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CHAOTIC DYNAMICS OF LASER DIODES WITH STRONGLY MODULATED OPTICAL INJECTION

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In this paper, a modulated optical injection system is proposed. It mainly consists of a semiconductor laser subject to a modulated optical injection. It is known that optical injection will force a laser intensity into high-speed periodic oscillation without modulation, while the oscillation will be locked with weak external modulation. However, the route to chaos in such a system is observed and reported for the first time. It is shown that the generated oscillation frequency shifts away from the external frequency when the modulation amplitude becomes sufficiently large. Due to the nonlinearities of the laser, the frequencies are mixed inside the laser cavity, which causes the dynamics to follow a quasi-periodic route to chaos. In addition, the resultant chaotic signal possesses a larger bandwidth than that attainable with constant optical injection alone. Since the system is capable of generating both narrowband photonic microwave signals with low phase variances and chaotic signals with large bandwidths, it is practically important for Doppler and chaotic lidar applications.

Keywords: Chaos; modulation; optical injection; semiconductor laser.

1. Introduction

The complex nonlinear dynamics of semiconductor lasers have aroused a lot of interest for many years, not only due to its rich variety of dynamics but also its practical applications in communications. As reported in [Dente *et al.*, 1988; Eriksson & Lindberg, 2001; Lariontsev, 2000; Green & Krauskopf, 2003], semiconductor lasers exhibit versatile interesting dynamical behaviors, including continuous-wave, periodic oscillations, regular pulsations, quasi-periodic pulsations, frequency-locked pulsations, chaotic oscillations, chaotic pulsations and so on, under different external perturbations.

Recently, the dynamics of semiconductor lasers subject to constant optical injection have been widely explored for bandwidth enhancement and noise reduction. It is also reported that an injected laser can be driven into a period-one oscillation state to generate a periodically modulated waveform. The waveform generally has a high modulation depth and the oscillation frequency can be widely tuned even beyond the original relaxation oscillation frequency of the laser [Simpson & Doft, 1999; Nizette *et al.*, 2001; Chan & Liu, 2004; Chan *et al.*, 2007].

A properly injected laser can be regarded as a simple tunable photonic microwave source. To

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reduce the associated phase noise of the oscillation, an addition of periodic current modulation from an external microwave source has been suggested. This method is known as the double-lock technique for high-quality photonic microwave generation [Simpson & Doft, 1999]. When the external modulation is weak, the fluctuation reduces as the depth of the modulation increases. Investigations on the *microwave* linewidth-narrowing, locking-range, and bistable behaviors have been reported [Simpson & Doft, 1999; Chan & Liu, 2004; Nizette *et al.*, 2002], while the mechanisms of double-locking under weak periodic modulations have also been explained in [Nizette *et al.*, 2001; Nizette *et al.*, 2002].

On the other hand, from the nonlinear dynamical point of view, it is common to observe quasi-periodicity, frequency locking, and broadband chaos when a system with a natural frequency is driven at an external frequency [Zhao, 1992]. Therefore, the complicated effects of strong modulation is the focus of this paper. A new modulated optical injection system is proposed, in which optical injection is amplitude-modulated by a pure microwave source. It is found that the laser follows a quasi-periodic route to chaos as the modulation depth increases. Unlike other injection schemes, such as those using chaotically modulated signal [Uchida *et al.*, 2003] or using simultaneous injections of two independent lasers [Al-Hosiny *et al.*, 2007], the proposed system is simpler and it is capable of generating very stable narrowband signals as well as very broadband signals. Moreover, the resultant chaotic bandwidth is found to be larger than that attainable with a constant optical injection alone. These kinds of signals are useful for improving chaotic lidar [Lin & Liu, 2004] and Doppler lidar measurements [Diaz *et al.*, 2002].

