Tutorial 3 on Week 4

1. Compute the output $y[n]$ if the input is $x[n] = a^n u[n]$ and the linear time-invariant system impulse response is $h[n] = u[n]$. Is the system stable? Why? Is the system causal? Why?

2. Determine $y[n] = x[n] \otimes h[n]$ where $x[n]$ and $h[n]$ are

$$x[n] = \begin{cases} 3, & n = -1 \\ 2, & n = 1 \\ 6, & n = 2 \\ 0, & \text{otherwise} \end{cases}$$

and

$$h[n] = \begin{cases} 2, & n = -1 \\ 4, & n = 0 \\ 7, & n = 1 \\ 0, & \text{otherwise} \end{cases}$$

3. Compute the output $y(t)$ if the input is $x(t) = u(t-3) - u(t-5)$ and the linear time-invariant system impulse response is $h(t) = e^{-3t}u(t)$. Is the system stable? Why? Is the system causal? Why?

4. Compute the impulse response $h[n]$ for a LTI system which is characterized by the following difference equation:

$$y[n] = x[n-1] + 2x[n-2] + 3x[n-3]$$

5. Define the area under a continuous-time signal $v(t)$ as:

$$A_v = \int_{-\infty}^{\infty} v(t)dv$$

Show that if $y(t) = x(t) \otimes h(t)$, then

$$A_y = A_x \cdot A_h$$

6. Denote $h[n]$ as the impulse response of a discrete-time linear time-invariant system. If the system is also memoryless, then determine the form of $h[n]$.