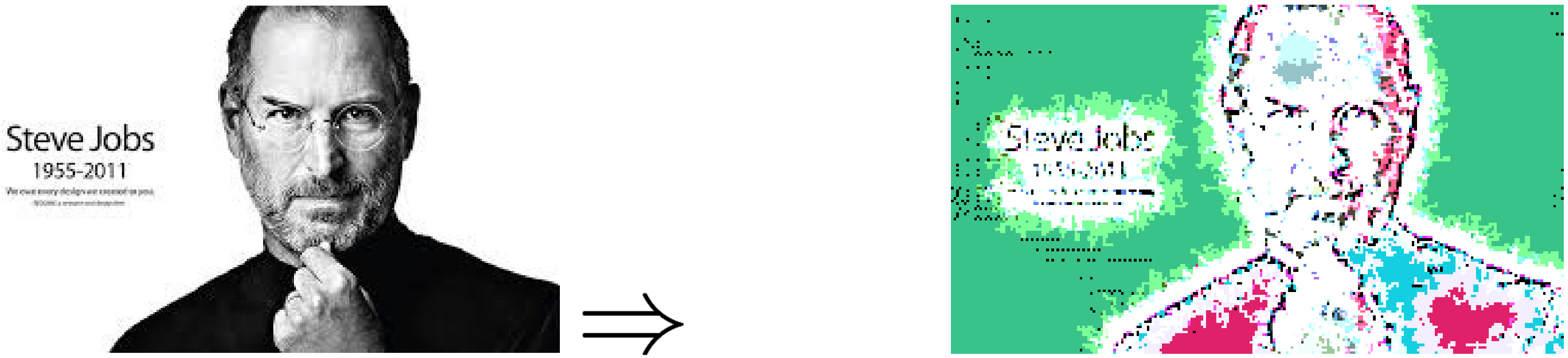


Fig.1.10: Pop-art production using an algorithm

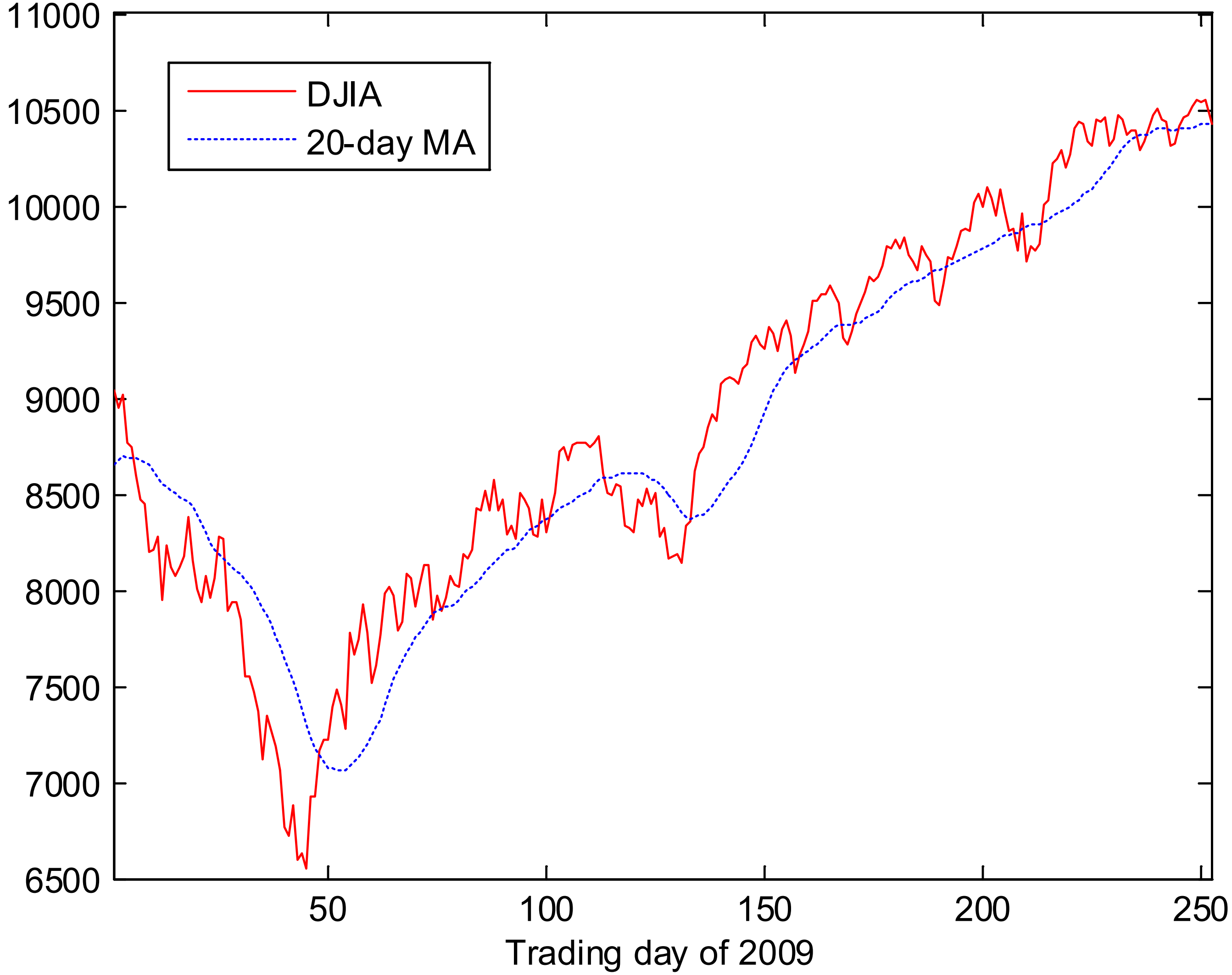


Fig.1.11: Software system for moving average of Dow Jones

Example 1.1

Consider an input signal $x(t)=\cos(\omega t)$ passing through a system. For $t<0$, the system amplifies the input by 5, while for $t \geq 0$, the system squares the input, to produce the output $y(t)$.