2. Modulated Optical Injection System

In this paper, a modulated optical injection system, as in Fig. 1, is configured and studied. The master laser (ML) optically injects into the slave laser (SL) through an optical isolator (OI), a mirror (M), and a beam splitter (BS). Both the injection strength ξ_i and the injection frequency detuning f_i are adjustable. Here, the injection strength is a normalized field quantity and the detuning is the optical frequency difference between the two lasers [Chan *et al.*, 2007]. The power spectrum of the slave laser is monitored by a photodetector (PD). Unlike the conventional optical injection system, an optical intensity modulator (MOD) is inserted into the optical path so that the injection strength is modulated as $\xi_i[1 + (a_m e^{-i2\pi f_m t} + \text{c.c.})]$, where a_m and f_m are the modulation amplitude and the modulation frequency, respectively. The c.c. stands for complex conjugate.

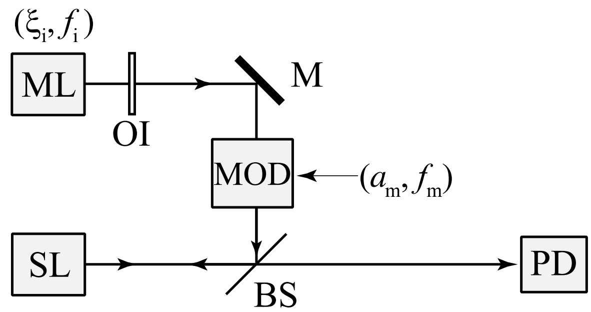


Fig. 1. Schematic of the modulated optical injection system. ML: master laser; SL: slave laser; OI: optical isolator; MOD: optical modulator; M: mirror; BS: beamsplitter; and PD: photodetector. The optical injection strength ξ_i and injection frequency detuning f_i are controlled at ML. The modulation amplitude a_m and modulation frequency f_m of the injection are controlled at MOD.

By including the modulation terms into a model of constant injection [Chan *et al.*, 2007], one can obtain the following rate equation model for the system in Fig. 1:

$$\frac{da_r}{dt} = \frac{1}{2}\left[\frac{\gamma_c\gamma_n}{\gamma_s\tilde{J}}\tilde{n} - \gamma_p(a_r^2 + a_i^2 - 1)\right](a_r + ba_i) + \xi_i[1 + (a_m e^{-i2\pi f_m t} + \text{c.c.})]\gamma_c\cos(2\pi f_i t) \quad (1)$$

$$\frac{da_i}{dt} = \frac{1}{2}\left[\frac{\gamma_c\gamma_n}{\gamma_s\tilde{J}}\tilde{n} - \gamma_p(a_r^2 + a_i^2 - 1)\right](-ba_r + a_i) - \xi_i[1 + (a_m e^{-i2\pi f_m t} + \text{c.c.})]\gamma_c\sin(2\pi f_i t) \quad (2)$$

$$\frac{d\tilde{n}}{dt} = -[\gamma_s + \gamma_n(a_r^2 + a_i^2)]\tilde{n} - \gamma_s\tilde{J}(a_r^2 + a_i^2 - 1) + \frac{\gamma_s\gamma_p}{\gamma_c}\tilde{J}(a_r^2 + a_i^2)(a_r^2 + a_i^2 - 1) \quad (3)$$

where $(a_r + ia_i)$ is the normalized complex intracavity electric field, $\tilde{n}$ is the normalized charge carrier density, γ_c is the cavity decay rate, γ_s is the spontaneous carrier relaxation rate, γ_n is the differential carrier relaxation rate, γ_p is the nonlinear carrier relaxation rate, b is the linewidth enhancement factor, and $\tilde{J}$ is the normalized bias above the threshold current.

Referring to the experimental characterizations of a typical laser [Hwang *et al.*, 2004], the following parameters are adopted for the simulations in this work: $\gamma_c = 5.36 \times 10^{11}\,\mathrm{s}^{-1}$; $\gamma_s = 5.96 \times 10^9\,\mathrm{s}^{-1}$; $\gamma_n = 7.53 \times 10^9\,\mathrm{s}^{-1}$; $\gamma_p = 1.91 \times 10^{10}\,\mathrm{s}^{-1}$; $b = 3.2$, and $\tilde{J} = 1.222$. The slave laser emits 4.5 mW when free-running and its relaxation oscillation frequency f_r is 10.25 GHz. The system output is the optical intensity $a_r^2 + a_i^2$ measured by the photodetector.

