Tutorial 10 on Week 11

1. Use z transform to compute the output $y[n]$ if the input is $x[n] = a^n u[n]$ with $|a| < 1$ and the linear time-invariant (LTI) system impulse response is $h[n] = b^n u[n]$ with $|b| \geq 1$. Is the system stable? Why? Is the system causal? Why?

2. Consider a stable and causal discrete-time LTI system with impulse response $h[n]$ and rational transfer function $H(z)$. It is known that $H(z)$ contains a pole at $z = 0.5$ and a zero somewhere on the unit circle. The precise numbers and locations of all other poles and zeros are unknown.

Discuss if each of the following statements is "true", "false" or "cannot be determined":

(a) The DTFT of $(0.5)^n h[n]$ exists.
(b) $H(e^{j\omega}) = 0$ for some ω.
(c) $h[n]$ is of finite duration.
(d) $h[n]$ is a real-valued signal.

3. Determine the Laplace transform of

$$x(t) = e^{-5t}u(t-1)$$

Specify its region of convergence (ROC). Determine all pole and zero location.

4. Determine the Laplace transform of

$$x(t) = -ae^{at}u(-t)$$

where a is complex number. Specify its ROC. Determine all pole and zero location.

5. Consider the continuous-time signal $x(t)$:

$$x(t) = e^{-5t}u(t) + e^{-\beta t}u(t)$$

Determine the constraints on the complex number β given that the ROC of $X(s)$ is $\Re\{s\} > -3$.

6. Determine the Fourier transforms of $x(t) = e^{-0.5t}u(t)$ and $y(t) = e^{2t}u(t)$.