Tutorial 9 on Week 10

1. Determine the z transform of

$$x[n] = \begin{cases} 0, & n < 0 \\ n, & 0 \le n \le N-1 \\ N, & n \ge N \end{cases}$$

   with $N \ge 1$. Specify its ROC.

2. Consider the discrete-time signal $x[n]$:

$$x[n] = (-1)^n u[n] + \alpha^n u[-n-n_0]$$

   Determine the constraints on the complex number α and integer n_0 given that the region of convergence (ROC) of $X(z)$ is $1 < |z| < 2$.

3. Let $h(t)$ be the impulse response of a linear time-invariant (LTI) continuous-time system and it has the form of:

$$h(t) = \begin{cases} e^{-at}, & t \geq 0, \quad a > 0 \\ 0, & t < 0 \end{cases}$$

(a) Determine the Fourier transform of $h(t)$, $H(j\Omega)$.

(b) The $h(t)$ is sampled with a sampling period of T to produce the discrete-time signal $h[n]$. Determine the discrete-time Fourier transform (DTFT) of $h[n]$, $H(e^{j\omega})$.

(c) Find the maximum values for $|H(j\Omega)|$ and $|H(e^{j\omega})|$.

4. When the input to a discrete-time LTI system is

$$x[n] = \left(\frac{1}{2}\right)^n u[n] + 2^n u[-n-1]$$

The corresponding output is

$$y[n] = 6\left(\frac{1}{2}\right)^n u[n] - 6\left(\frac{3}{4}\right)^n u[n]$$

(a) Find the transfer function $H(z)$ of the system and specify its ROC.

(b) Determine the pole(s) and zero(s) of $H(z)$.

(c) Find the impulse response $h[n]$ of the system.

(d) Determine the DTFT of $h[n]$.

(e) Write down the difference equation that relates $x[n]$ and $y[n]$.

(f) Is the system stable? Why?

(g) Is the system causal? Why?