3. Results and Discussions

For the case of optical injection without modulation, the slave laser can be stably locked by the master laser with sufficiently high injection strength. It is known that, by properly adjusting (ξ_i, f_i), the slave laser exhibits periodic oscillation through undamping of the relaxation oscillation. Equivalently, the laser is said to be in a nonlinear dynamical period-one oscillation state through a Hopf bifurcation [Simpson *et al.*, 1996].

In general, the oscillation frequency f_0 can be tuned by varying the injection parameters. For demonstrative purpose, the injection is kept constant at $(\xi_i, f_i) = (0.06, 20.00\,\mathrm{GHz})$, where the period-one oscillation is generated at $f_0 = 21.545$ GHz. Figure 2 shows the power spectra and the time series obtained when the modulation amplitude a_m is varied from 0 to 0.9. The modulation frequency is chosen to be $f_m = f_0/2 = 10.7725$ GHz. Such subharmonic modulation has shown to be capable of frequency stabilization [Simpson *et al.*, 1997]. It is preferred over modulation at the fundamental due to practical reasons such as the costs of microwave sources and optical modulators.

When no modulation is applied, i.e. $a_m = 0$, period-one oscillation is obtained at $f_0 = 21.545$ GHz, as shown in Fig. 2(a). To simulate a realistic case with spontaneous emission noise, the Langevin terms are incorporated into the simulations [Hwang *et al.*, 2004]. As observed in Fig. 2(a), the resultant output background noise is enhanced around the subharmonic frequency $f_0/2$. Nonetheless, it is still very weak as compared with the main peak at f_0 so that the corresponding time series shows a nearly sinusoidal oscillation. The quality of the oscillation is quantified by the associated statistical phase variance. It is estimated to be $0.59\,\mathrm{rad}^2$ by integrating the normalized single sideband noise power.

A weak external modulation is then applied to lock the generated oscillation. The quality of the oscillation improves as the modulation amplitude increases. For example, Fig. 2(b) shows the signal at $a_m = 0.17$. The oscillation is locked to the second harmonic of the external microwave source, which is observed as the narrowing of the linewidth at f_0. The improvement of signal quality is quantified by a reduction of phase variance to $0.029\,\mathrm{rad}^2$, well demonstrating the double-lock technique for high-quality photonic microwave generation.

However, when a_m is increased further, the reduction of phase noise does not continue and the system enters into some other dynamical states. Figure 2(c) shows that, under a strong modulation amplitude of $a_m = 0.50$, the period-one oscillation frequency is shifted to about 22.58 GHz. The oscillation is no longer locked by the modulation at $f_m = 10.7725$ GHz. Instead, the oscillation frequency and the modulation frequency are mixed through the laser nonlinearities to produce a much more complicated spectrum. Since the frequencies are incommensurate, a quasi-periodic state is reached. When a_m increases to 0.74, as shown in Fig. 2(d), the frequency mixing is even more pronounced and the peaks are broadened.

As a_m reaches 0.90, the state becomes chaotic as shown by the broadband output in Fig. 2(e). The power spectrum consists of a broadband background from the frequency mixings and a few narrow peaks at the multiples of f_m. The associated time series exhibits random-like oscillations that are characteristic of chaotic signals. The existence of chaos is further confirmed by the calculation of Lyapunov exponents, which are $0.012\gamma_c, 0$, and $-0.055\gamma_c$. The positive largest Lyapunov exponent signifies that the state is chaotic.

Therefore, the evolution of states in Fig. 2 demonstrates that the double-lock technique works for the proposed system when the modulation is weak. As the modulation amplitude increases, the system clearly follows a quasi-periodic route to chaos. Broadband chaos can be generated through nonlinear frequency mixings when the modulation is sufficiently strong.

