

## Tutorial 11 on Week 12

1. Determine the Laplace transform of

$$x(t) = \begin{cases} e^t \sin(2t), & t \leq 0 \\ 0, & t > 0 \end{cases}$$

Specify its region of convergence (ROC). Find all the pole(s).

2. Determine the Laplace transform of

$$x(t) = e^{-t}u(t) \otimes \sin(3\pi t)u(t)$$

Specify its ROC. Find all the pole(s).

3. Given the Laplace transform of a continuous-time signal  $h(t)$ :

$$H(s) = \frac{s + 10}{(s + 2)^2(s - 1)(s - 10)(s - 20)}$$

Determine all the possible ROCs for  $H(s)$ .

4. Consider an absolutely integrable signal  $x(t)$ . Its Laplace transform  $X(s)$  is a rational function and is known to have a pole at  $s = 2$ .  $X(s)$  may have other poles. Answer the following questions:

- (a) Can  $x(t)$  be of finite duration? Why?
- (b) Can  $x(t)$  be left-sided? Why?
- (c) Can  $x(t)$  be right-sided? Why?
- (d) Can  $x(t)$  be two-sided? Why?

5. Prove the convolution property of Laplace transform:

$$x(t) \otimes y(t) \leftrightarrow X(s)Y(s)$$

6. Let

$$g(t) = x(t) + \alpha x(-t)$$

where

$$x(t) = \beta e^{-t}u(t)$$

It is known that the Laplace transform of  $g(t)$  is:

$$G(s) = \frac{s}{s^2 - 1}, \quad -1 < \Re\{s\} < 1$$

Determine the values of  $\alpha$  and  $\beta$ .