A Power Wave Theory of Antennas

Everett G. Farr
Farr Fields, LC

City University of Hong Kong
August 26, 2014
Hong Kong

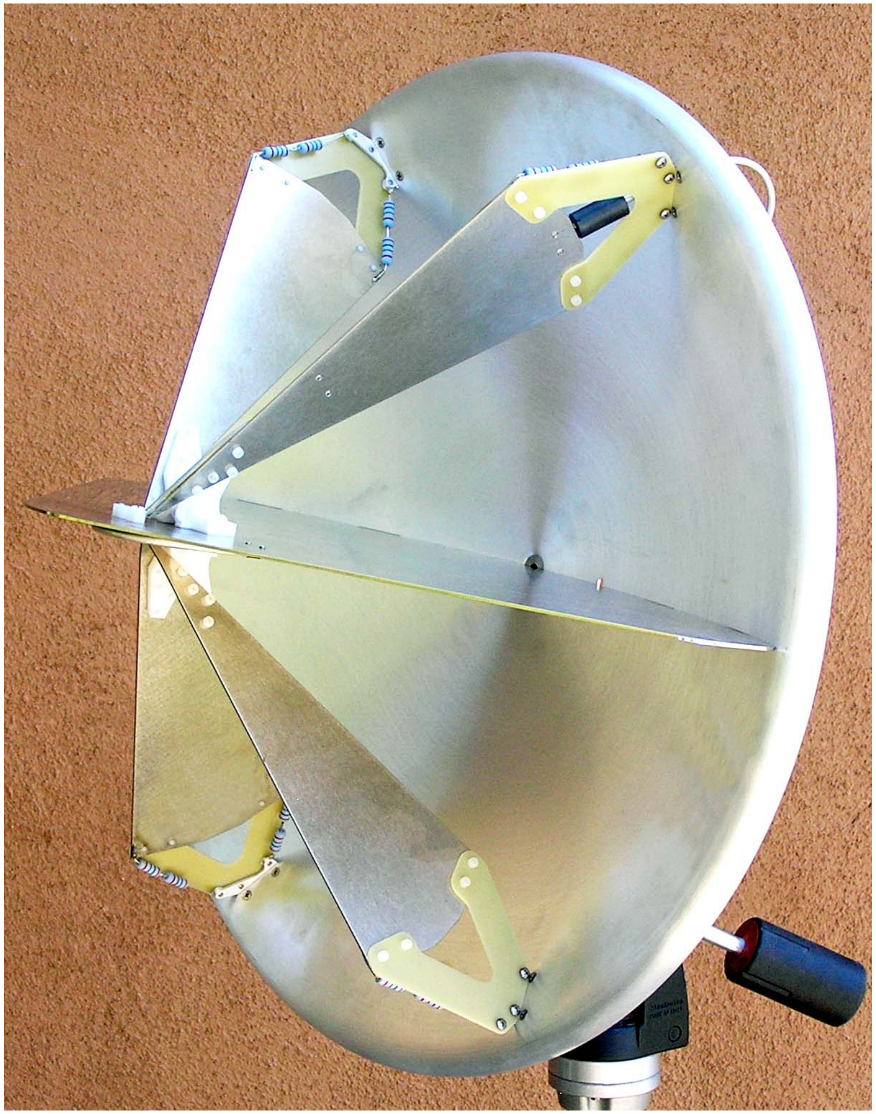
Overview

Part 1: Some UWB Antennas We've Worked On

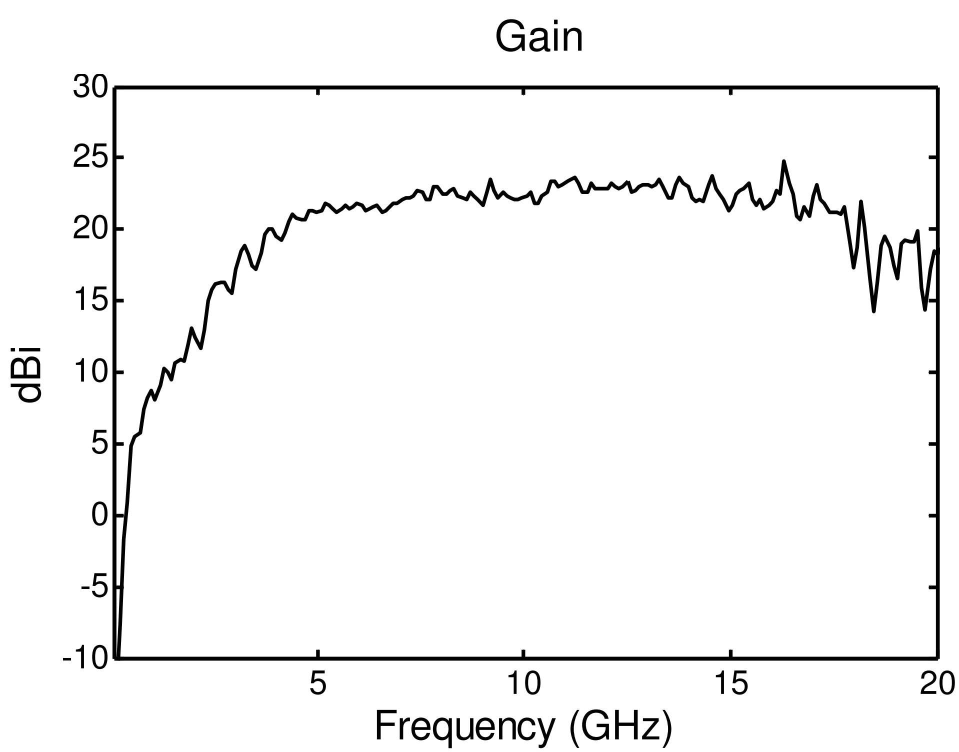
Part 2: The Power Wave Theory of Antennas

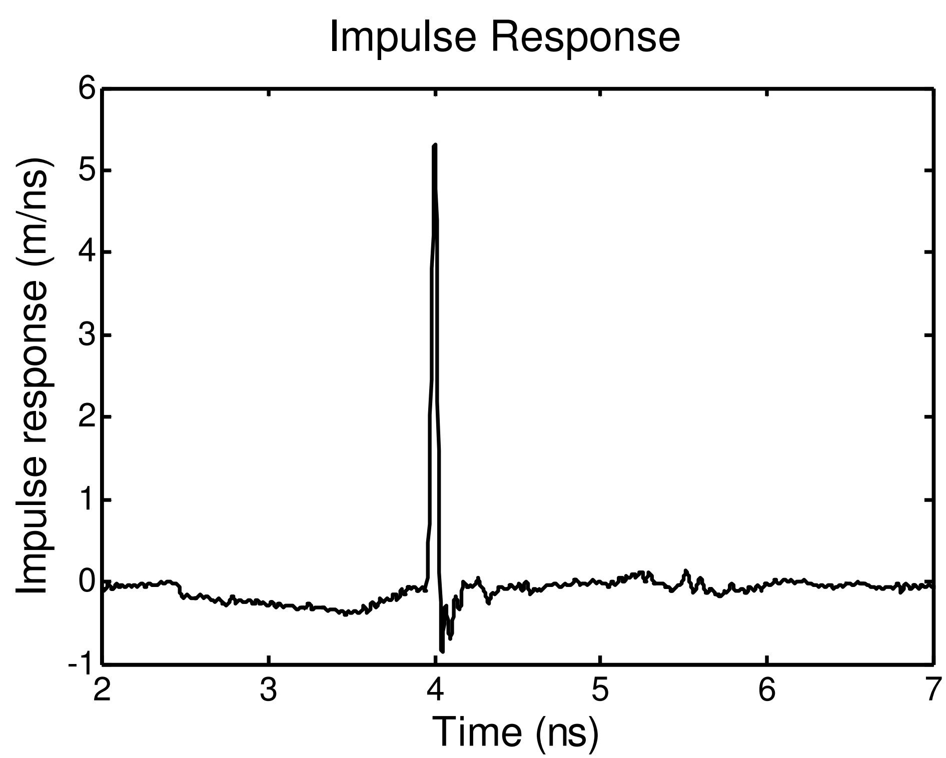
Example of UWB Antenna: IRA-3Q

- Diameter: 18 in. (46 cm)
- Radiates a Clean Impulse, with FWHM = 38 ps.
- Frequency range 250 MHz – 20 GHz.
- Excellent impedance match across entire frequency range.