3.1. *The route to chaos*

The route to chaos is more clearly shown by the portraits of peak sequences given in Fig. 3. Denoting the nth peak of the intensity time series as $P(n)$, consecutive intensity peaks $P(n)$ verses $P(n + 1)$ are plotted to determine the dynamical states. For clarity, the Langevin noise is omitted in this figure. The portrait of the original period-one oscillation

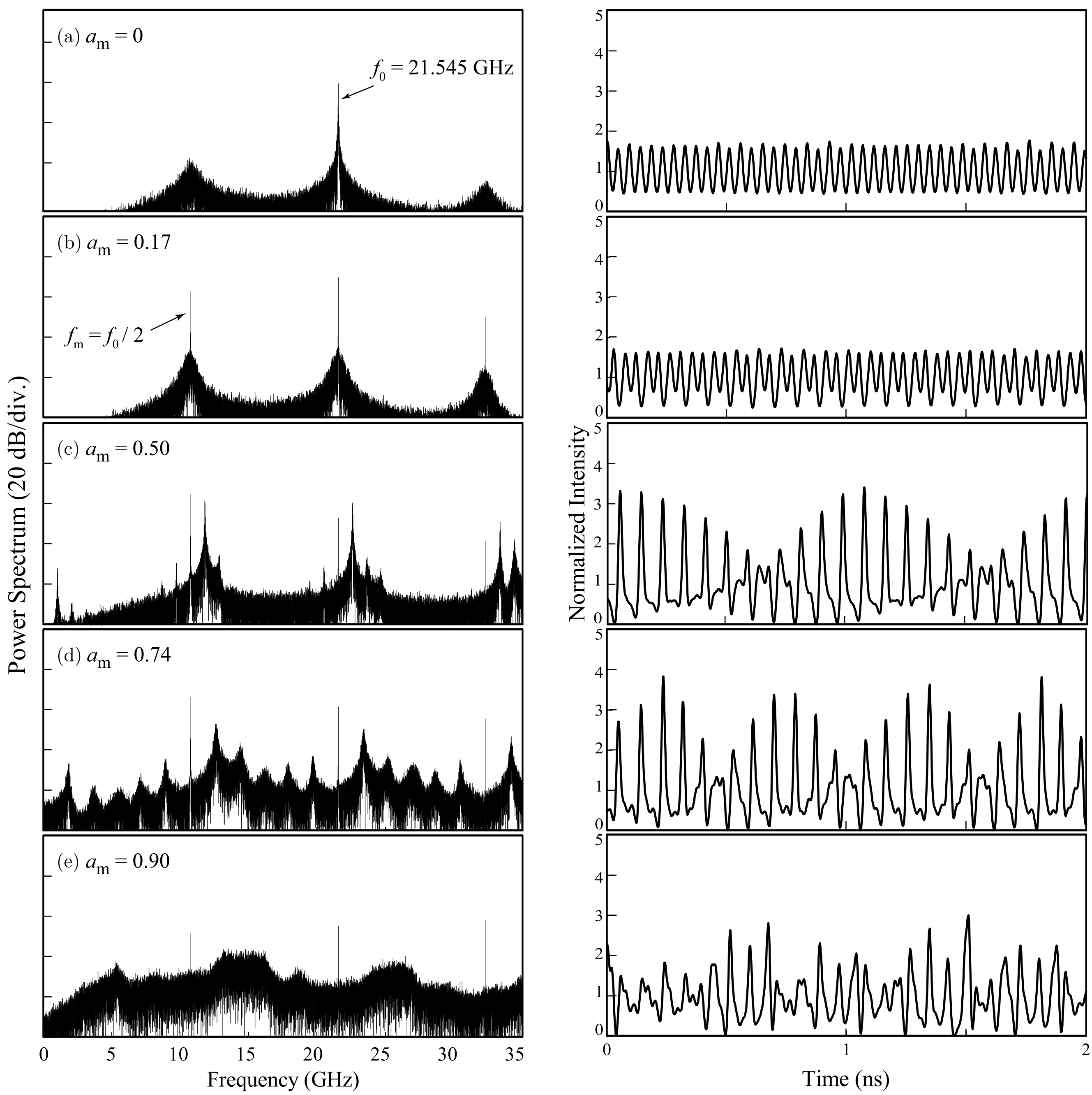


Fig. 2. Power spectra and time series of the slave laser under injection of $(\xi_i, f_i) = (0.06, 20.00\,\text{GHz})$. (a) The slave laser enters period-one oscillation at $f_0 = 21.545\,\text{GHz}$ when there is no modulation on the injection. (b)–(e) The injection is modulated at $f_m = f_0/2$ as the modulation amplitude a_m increases. The respective dynamical states are (a) free period-one, (b) locked period-one, (c), (d) quasi-periodic, and (e) chaos.

