Asymptotic Rate Analysis of Large-Scale Distributed Antenna System (DAS): From Cellular DAS to Virtual-Cell based DAS

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Distributed Antenna System (DAS)

- Originally proposed to cover the dead spots for indoor wireless communication systems.
- Implemented in cellular systems to improve cell coverage.
- Included into the 4G LTE standard.
- Key technology for C-RAN and 5G.
**Base Station (BS) Antennas: Co-located or Distributed?**

<table>
<thead>
<tr>
<th></th>
<th>Co-located Antennas (CA)</th>
<th>Distributed Antennas (DA)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Implementation cost</strong></td>
<td>Lower</td>
<td>How much higher?</td>
</tr>
<tr>
<td><strong>Sum rate</strong></td>
<td></td>
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</tbody>
</table>

*X*: user  
*Y*: BS antenna
Asymptotic Capacity Analysis for Large-Scale MIMO

- Consider a single user with $N$ co-located antennas transmits to $M$ BS antennas. No CSIT.

  ✓ CA: As $M$ and $N$ go to infinity but with a fixed ratio of $\beta$, the ergodic capacity converges to a function of $\beta$ and the mean received SNR.

  ✓ DA:
  
  - Assume that $M$ BS antennas are grouped into $L$ distributed clusters. Each cluster has $U$ co-located antennas.

  - As $U$ and $N$ go to infinity but with a fixed ratio of $\beta$, the asymptotic ergodic capacity is determined by $\beta$ and $L$ mean received SNRs (or equivalently, the large-scale fading gains from the user to $L$ BS antenna clusters).

  - $L+1$ fixed-point equations need to be jointly solved to obtain the asymptotic ergodic capacity.
Asymptotic Capacity Analysis for Large-Scale MIMO

• In general, for a multiuser DAS with K users and L distributed BS antenna clusters, \(L+K\) fixed-point equations need to be jointly solved to calculate the asymptotic ergodic capacity.

• To reduce computational complexity, approximations have been developed to obtain the asymptotic ergodic capacity as an explicit function of \(L\times K\) mean received SNRs (large-scale fading gains) by assuming
  
  o a doubly-regular large-scale fading gain matrix, or
  o \(LU \gg KN\) (\(U\) and \(N\) co-located antennas at each cluster and user, respectively).

✓ Not accurate especially when \(LU \gg KN\) is not satisfied
✓ Errors \(\uparrow\) as \(L\) \(\uparrow\)
Questions to be Answered

• With a large number of distributed BS antenna clusters $L$:
  
  - How does the average ergodic capacity (i.e., averaged over the large-scale fading gains) scale with $L$?
    
    Bounds will be developed which share the same scaling behavior as the average ergodic capacity.
  
  - Does the capacity gain over CA increase with $L$?
    
    Yes, but with cautions.
  
  - Is the cellular structure still suitable?
## Our Work

<table>
<thead>
<tr>
<th>Uplink</th>
<th>Downlink</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Single-user</strong></td>
<td></td>
</tr>
<tr>
<td>[Dai'11]</td>
<td>[Liu-Dai'14]</td>
</tr>
<tr>
<td><strong>Multi-user</strong></td>
<td></td>
</tr>
<tr>
<td>Single-cell</td>
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<tr>
<td>Multi-cell</td>
<td>[Dai'14]</td>
</tr>
<tr>
<td>Virtual-cell</td>
<td>[Dai'14]</td>
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<td>[Wang-Dai'15]</td>
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<td>[Liu-Dai'14]</td>
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<td>[Wang-Dai'16]</td>
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Cellular DAS

Part I: Single-Cell

Assumptions

- K single-antenna users are uniformly distributed within a circular cell.
- L BS antennas are either co-located at the center of the cell, or uniformly distributed over the cell.
- Downlink transmission with equal transmission power among users.
- Linear precoding
  - Nonorthogonal: Maximum Ratio Transmission (MRT)
  - Orthogonal: Zero-forcing Beamforming (ZFBF)
Downlink Asymptotic Average User Rate

- **Achievable ergodic rate of user** $k$: 
  \[ R_k = \mathbb{E}_H \left[ \log_2 \left( 1 + \frac{P_k \|\gamma_k\|^2 \tilde{g}_k w_k w_k^\dagger \tilde{g}_k^\dagger}{N_0 + I_k^{\text{intra}}} \right) \right] \]

- **Average user rate**: 
  \[ \tilde{R} = \mathbb{E}_{\{r_k^U\}_{k \in \mathcal{K}}, \{r_l^B\}_{l \in \mathcal{B}}} [R_k] \]

- **Asymptotic average user rate**: 
  \[ \tilde{R} = \lim_{L,K \to \infty, \ L/K \to \nu} \tilde{R} \]
Downlink Asymptotic Average User Rate

Assume L, K go to infinity and L/K \( \to \nu \).

- MRT
  - CA: \( \tilde{R}^{MC} = \log_2 (1 + \nu) \)
  - DA: \( \tilde{R}^{MD-ub} \sim \frac{\alpha}{2} \log_2 \nu \)
    
    \( \alpha \gg 2 \) is the path-loss factor.
  - Rate gains of DA over CA increase with \( \nu \).
Downlink Asymptotic Average User Rate

Assume $L, K$ go to infinity and $L/K \to \nu$.