Data for the IRA-3Q



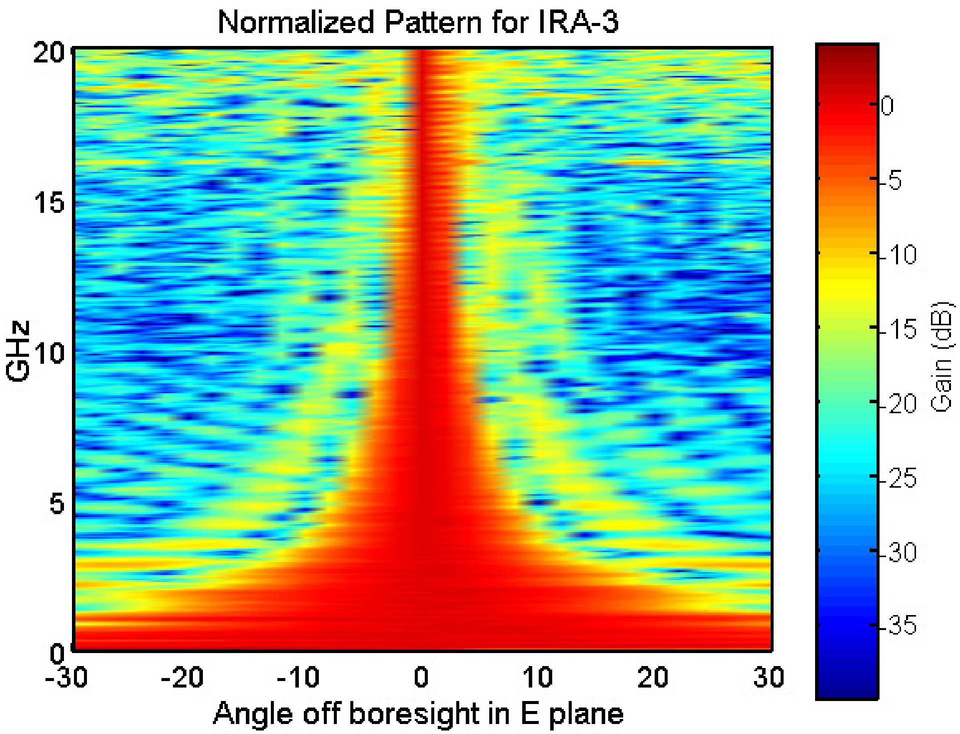


Applications of IRAs

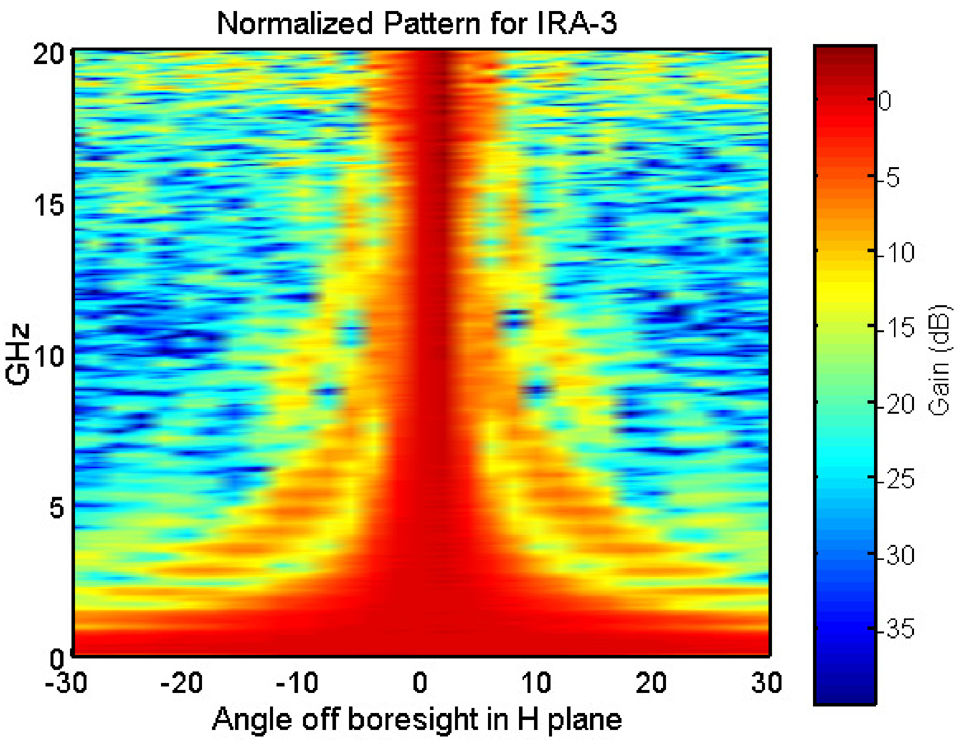
- Broadband EMC/EMI or RCS testing with single antenna
- Intentional EMI
- Impulse Radar to locate weapons, tanks under trees, mines, or unexploded ordnance
- Broadband communications or surveillance

Normalized Antenna Pattern
IRA-3M

E-Plane



H-Plane



Radome on IRA

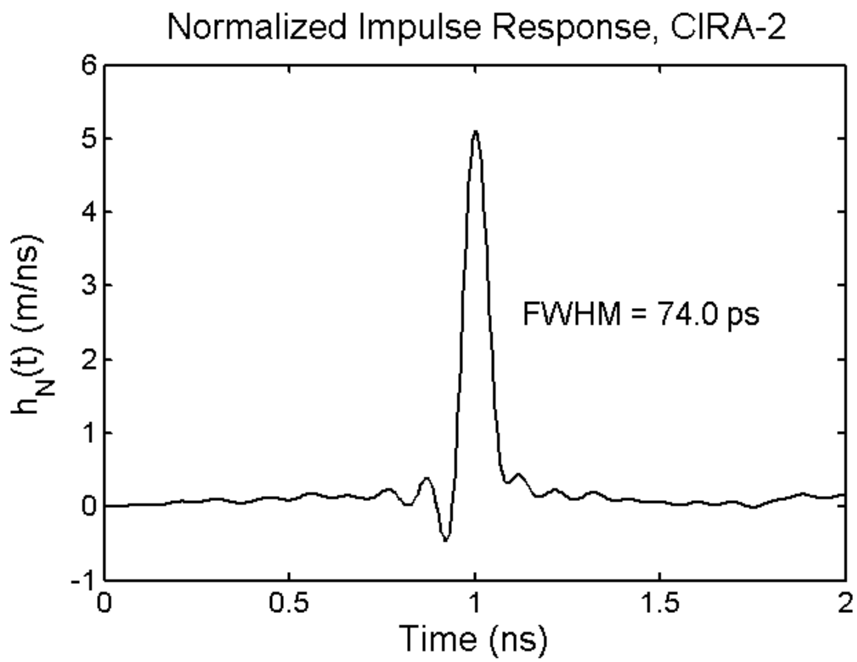
IRA-3

Collapsible IRA

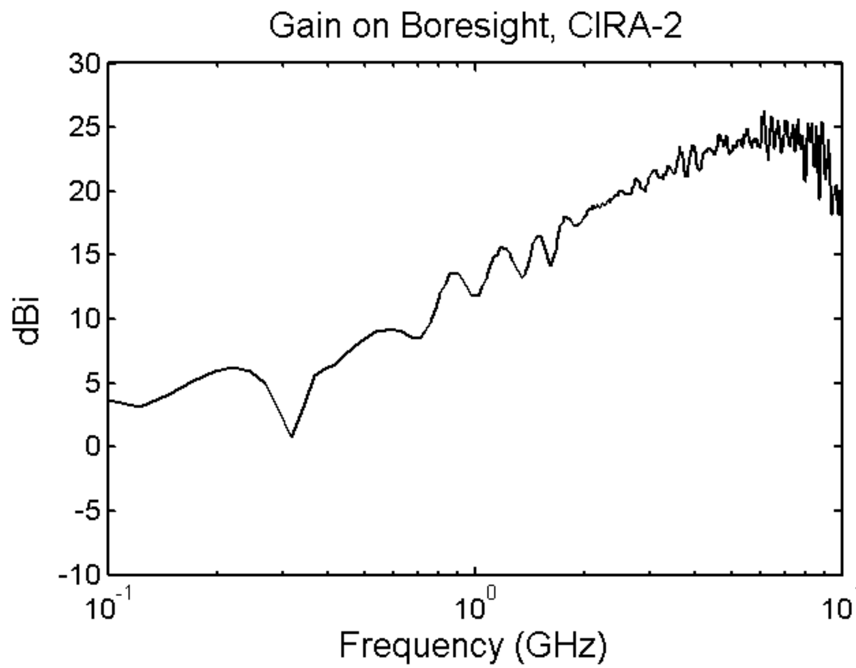
- Compact, Lightweight, rapidly deployable design
- Metallized nylon and resistive fabric
- When collapsed: Length=81 cm, Diam=10 cm
- Suitable for broadband communications in field
- Impulse response FWHM = 70 ps
- Peak G_r = 23 dB at 4 GHz
- Useful from 150 MHz to 8 GHz