for $a_m = 0$ is shown in Fig. 3(a). As expected, only a single dot is observed because the intensity peaks of the period-one oscillation are uniform and the laser remains in the period-one state. The portrait remains nearly the same when a_m increases to 0.17 in Fig. 3(b), as the modulation at this amplitude merely helps to reduce the phase variance of the oscillation.

However, when a_m is increased to 0.50, a loop is formed in the portrait as given in Fig. 3(c). This signifies that the system is in the two-frequency quasi-periodic state, where the peak series is itself oscillating with a long period. As a_m increases to 0.72, the loop becomes very much distorted as shown in Fig. 3(d). Although a torus structure cannot be clearly identified, the associated power

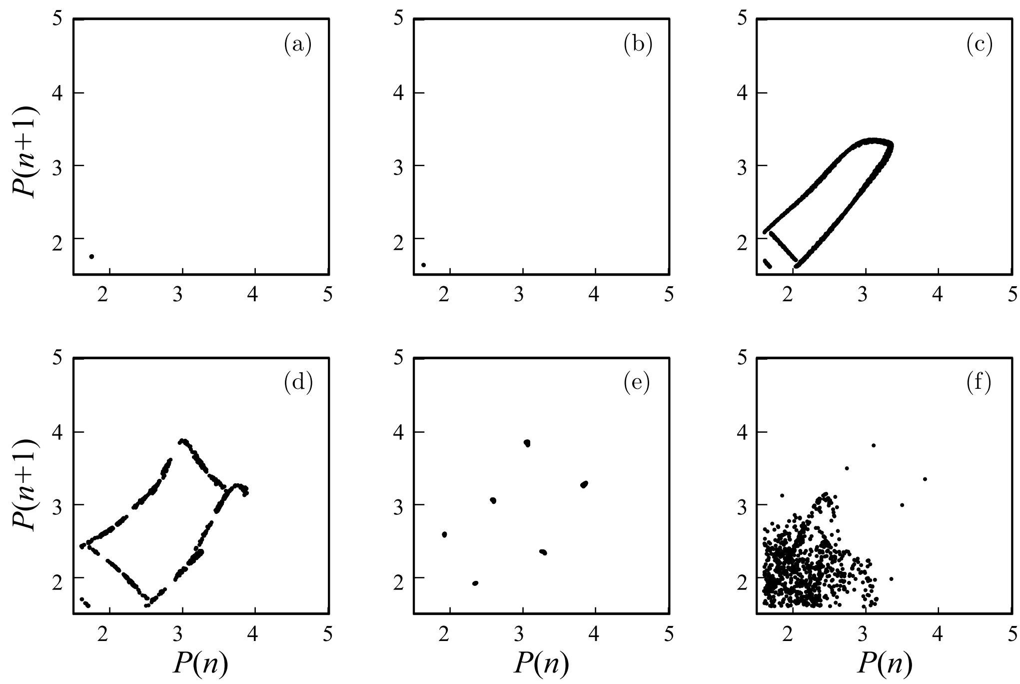


Fig. 3. Portraits of the peak series as $a_{\rm m}$ varies. The states are (a) free period-one, (b) locked period-one, (c) two-frequency quasi-periodic, (d) three-frequency quasi-periodic, (e) frequency locking, and (f) chaos; where $a_{\rm m}$ = 0, 0.17, 0.50, 0.72, 0.74, and 0.90, respectively.

spectrum (without noise) confirms that the state is three-frequency quasi-periodic. Moreover, similar to many nonlinear systems that follow the quasi-periodic route to chaos, we can identify a few ranges of $a_{\rm m}$ that result in the frequency-locked state. Figure 3(e) shows such a case at $a_{\rm m} = 0.74$. A frequency-locked state of 1:6 is observed from the portrait of 6 dots. Although the frequency-locked states are readily observed without noise, they are not as easily identified in the presence of noise as the portraits are blurred.

