Tutorial 7 on Week 8

1. The continuous-time signal $x(t) = \sin(100\pi t + 1)$ is passed through an ideal continuous-time to discrete-time converter with the sampling period $T = 1/50$ s to produce a discrete-time signal $x[n]$. Find $x[n]$. Can $x[n]$ uniquely represent $x(t)$?

2. Prove the multiplicative property of Fourier transform:

$$x_1(t) \cdot x_2(t) \leftrightarrow \frac{1}{2\pi} X_1(j\Omega) \otimes X_2(j\Omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_1(j\tau) X_2(j(\Omega - \tau)) d\tau$$

3. The continuous-time signal $x(t) = \sin(20\pi t) + \cos(40\pi t)$ is sampled at a sampling period T to obtain the discrete-time signal $x[n]$:

$$x[n] = \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right)$$

(a) Determine a choice for T consistent with this information.

(b) Is your choice for T in Part (a) unique? If so, explain why. If not, specify another choice of T consistent with the information given.

4. Consider sampling $x(t) = \cos(\Omega_0 t)$ with a sampling period T to produce $x[n]$. Determine the condition of Ω_0 and T if $x[n]$ is periodic.

5. The frequency which, under the sampling theorem, must be exceeded by the sampling frequency is called the Nyquist rate. Determine the Nyquist rate corresponding to each of the following signals:

(a) $x(t) = 1 + \sin(2000\pi t) + \cos(4000\pi t)$

(b) $x(t) = \dfrac{\sin(8000t)}{\pi t}$