CIRA-2 Data

Impulse Response

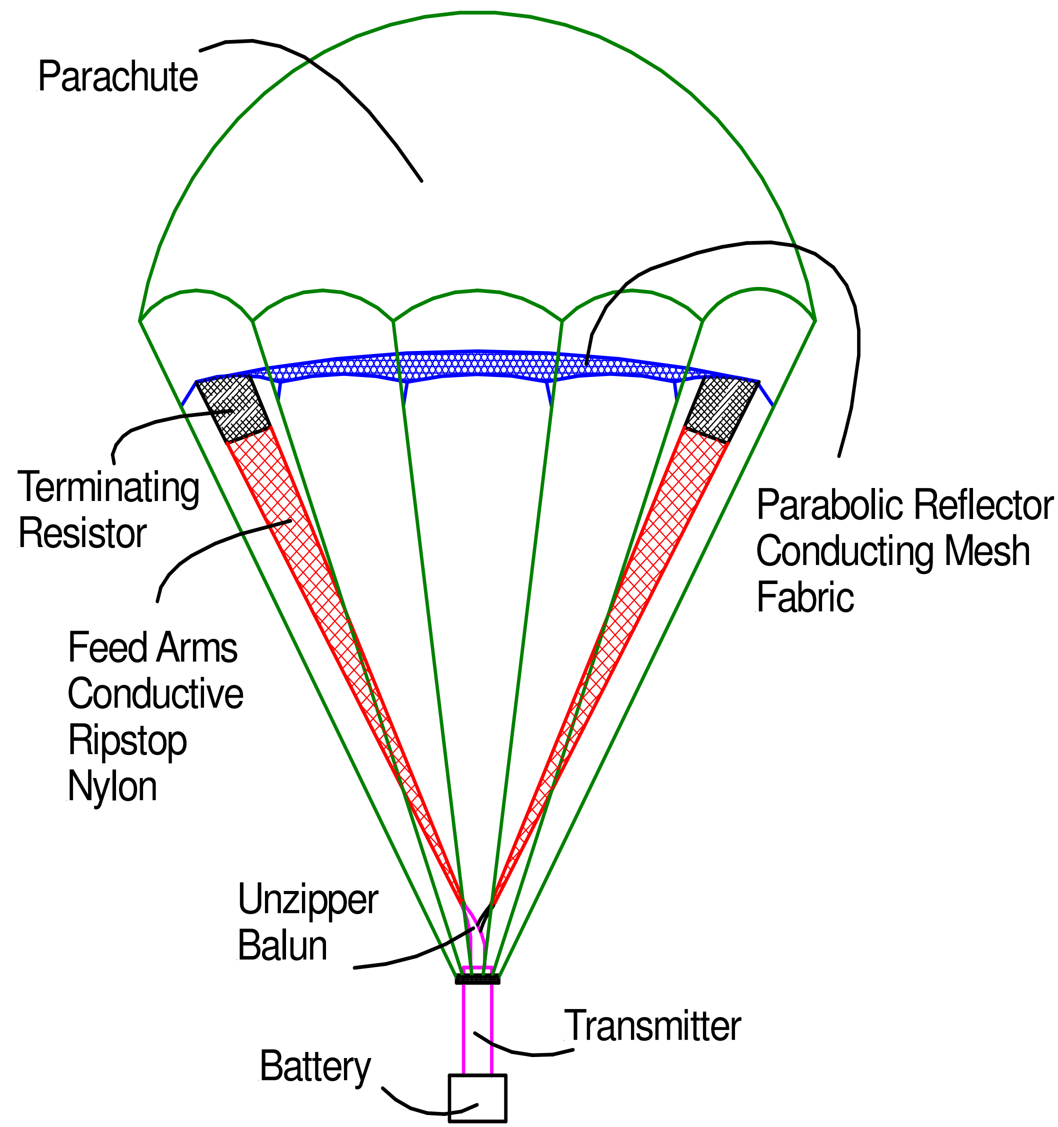


Gain

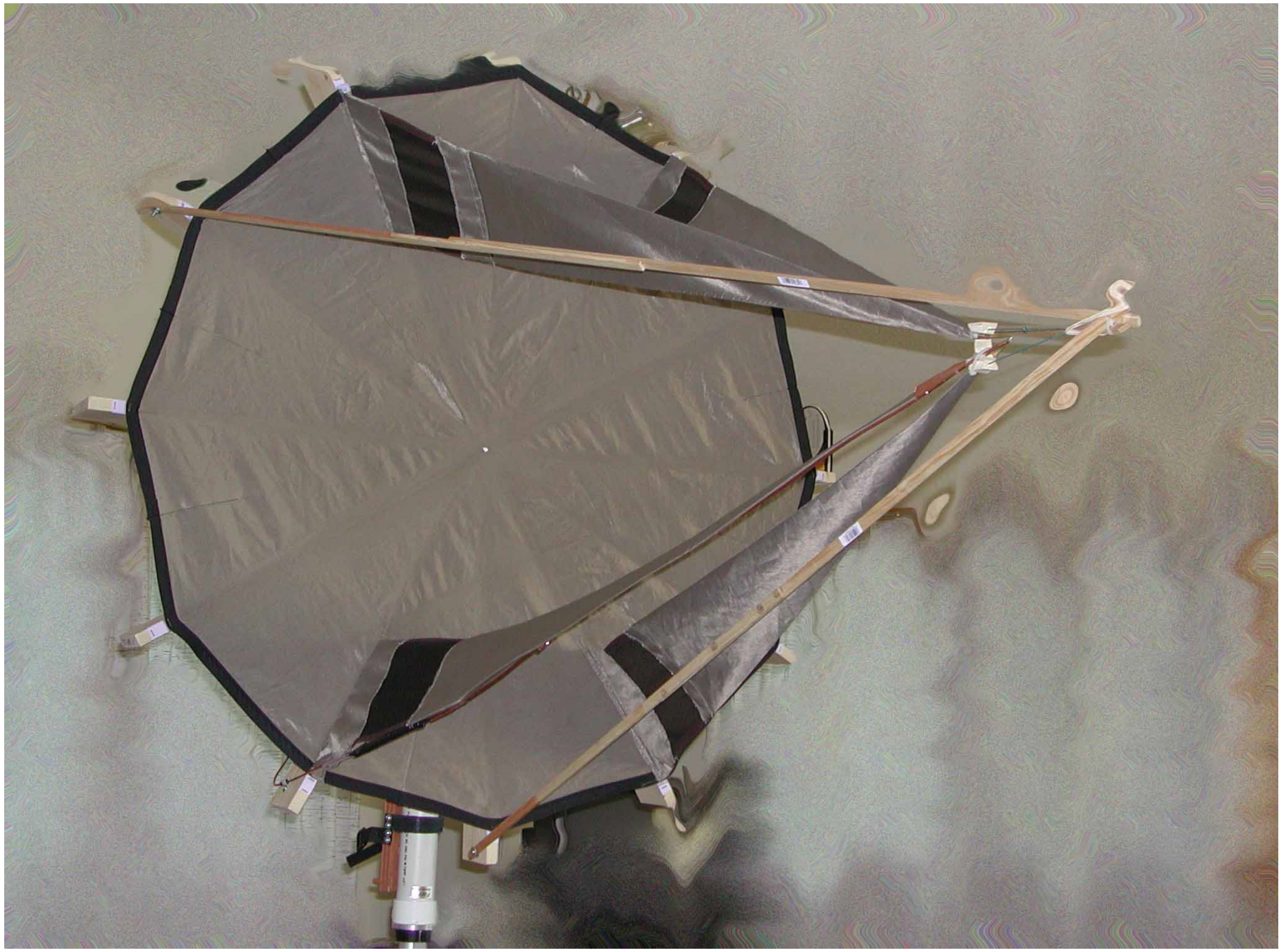


Para-IRA Concept

- **Para**chute Delivered
- Impulse Radiating Antenna
- Goal is to Illuminate 100-Meter Radius Area with a Wideband Pulse
- Parachute Allows Rapid and Flexible Deployment



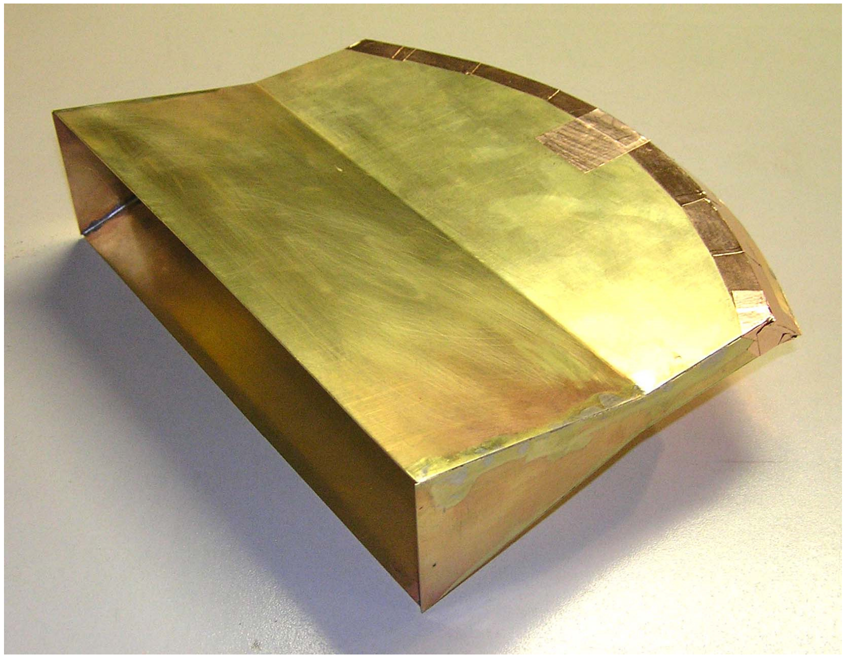
Phase I Antenna Mounted Onto Frame for Testing