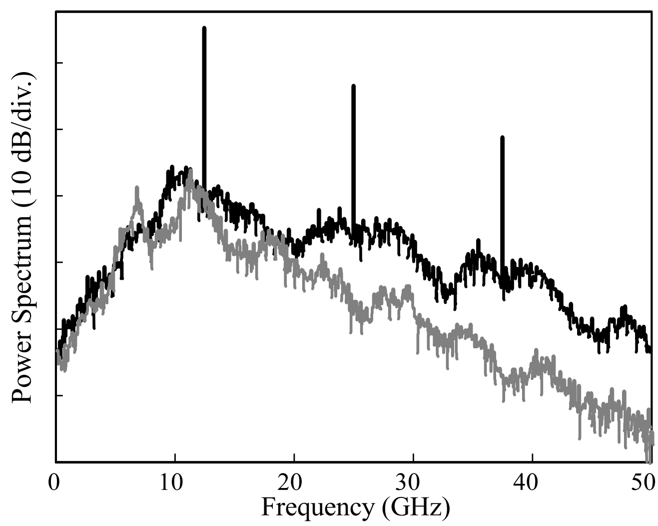
Finally, when $a_{\rm m}$ is further increased to 0.90, Fig. 3(f) shows a very complicated portrait signifying chaotic oscillations. As a conclusion, the system follows a quasi-periodic route to chaos while windows of frequency-locked states are found occasionally.

3.2. *Signal characteristics*

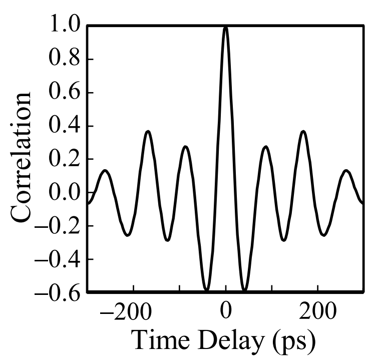
The chaos obtained by the proposed system (as given in Fig. 1) possesses a wider bandwidth than the conventional system without modulation. A detailed comparison has been carried out. Figure 4(a) depicts the power spectra for systems with and without modulation using the same slave laser. The dark curve represents the power spectrum obtained under modulated injection at ($\xi_{\rm i}$, $f_{\rm i}$, $a_{\rm m}$, $f_{\rm m}$) = (0.05, 17.50 GHz, 1.0, 12.50 GHz). Several narrow peaks at multiples of $f_{\rm m}$ are noticed, but they contribute to only 13% of the total power contained in the spectrum. The gray curve is obtained under injection of ($\xi_{\rm i}$, $f_{\rm i}$) = $(0.05, 8.75\,\text{GHz})$ without modulation. The parameters are optimized for maximizing the chaotic bandwidth under constant optical injection. Both states are verified to be chaotic as their largest Lyapunov exponents are both positive. The exponents are $0.018\gamma_{\rm c}$ and $0.015\gamma_{\rm c}$ with and without modulation, respectively.

For comparison, both spectra in Fig. 4(a) are normalized to their chaotic peaks. It is observed that the bandwidth of the signal generated with modulation is much larger than that generated without modulation. To quantify the observation, the signal bandwidth B is calculated for both cases,[1] where $B = 27.4$ GHz and 18.4 GHz with and without modulation, respectively.

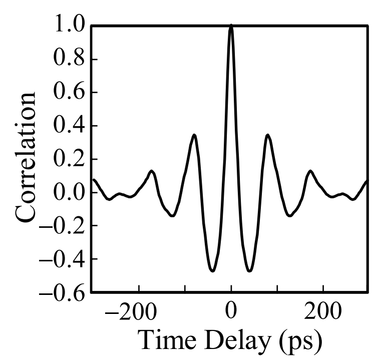
[1]Due to the irregular shapes of the chaotic spectra, B is defined as the frequency that 90% of the spectral power is contained below B.



