Proof of (4.8):

As $x(t)$ and $y(t)$ have the same fundamental period of T_p or fundamental frequency Ω_0, we can write:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}, \qquad y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\Omega_0 t}$$

Multiplying $x(t)$ and $y(t)$ by A and B, respectively, yields:

$$Ax(t) = A \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}, \qquad By(t) = B \sum_{k=-\infty}^{\infty} b_k e^{jk\Omega_0 t}$$

Summing $Ax(t)$ and $By(t)$, we get:

$$Ax(t) + By(t) = \sum_{k=-\infty}^{\infty} (Aa_k + Bb_k) e^{jk\Omega_0 t} \leftrightarrow Aa_k + Bb_k$$

Proof of (4.9) and (4.10):

Recall (4.3):

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}$$

Substituting t by $t - t_0$, we obtain:

$$x(t-t_0) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 (t-t_0)} = \sum_{k=-\infty}^{\infty} \left(e^{-jk\Omega_0 t_0} a_k\right) e^{jk\Omega_0 t} \leftrightarrow e^{-jk\Omega_0 t_0} a_k$$

Substituting t by $-t$ yields:

$$x(-t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 (-t)} = \sum_{l=-\infty}^{\infty} a_{-l} e^{jl\Omega_0 t} = \sum_{k=-\infty}^{\infty} a_{-k} e^{jk\Omega_0 t} \leftrightarrow a_{-k}$$

Proof of (4.12):

Applying (4.3) again, the product of $x(t)$ and $y(t)$ is:

$$
\begin{aligned}
x(t)y(t) &= \sum_{l=-\infty}^{\infty} a_l e^{jl\Omega_0 t} \sum_{n=-\infty}^{\infty} b_n e^{jn\Omega_0 t} \\
&= \sum_{l=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} a_l b_n e^{j(l+n)\Omega_0 t} \\
&= \sum_{l=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} a_l b_{l-k} e^{jk\Omega_0 t}, \quad k = l+n \\
&= \sum_{k=-\infty}^{\infty} \left(\sum_{l=-\infty}^{\infty} a_l b_{l-k} \right) e^{jk\Omega_0 t} \leftrightarrow \sum_{l=-\infty}^{\infty} a_l b_{k-l}
\end{aligned}
$$