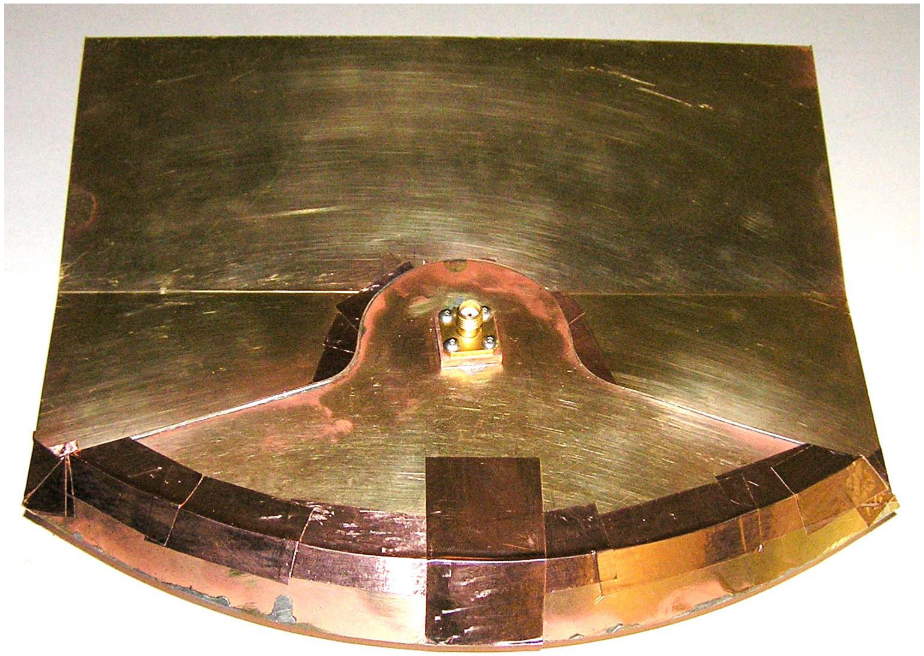


Phase I Tow Test Results

- Measure force on scale to correlate terminal velocity with weight
- Descent Rate Results: A 20 pound package falls at 58 kph

Folded Horn Antenna

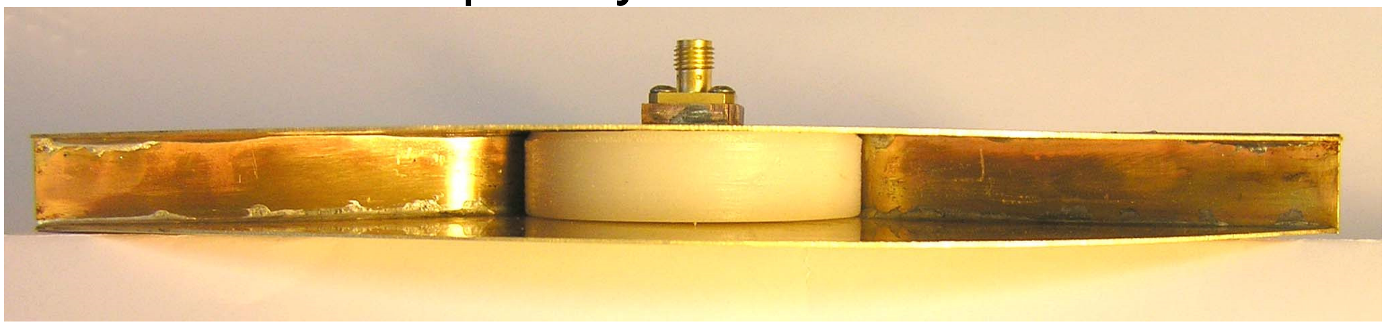




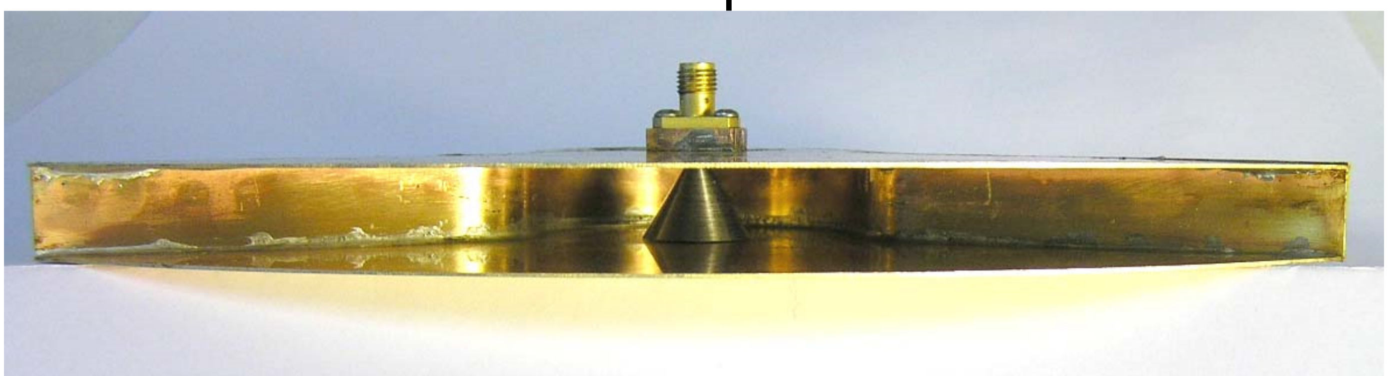
- Useful for medium bandwidth (3-5 GHz) at high power
- Could be scaled X10 to reach 300-500 MHz, and mounted onto truck.
- Nearly flat phase front in aperture

Feed Point Modifications in FH-1E

- Add dielectric disk: Simulates oil tank near feed, and shifts the dip in S_{11} to lower frequency



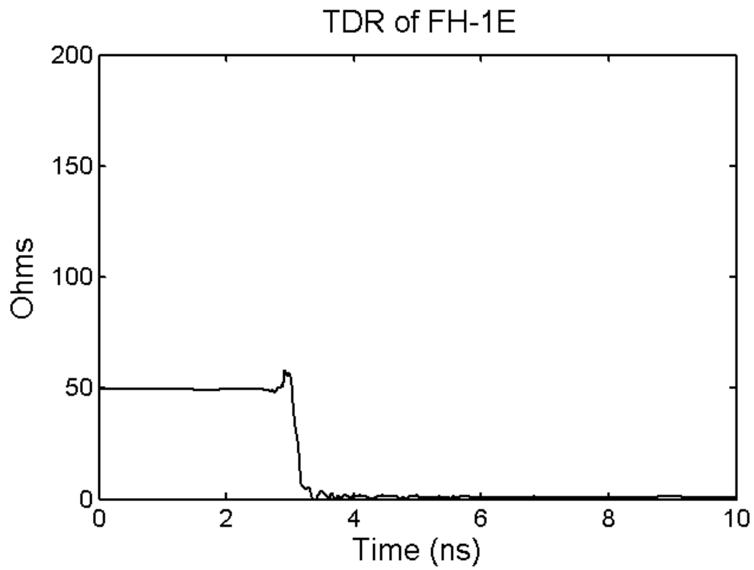
- Add cone: to maintain 50-Ω impedance

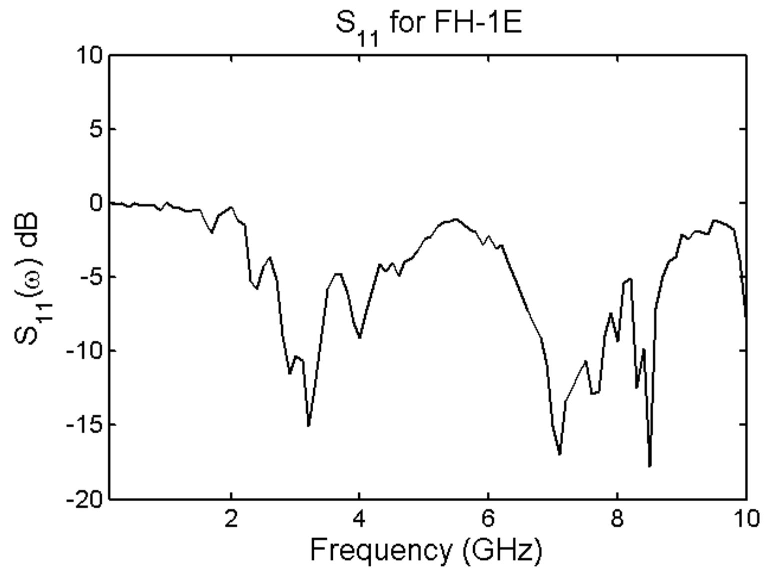


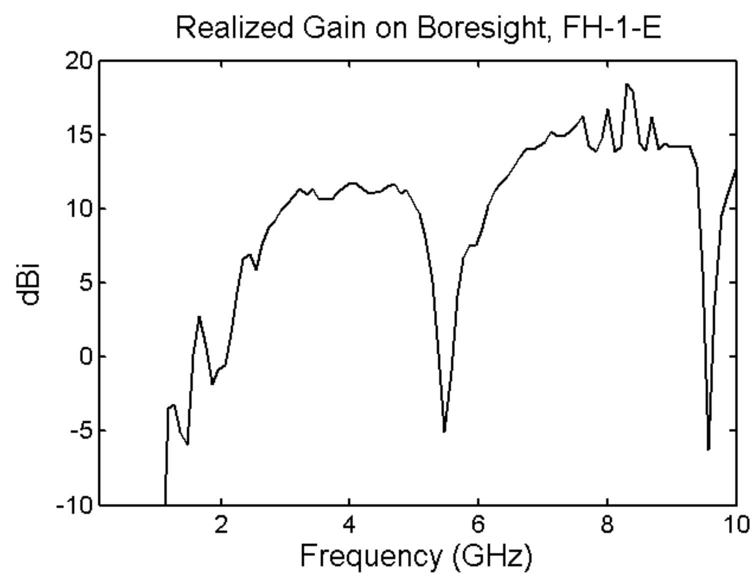
$$Z_o = \frac{\eta}{2\pi} \ln\left(\cot \theta_h / 2\right)$$

- We needed both!