- **ZFBF**
  - **CA**: $\tilde{R}^{ZC} = \log_2 \left( \frac{P_t}{N_0} (\nu - 1) \right) + \frac{\alpha}{2} \log_2 e$
  - **DA**: $\tilde{R}^{ZD-lb} = \infty$
  - For given ratio $\nu$, rate gains of DA over CA increase with $L$. 

\[ \tilde{R}^{ZD-lb} \sim \log_2 \frac{(L - K + 1)^{\alpha/2}}{K} \]
Simulation Results

- **DA**
  - Bounds well indicate the scaling behavior of average user rate.
  - Scaling behavior depends on the precoding schemes. Gains over CA are more significant with orthogonal precoding.
Cellular DAS

Part II: Multi-Cell

System Model

• A total number of 7 cells share the same frequency band. No cooperation is adopted among BSs.

• K users are uniformly distributed within each cell, each with N co-located antennas.

• In each cell, M BS antennas are either co-located at the cell center, or divided into L uniformly distributed clusters each with a cluster size of N.

• Linear precoding: Block diagonalization (BD).
### Downlink Asymptotic Average User Rate

- **Achievable ergodic rate of user** $k$:

\[ R_k = \frac{1}{N} \mathbb{E}_{H_{k,B_0}} \left[ \log_2 \det \left( I_N + \frac{\bar{P}_k \| \gamma_{k,B_0} \|^2 \tilde{G}_{k,B_0} W_k W_k^\dagger \tilde{G}_{k,B_0}^\dagger}{N_0 I_N + Q_k^{\text{intra}} + Q_k^{\text{inter}}} \right) \right] \]

- **Average user rate**:

\[ \bar{R} = \mathbb{E} \left\{ r_k^U \right\}_{k \in K}, \left\{ r_l^B \right\}_{l \in B} \left[ R_k \right] \]

- **Asymptotic average user rate**:

\[ \tilde{R} = \lim_{M,N \to \infty, \atop M/N \to L \geq K} \bar{R} \]
Assume $M, N$ go to infinity and $M/N \rightarrow L \geq K$.

- **CA:** $\tilde{R}^{MC} \sim \log_2 \frac{L}{K}$
- **DA:** $\tilde{R}^{MD-lb} \sim \log_2 \frac{(L-K+1)^{\alpha/2}}{K}$

For given ratio $L/K$, rate gains of DA over CA increase with $L$. 
Downlink User Rate versus User Location at Cell Edge

- **CA**: Insensitive to the user’s location at the cell edge.
- **DA**: Significantly larger variance despite higher average rate.

A few “unlucky” users may suffer from strong inter-cell interference if they are close to some BS antennas in the neighboring cells!

Despite gains in average user rate, the cell-edge problem is worsened in the DA case!
Virtual-Cell based DAS

To Cellular or Not to Cellular?

- By splitting a large area into small ones, there are always a certain number of users/BS antennas located at the border and closer to the neighboring cells.

- With distributed BS antennas, the geographic division of cells becomes less justified.
Virtual Cell

- Each user chooses a few surrounding BS antennas as its virtual cell, i.e., its own serving BS antenna set.
- Different from the conventional cellular structure where cells are divided according to the coverage of each BS, here the virtual cell is formed in a user-centric manner.

With virtual-cell based DAS:

- Uniform rate performance among users
- Low CSI measurement overhead
- Scalable signal processing
Virtual-cell based DAS for 5G

- **C-RAN**
  - Scalable signal processing

- **Millimeter wave**
  - Reduced cell coverage

- **Small cells**
  - Cooperation among BSs
**BS Clustering versus Virtual-Cell based User Grouping**

- **BS clustering**
  - Based on the cellular structure where each user is associated with its closest BS.
  - Clusters are formed according to BSs’ locations.

- **Virtual-cell based user grouping**
  - Each user chooses V closest BS antennas to form its virtual cell.
  - Users are grouped together if their virtual cells overlap with each other.
BS Clustering versus Virtual-Cell based User Grouping

- **BS clustering**: Cluster-edge users still suffer from significant rate degradation.

- **Virtual-cell based user grouping**:
  - Rate performance is less sensitive to the user’s location.
  - Both the lowest user rate and the average user rate are improved.
Effect of Virtual-Cell Size $V$ on Average User Rate

- **Without user grouping (MRT is adopted in each user’s virtual cell):**
  - The optimal virtual cell size to maximize the average user rate: $V^* = \left[ 0.2 \frac{L}{K} \right]$
  - A small virtual cell size should be chosen to avoid sharing BS antennas for different user which would otherwise cause strong interference.

- **With user grouping (ZFBF is adopted in each group):**
  - The larger virtual cell size, the more users grouped together, and the lower inter-group interference.
  - The virtual cell size $V$ determines a rate-complexity tradeoff: with a larger $V$, the average user rate is improved at the cost of higher signal processing complexity.
Summary

• In a cellular system:
  - Substantial rate gains can be achieved by DA over CA, and the gains increase with the number of BS antennas due to distinct scaling orders.
  - Gains are more pronounced with orthogonal precoding.
  - Despite a higher downlink average user rate, the rate of each user with DA becomes more sensitive to its position.

• In a virtual-cell based DAS:
  - Uniform rate performance can be achieved among users.
  - The virtual cell size is an important design parameter which should be properly chosen based on the system setting.
Ongoing and Future Work

• Simple and accurate approximations for average rate/capacity
• Optimal network decomposition
• Innovative PHY-layer and MAC-layer designs for future mobile communication systems
The End

Thank you!