Tutorial 12 on Week 13

1. Determine the inverse Laplace transform of

$$X(s) = \frac{2(s+2)}{s^2+7s+12}, \quad \Re\{s\} > -3$$

2. Determine the Laplace transform of

$$x(t) = e^{2t}u(-t+2)$$

Specify the region of convergence (ROC) and find all poles of $X(s)$.

3. Consider a signal $y(t)$ which is related to two signals $x_1(t)$ and $x_2(t)$ by

$$y(t) = x_1(t-2) \otimes x_2(-t+3)$$

where

$$x_1(t) = e^{-2t}u(t) \quad \text{and} \quad x_2(t) = e^{-3t}u(t)$$

Determine the Laplace transform of $y(t)$.

4. A discrete-time signal $x[n]$ is passed through a causal linear time-invariant (LTI) system with transfer function $H(z) = 1 - az^{-1}$ to produce an output $y[n]$. Determine the condition if we can obtain $x[n]$ from $y[n]$.

5. Consider a causal LTI system with transfer function:

$$H(s) = \frac{3s^2 + 8s + 5}{(s+2)(s^2 + 2s + 1)}$$

(a) Determine the ROC of $H(s)$.
(b) Find the impulse response of the system.
(c) Is the system stable? Why?
(d) Find the system frequency response.
(e) Write down the differential equation that relates the input $x(t)$ and output $y(t)$.