Data on the Optimized Feed Horn, FH-1E



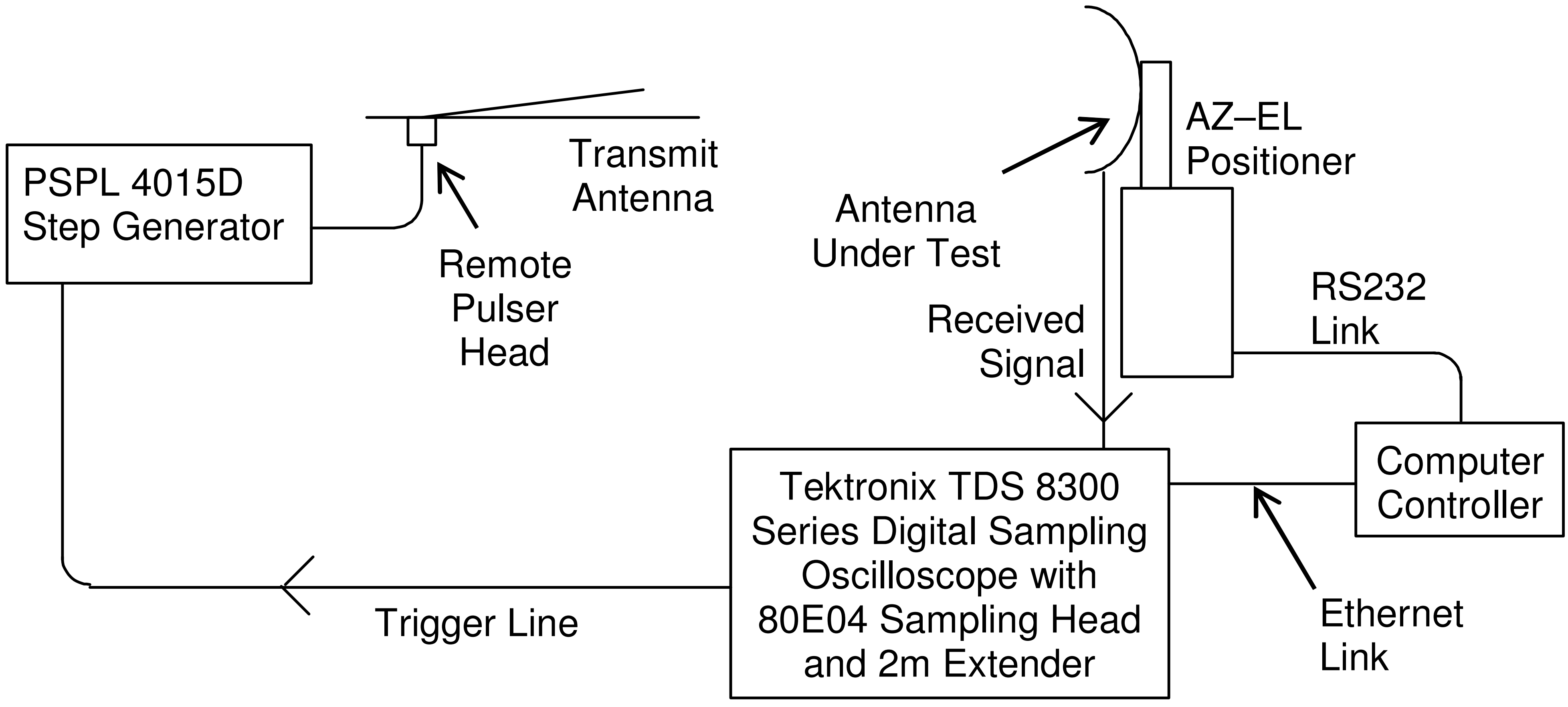




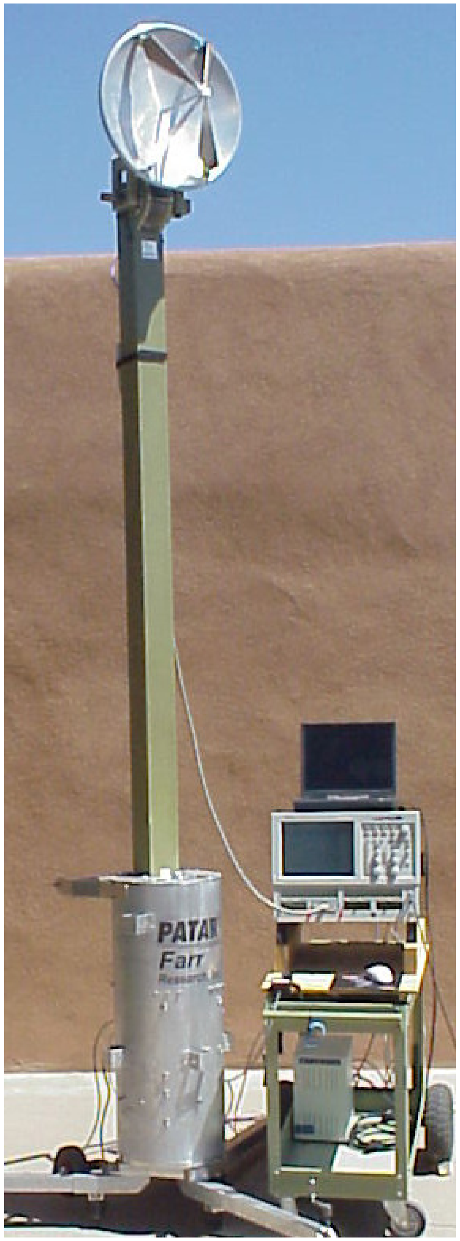
Time Domain Antenna Measurement System

- With the *PATAR*® system one person can set up an antenna range, take and process data, then tear down and store the equipment all within 4 hours.
- Equipment fits into a shed
- No anechoic chamber needed due to time gating and temperature stability of scopes
- Bandwidth of 900 MHz to 20 GHz for arbitrary antennas
- For impulse antennas, bandwidth reaches as low as 200 MHz
- Works as well for narrowband antennas as for UWB antennas
- Introduces concept of "Personal Antenna Range"

Measurement Setup



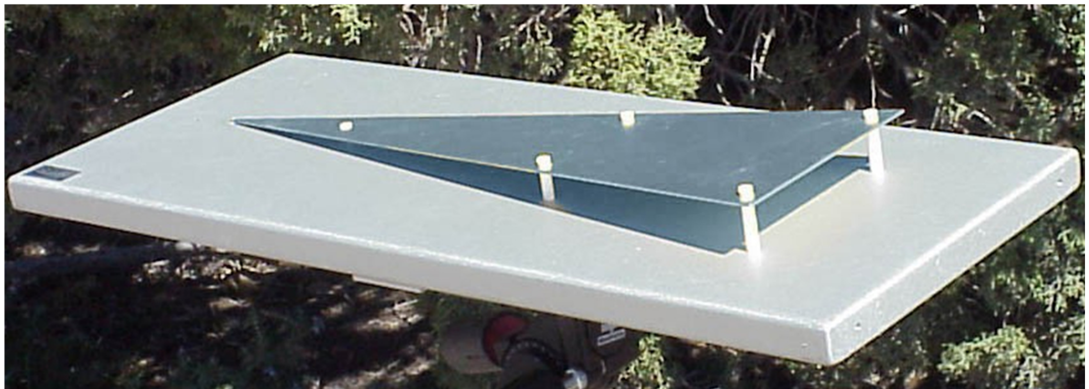
Custom Elevation / Azimuth Positioner



- Easy setup, teardown, stowage
- Mast and legs removable
- Easy leveling, aiming
- Precision better than ±0.2 degrees in both azimuth and elevation

