Tutorial 4 on Week 5

1. A continuous-time periodic real-valued signal $x(t)$ has a fundamental period of $T = 8$. Its Fourier coefficients are:

$$a_k = \begin{cases} -4j, & k = -3 \\ 2, & k = -1 \\ 2, & k = 1 \\ 4j, & k = 3 \\ 0, & \text{otherwise} \end{cases}$$

Express $x(t)$ in the form:

$$x(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_k t + \phi_k)$$

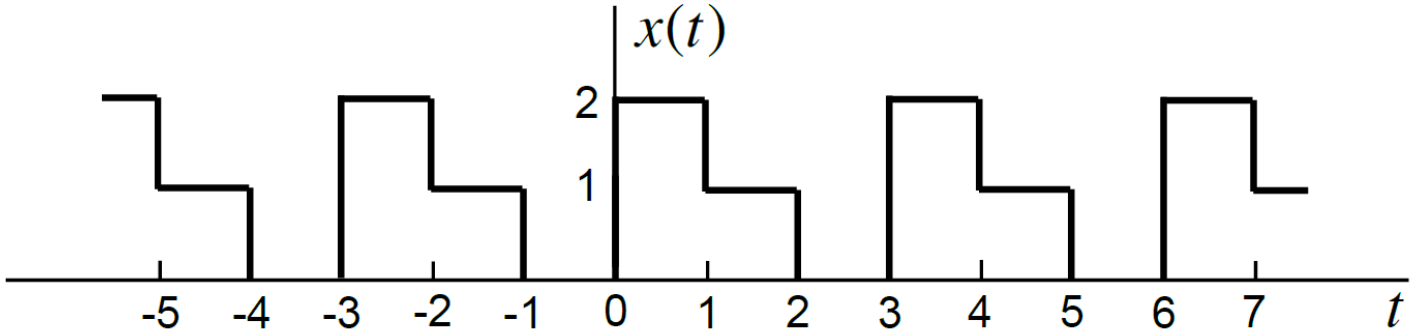
2. Prove the conjugation property of Fourier series:

$$x(t) \leftrightarrow a_k \Rightarrow x^*(t) \leftrightarrow a^*_{-k}$$

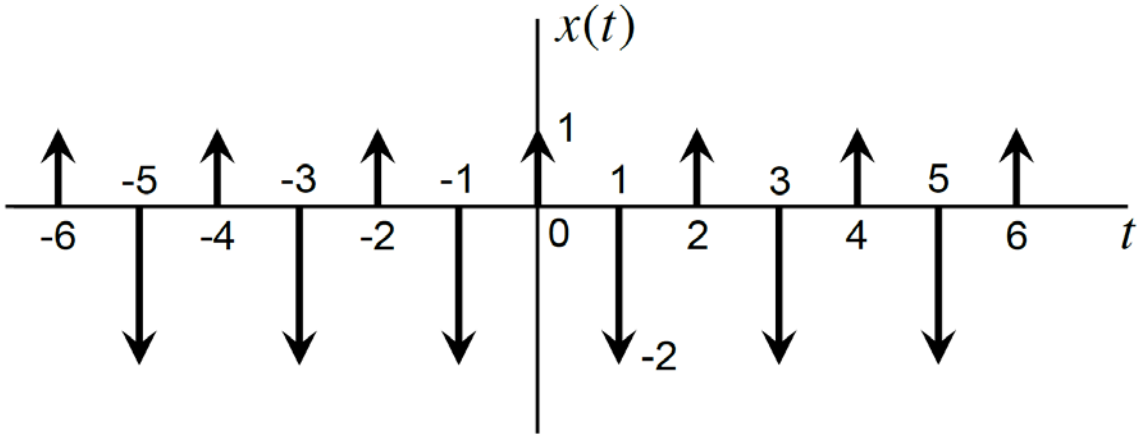
Then show that if $x(t)$ is real-valued, then the magnitudes of Fourier series coefficients are symmetric around $k = 0$:

$$|a_k| = |a_{-k}|$$

3. Determine the Fourier series coefficients of the following continuous-time periodic signal $x(t)$:



4. Determine the Fourier series coefficients of the following continuous-time periodic signal $x(t)$:



5. Consider a continuous-time linear time-invariant system whose frequency response is:

$$H(j\Omega) = \int_{-\infty}^{\infty} h(t)e^{-j\Omega t}dt = \frac{\sin(4\Omega)}{\Omega}$$

where $h(t)$ is the impulse response.

If a periodic signal $x(t)$ with fundamental period $T = 8$ and within the interval $(0, 8)$, $x(t)$ is:

$$x(t) = \begin{cases} 1, & 0 < t < 4 \\ -1, & 4 < t < 8 \end{cases}$$

Determine the system output $y(t) = x(t) \otimes h(t)$.