Write down the mathematical expression of the system that relates $x(t)$ and $y(t)$. Is the input $x(t)=\cos(\omega t)$ continuous-time or discrete-time signal? Is the system continuous-time or discrete-time?

According to the system description, we easily obtain:

$$y(t)=\begin{cases} 5x(t), \ t<0 \\ x^2(t), \ t \geq 0 \end{cases}$$

It is also clear that the input is a continuous-time signal and the system is continuous-time.

What will You Learn?

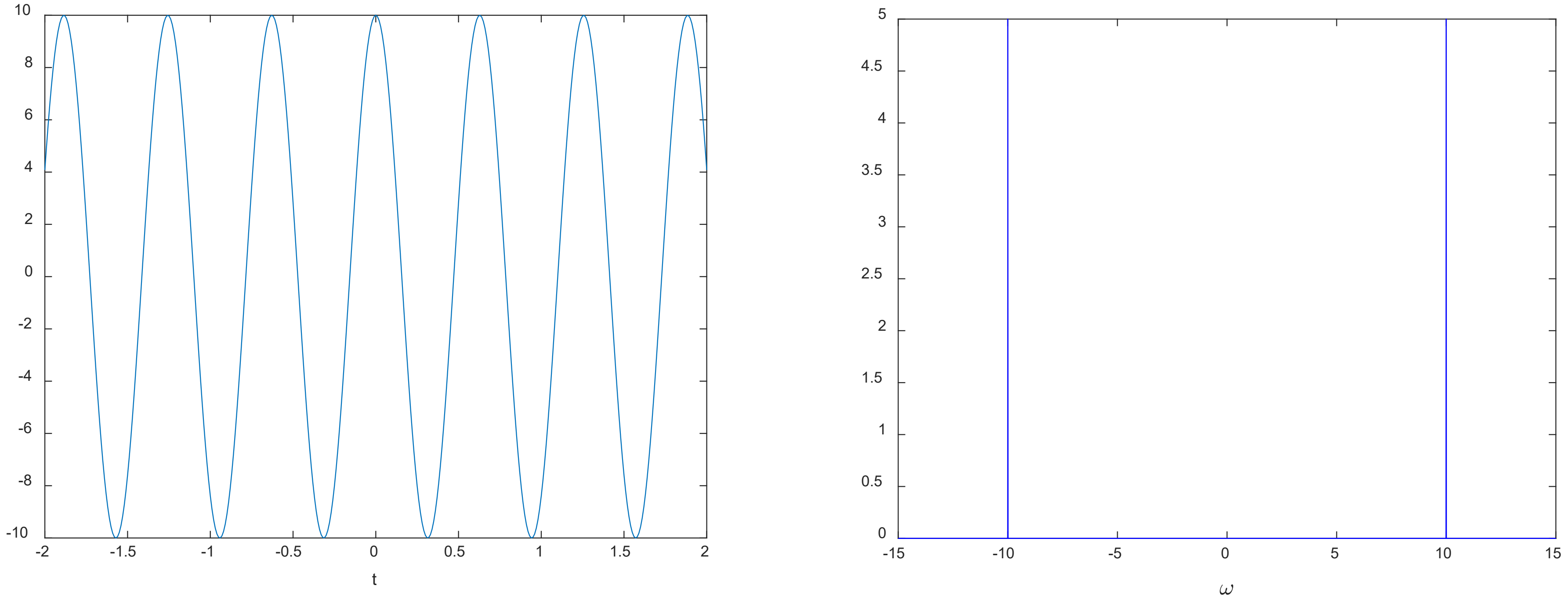
- Signal representation and characterization, which includes generating signals, understanding signal types and properties, and performing operations on signals.
- System classification and analysis, which includes classifying system types, and calculating impulse response, frequency response, input and/or output for linear time-invariant (LTI) systems.
- Transform tools include Fourier series and Fourier transform as well as their applications in signal and LTI system analysis, e.g.: a periodic continuous-time signal $x(t)$ can be represented as sum of complex exponentials:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}, \qquad t \in (-\infty, \infty) \tag{1.2}$$

Why Important?

- Signals and systems arise in our daily life, studying it will lay a good foundation for you in other relevant/higher-level courses and to solve real-world problems:
  - Generate signals which meet certain specifications, e.g., synthesized speech and music.
  - Design systems which produce desired outputs, e.g., a system which suppresses noise in the measured data.
  - New signal representation for efficient data processing, e.g., David Donoho proposed sparse representation and obtained the Shaw Prize 2013 (邵逸夫數學科學獎). Sparsity means containing many zero elements. https://www.youtube.com/watch?v=5wv4grOMgIU

A sine wave $x(t) = 10\cos(10t)$ is not sparse in time domain but it is sparse in frequency domain.



How to Study?

Make sure you have a clear concept and then practice.