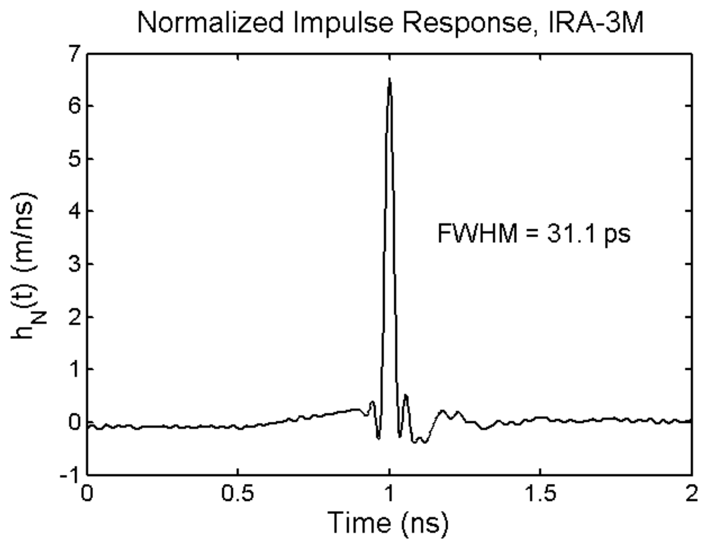
Source End of Range

Includes Pulser, mounting Bracket, and TEM sensor on fixed tripod

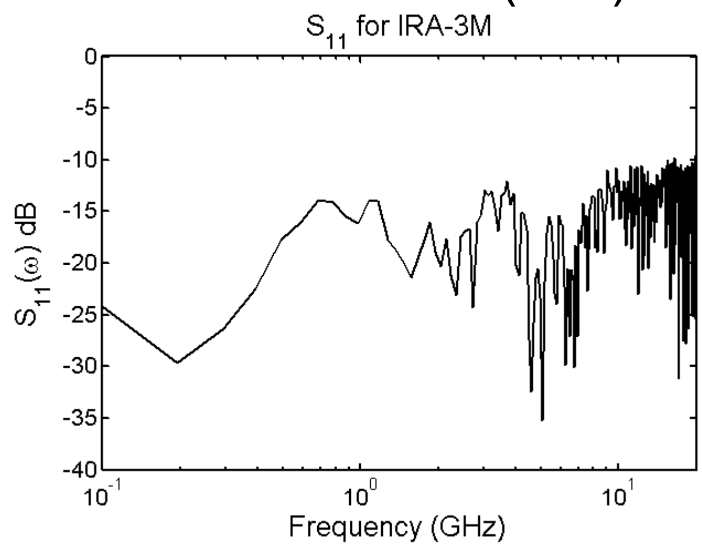


Parameters Calculated

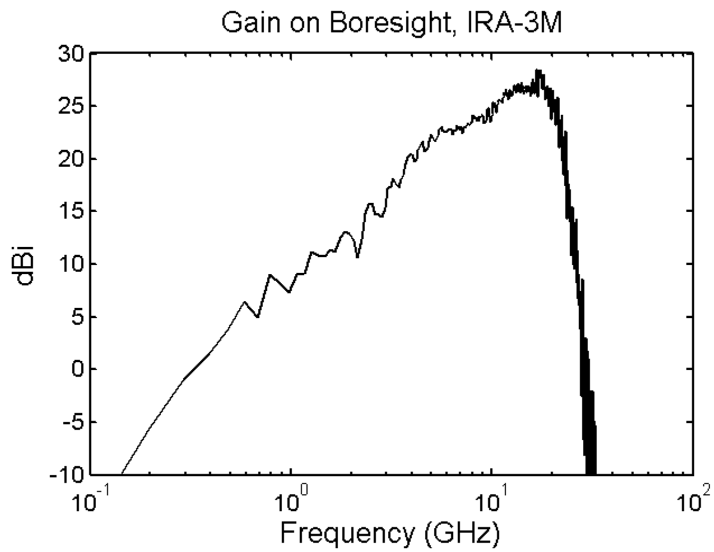
Impulse Response



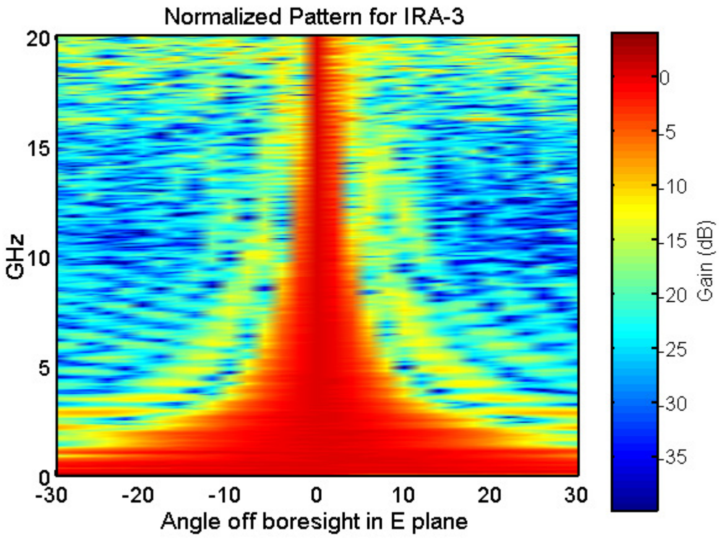
Return Loss (S_{11})

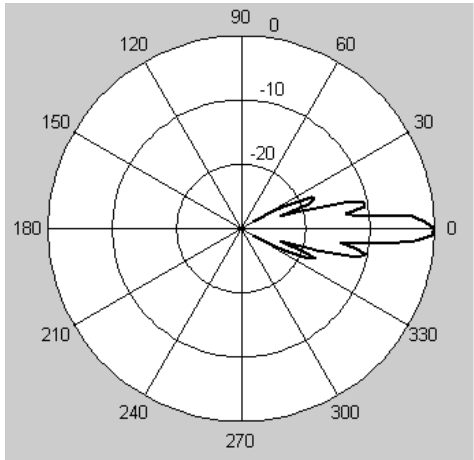


Gain, Realized Gain



Antenna Pattern



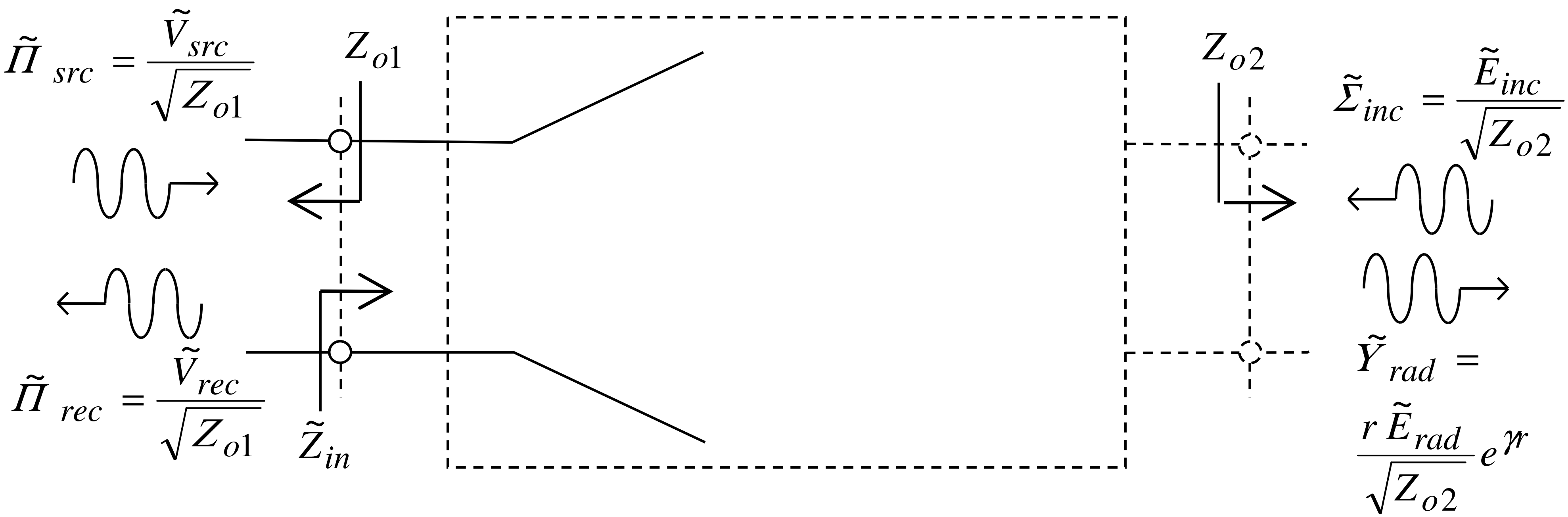


Part 2: The Power Wave Theory of Antennas

Farr
Fields, LC

The Antenna Equation and the Generalized Antenna Scattering Matrix (GASM)

Dominant Polarization on Boresight



$$\begin{bmatrix} \tilde{\Pi}_{rec} \\ \tilde{Y}_{rad} \end{bmatrix} = \begin{bmatrix} \tilde{\Gamma} & \tilde{h} \\ s\,\tilde{h}/(2\pi v) & \tilde{\ell} \end{bmatrix} \begin{bmatrix} \tilde{\Pi}_{src} \\ \tilde{\Sigma}_{inc} \end{bmatrix}$$

➔ GASM completely specifies response of any antenna, including those with waveguide feeds.

Relationship to Currently Defined Quantities

Realized Gain

$$\tilde{G}_r = \frac{4\pi}{\lambda^2} |\tilde{h}|^2$$

$\tilde{h}$ transfer function

$h(t)$ impulse response

Effective Length

$$\tilde{h}_V = \frac{\tilde{V}_{oc}}{\tilde{E}_{inc}} = \frac{\tilde{Z}_{in} + Z_{o1}}{Z_{o1}} \sqrt{\frac{Z_{o1}}{Z_{o2}}}\ \tilde{h}$$

Impedance Mismatch Factor

$$1 - |\tilde{\Gamma}|^2$$

$\tilde{\Gamma}$ reflection coefficient

$\Gamma(t)$ reflection impulse response

RCS

$$\sigma = 4\pi \left|\tilde{\ell}\right|^2$$

$\tilde{\ell}$ scattering coefficient

$\ell(t)$ scattering impulse response

New Definitions and Symbols

$$\tilde{\Pi}_{src} = \frac{\tilde{V}_{src}}{\sqrt{Z_{o1}}} = \text{source power wave}$$

$$\tilde{\Pi}_{rec} = \frac{\tilde{V}_{rec}}{\sqrt{Z_{o1}}} = \text{received power wave}$$

$$\tilde{\Sigma}_{inc} = \frac{\tilde{E}_{inc}}{\sqrt{Z_{o2}}} = \text{incident power flux density wave}$$

$$\tilde{Y}_{rad} = \frac{r\,\tilde{E}_{rad}}{\sqrt{Z_{o2}}} e^{\gamma r} = \text{radiated radiation intensity wave}$$

➔ Π, Σ, and Y are Greek for *P*, *S*, and *U*, which are the commonly used symbols for power, power flux density, and radiation intensity.

