Tutorial 6 on Week 7

1. Given the inverse Fourier transform formula:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega)e^{j\Omega t}d\Omega$$

   Show that

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}dt$$

   which is the Fourier transform formula.

2. Determine the discrete-time Fourier transform (DTFT) of $x[n] = (0.5)^n u[n]$.

3. Consider a discrete-time linear time-invariant system with input $x[n]$, output $y[n]$ and impulse response $h[n]$. Let the DTFT of $h[n]$ be $H(e^{j\omega})$.

(a) If $x[n] = e^{j\omega_1 n}$, determine $y[n]$ in terms of $H(e^{j\omega})$.

(b) Extend the result of (a) when

$$x[n] = \sum_{k=1}^{K} \alpha_k e^{j\omega_k n}$$

4. Let $X(e^{j\omega})$ denote the DTFT of $x[n]$. Prove:

(a) The DTFT of $x^*[n]$ is $X^*(e^{-j\omega})$.

(b) If $x[n]$ is real-valued, then:

$$|X(e^{j\omega})| = |X(e^{-j\omega})|$$

(c) The DTFT of $x^*[-n]$ is $X^*(e^{j\omega})$.

5. Prove that

$$x[n] = e^{j\omega_0 n} \leftrightarrow X(e^{j\omega}) = \sum_{l=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 - 2\pi l)$$

is a DTFT pair where $\omega_0 \in (-\pi, \pi)$. Then determine the DTFTs of $x[n] = 1$ and $x[n] = \cos(0.5n)$ in $(-\pi, \pi)$.

6. Consider a discrete-time linear time-invariant system with input:

$$x[n] = \alpha^n u[n], \quad |\alpha| < 1$$

and impulse response:

$$h[n] = \beta^n u[n], \quad |\beta| < 1$$

Determine the DTFT of the system output $y[n]$.