(a)



(b)



(c)

Fig. 4. Effects of injection modulation. (a) Power spectra of the slave laser with modulated injection (dark curve) and constant injection (gray curve). Modulated injection gives a larger chaotic bandwidth. (b) Autocorrelation of output intensity under constant injection. (c) Autocorrelation of output intensity under modulated injection.

The broadening of chaotic bandwidth is further investigated by performing autocorrelations on the corresponding intensity time series, as given in Figs. 4(b) and 4(c). By comparison, the modulated system has a narrower autocorrelation peak and smaller adjacent sidelobes. The central peak has a full width at half-maximum (FWHM) of 24 ps in Fig. 4(b), where it is reduced to 18 ps in Fig. 4(c). The results show that strong external modulation can increase the chaotic bandwidth of the optical injection system.

The broadening of chaotic bandwidth has also been measured under various operating conditions. Figure 5 shows the chaotic bandwidth B as a function of the injection frequency detuning f_{i}. The injection strength ξ_{i} is kept at 0.05 and the open circles show the data obtained without external modulation. The gray triangles, gray squares, dark triangles, and dark squares represent the data obtained with modulation $(a_{\mathrm{m}}, f_{\mathrm{m}}) = (0.5$, 10.72 GHz), (0.5, 12.50 GHz), (1.0, 10.72 GHz), and (1.0, 12.50 GHz), respectively.

Since chaos is observed only within a limited range of operating conditions, the data points are not linked continuously and do not cover the whole range of f_{i}. For instance, without modulation, chaos only exists for a range of f_{i} between 4.5 GHz and 9 GHz. However, we observe that the range is significantly increased when modulation is applied. Broadening of chaotic bandwidth is generally observed for different operating conditions.

Therefore, the proposed modulation scheme is useful for improving the precision of chaotic lidar systems, which require broadband signals for

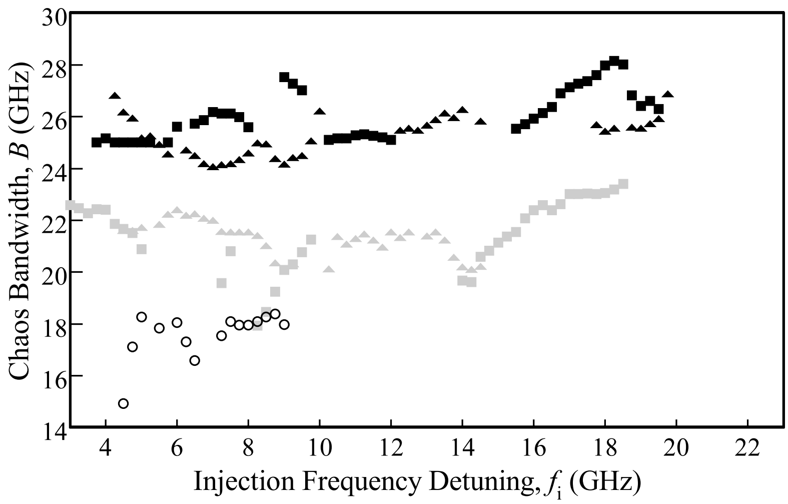


Fig. 5. Chaos bandwidth broadening by strong modulation over a range of f_i. The modulation parameters (a_m, f_m) are varied: (a) open circles (0,−), (b) gray triangles (0.5, 10.72 GHz), (c) gray squares (0.5, 12.50 GHz), (d) dark triangles (1.0, 10.72 GHz), and (e) dark squares (1.0, 12.50 GHz).

autocorrelation. The associated narrow spectral peaks of the spectra generated from the proposed system are also potentially applicable to Doppler lidar experiments [Lin & Liu, 2004; Diaz *et al.*, 2002].

4. Conclusions

In this paper, the observation of chaos in a strongly modulated optical injection system is reported. By varying the modulation amplitude, the quasi-periodic route to chaos is identified and explained. Based on the proposed system, signals with high-quality narrowband microwave oscillation or broadband chaos can be easily obtained through proper adjustments of the modulation parameters. The system offers a simple method of generating broadband chaotic signals with increased bandwidths over the constant injection systems, which are useful for chaotic and Doppler lidar applications.

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