Relationships between Power Expressions and Power Wave Expressions

Power | Power Wave

$$\tilde{P}_{src} = \tfrac{1}{2}\,\mathrm{Re}\left(\tilde{V}_{src}\,\tilde{I}_{src}^{\;*}\right) = \left|\tilde{\Pi}_{src}\right|^2$$

$$\tilde{P}_{rec} = \tfrac{1}{2}\,\mathrm{Re}\left(\tilde{V}_{rec}\,\tilde{I}_{rec}^{\;*}\right) = \left|\tilde{\Pi}_{rec}\right|^2$$

Power Flux Density | Power Flux Density Wave

$$\tilde{S}_{inc} = \tfrac{1}{2}\iint \mathrm{Re}\left(\tilde{\vec{E}}_{inc}\times\tilde{\vec{H}}_{inc}^{\;*}\right)\bullet d\vec{A} = \left|\tilde{\Sigma}_{inc}\right|^2$$

Radiation Intensity | Radiation Intensity Wave

$$\tilde{U}_{rad} = \tfrac{1}{2}\,\mathrm{Re}\left(\tilde{\vec{E}}_{rad}\times\tilde{\vec{H}}_{rad}^{\;*}\right)\bullet r^2\hat{r} = \left|\tilde{Y}_{rad}\right|^2$$

➔ Power waves add phase to well-known power expressions.

Antenna Equation and GASM in the Time Domain

- Antenna equation and GASM in the time domain

$$\begin{bmatrix} \Pi_{rec}(t) \\ Y_{rad}(t) \end{bmatrix} = \begin{bmatrix} \Gamma(t) & h(t) \\ h'(t)/2\pi v & \ell(t) \end{bmatrix} \overset{*}{\bullet} \begin{bmatrix} \Pi_{src}(t) \\ \Sigma_{inc}(t) \end{bmatrix}$$

where " ' " indicates a time derivative and the "$\overset{*}{\bullet}$" operator is a matrix-product convolution operator, defined as

$$\begin{bmatrix} s_{11}(t) & s_{12}(t) \\ s_{21}(t) & s_{22}(t) \end{bmatrix} \overset{*}{\bullet} \begin{bmatrix} a_1(t) \\ a_2(t) \end{bmatrix} = \begin{bmatrix} s_{11}(t) * a_1(t) + s_{12}(t) * a_2(t) \\ s_{21}(t) * a_1(t) + s_{22}(t) * a_2(t) \end{bmatrix}$$

Antenna Equation for Two Polarizations and Arbitrary Angles

Frequency Domain

$$\begin{bmatrix} \tilde{\Pi}_{rec}(\theta',\phi') \\ \tilde{Y}_{\theta,rad}(\theta,\phi,\theta',\phi') \\ \tilde{Y}_{\phi,rad}(\theta,\phi,\theta',\phi') \end{bmatrix} = \begin{bmatrix} \tilde{\Gamma} & \tilde{h}_\theta(\theta',\phi') & \tilde{h}_\phi(\theta',\phi') \\ s\,\tilde{h}_\theta(\theta,\phi)/(2\pi v) & \tilde{\ell}_{\theta\theta}(\theta,\phi,\theta',\phi') & \tilde{\ell}_{\theta\phi}(\theta,\phi,\theta',\phi') \\ s\,\tilde{h}_\phi(\theta,\phi)/(2\pi v) & \tilde{\ell}_{\phi\theta}(\theta,\phi,\theta',\phi') & \tilde{\ell}_{\phi\phi}(\theta,\phi,\theta',\phi') \end{bmatrix} \begin{bmatrix} \tilde{\Pi}_{src} \\ \tilde{\Sigma}_{\theta,inc}(\theta',\phi') \\ \tilde{\Sigma}_{\phi,inc}(\theta',\phi') \end{bmatrix}$$

(θ',ϕ') source coordinates (θ,ϕ) observation coordinates

Time Domain

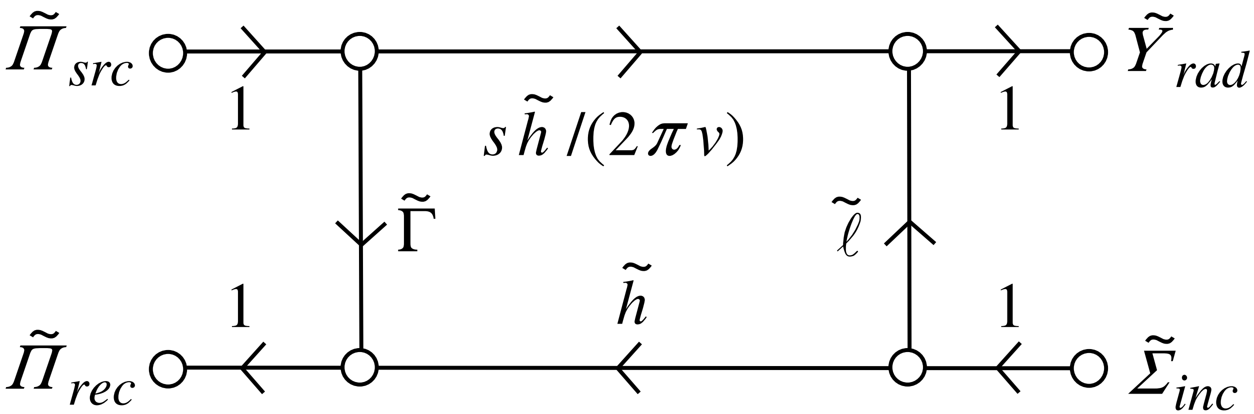
$$\begin{bmatrix} \Pi_{rec}(\theta',\phi',t) \\ Y_{\theta,rad}(\theta,\phi,\theta',\phi',t) \\ Y_{\phi,rad}(\theta,\phi,\theta',\phi',t) \end{bmatrix} = \begin{bmatrix} \Gamma(t) & h_\theta(\theta',\phi',t) & h_\phi(\theta',\phi',t) \\ h'_\theta(\theta,\phi,t)/(2\pi v) & \ell_{\theta\theta}(\theta,\phi,\theta',\phi',t) & \ell_{\theta\phi}(\theta,\phi,\theta',\phi',t) \\ h'_\phi(\theta,\phi,t)/(2\pi v) & \ell_{\phi\theta}(\theta,\phi,\theta',\phi',t) & \ell_{\phi\phi}(\theta,\phi,\theta',\phi',t) \end{bmatrix} \overset{*}{\circ} \begin{bmatrix} \Pi_{src}(t) \\ \Sigma_{\theta,inc}(\theta',\phi',t) \\ \Sigma_{\phi,inc}(\theta',\phi',t) \end{bmatrix}$$

More Compact Frequency Domain Expression

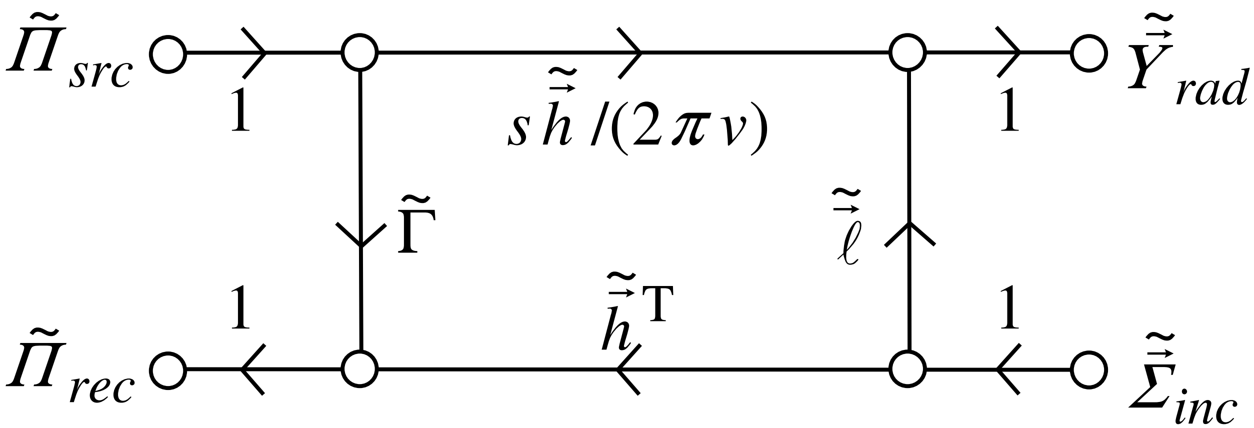
$$\begin{bmatrix} \tilde{\Pi}_{rec} \\ \tilde{\vec{Y}}_{rad} \end{bmatrix} = \begin{bmatrix} \tilde{\Gamma} & \tilde{\vec{h}}^{\mathrm{T}} \\ j\omega\tilde{\vec{h}}/(2\pi v) & \tilde{\overleftrightarrow{\ell}} \end{bmatrix} \begin{bmatrix} \tilde{\Pi}_{src} \\ \tilde{\vec{\Sigma}}_{inc} \end{bmatrix}$$

Signal Flow Graphs

Dominant polarization, on boresight

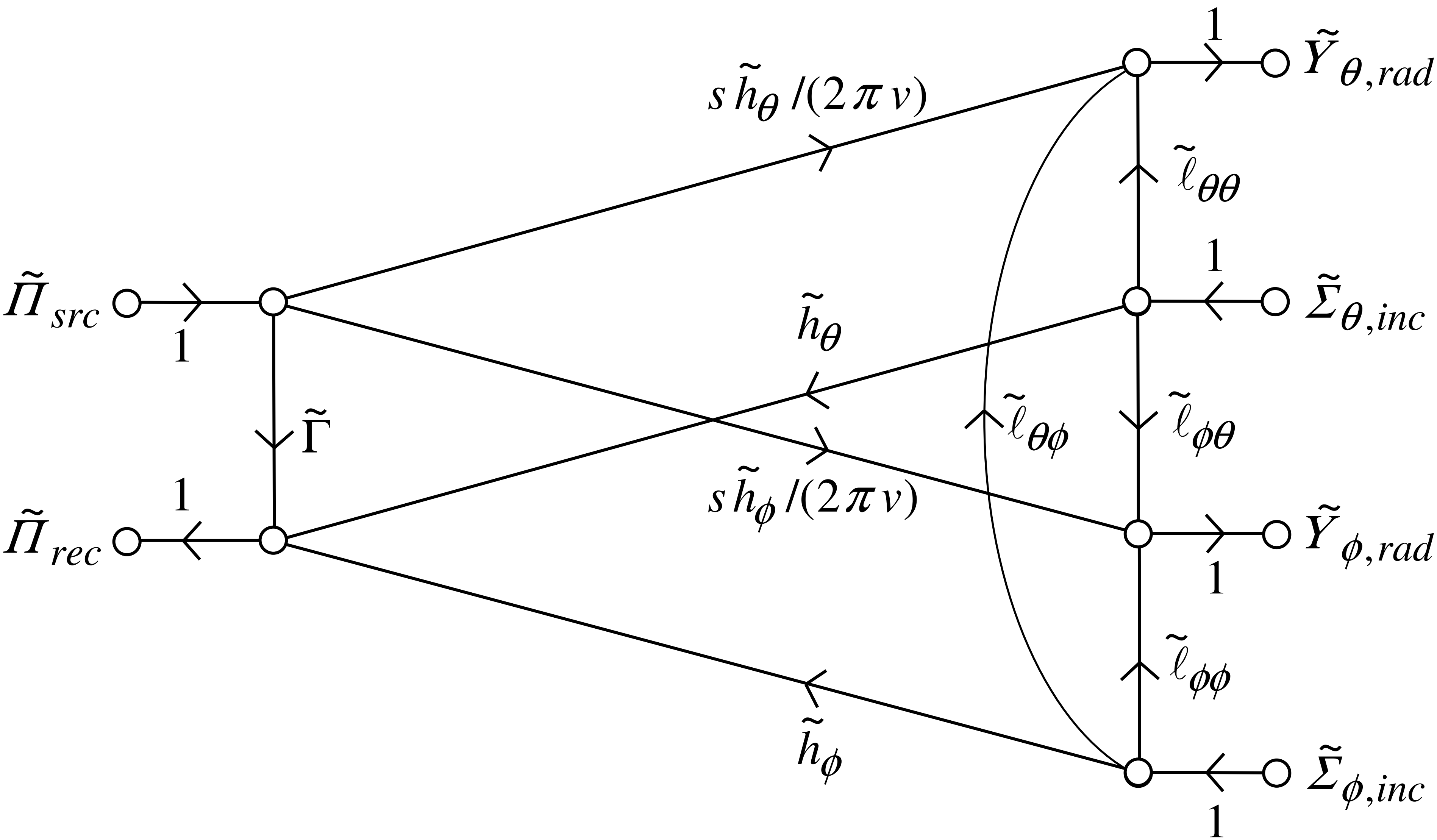


Both polarizations, arbitrary angles, vectorized 2-port version

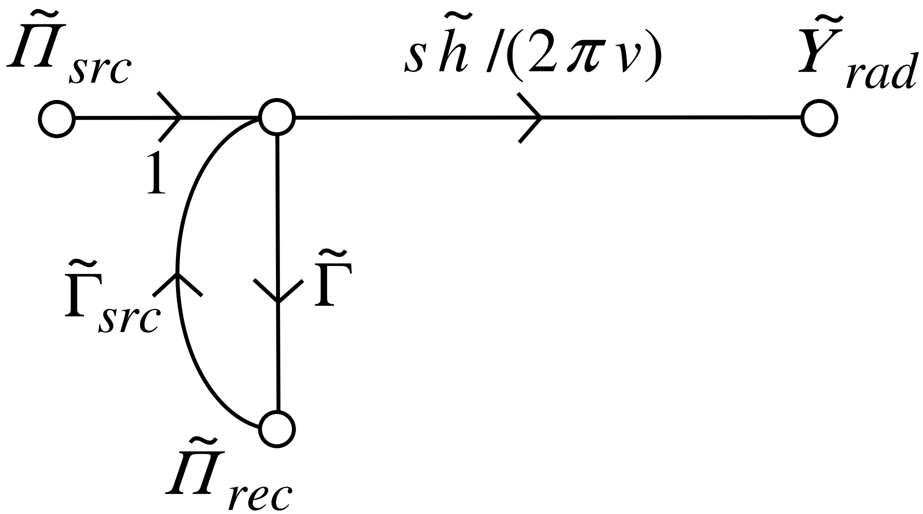


Signal Flow Graphs (cont'd)

Both polarizations, arbitrary angles, scalar 3-port version



Solve Arbitrary Source with Signal Flow Graph



$$\tilde{\Gamma}_{src} = \frac{\tilde{Z}_{src} - Z_{o1}}{\tilde{Z}_{src} + Z_{o1}}$$

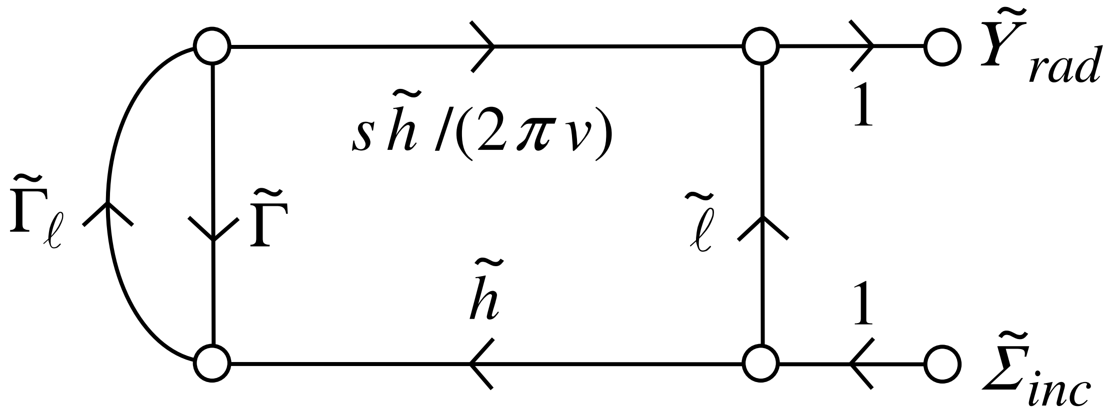
Dominant polarization, on boresight

$$\tilde{Y}_{rad} = \frac{1}{1 - \tilde{\Gamma}\,\tilde{\Gamma}_{src}} \, \frac{s\,\tilde{h}}{2\pi\,v} \, \tilde{\Pi}_{src}$$

Both polarizations, arbitrary angles

$$\tilde{\vec{Y}}_{rad}(\theta,\phi) = \frac{1}{1 - \tilde{\Gamma}\,\tilde{\Gamma}_{src}} \, \frac{s\,\tilde{\vec{h}}(\theta,\phi)}{2\pi\,v} \, \tilde{\Pi}_{src}$$

Scattering from an Antenna with Arbitrary Load



Dominant polarization, on boresight

$$\tilde{Y}_{rad} = \left[\frac{\tilde{\Gamma}_\ell}{1-\tilde{\Gamma}\tilde{\Gamma}_\ell} \frac{s\,\tilde{h}^2}{2\pi v} + \tilde{\ell} \right] \tilde{\Sigma}_{inc}$$

Both polarizations, arbitrary angles

$$\tilde{\vec{Y}}_{rad} = \left[\frac{\tilde{\Gamma}_\ell}{1-\tilde{\Gamma}\tilde{\Gamma}_\ell} \frac{s}{2\pi v}\, \tilde{\vec{h}}(\theta,\phi)\, \tilde{\vec{h}}^T(\theta',\phi') + \tilde{\overleftrightarrow{\ell}}(\theta',\phi',\theta,\phi) \right] \cdot \tilde{\vec{\Sigma}}_{inc}(\theta',\phi')$$

Array of *N* Antennas or *N* Modes in Multimoded Waveguides

Need an $h(t)$ for each array element or mode

, $$\tilde{h}_{i\,j}, \quad i = 1,2; \quad j = 1,\ldots, N$$

$$\begin{bmatrix} \tilde{\tilde{\vec{\Pi}}}_{rec} \\ \tilde{\tilde{\vec{Y}}}_{rad} \end{bmatrix} = \begin{bmatrix} \tilde{\tilde{\overleftrightarrow{\Gamma}}} & \tilde{\tilde{\overleftrightarrow{h}}}^{\mathrm{T}} \\ s\,\tilde{\tilde{\overleftrightarrow{h}}}/(2\pi v) & \tilde{\tilde{\overleftrightarrow{\ell}}} \end{bmatrix} \begin{bmatrix} \tilde{\tilde{\vec{\Pi}}}_{src} \\ \tilde{\tilde{\vec{\Sigma}}}_{inc} \end{bmatrix}$$

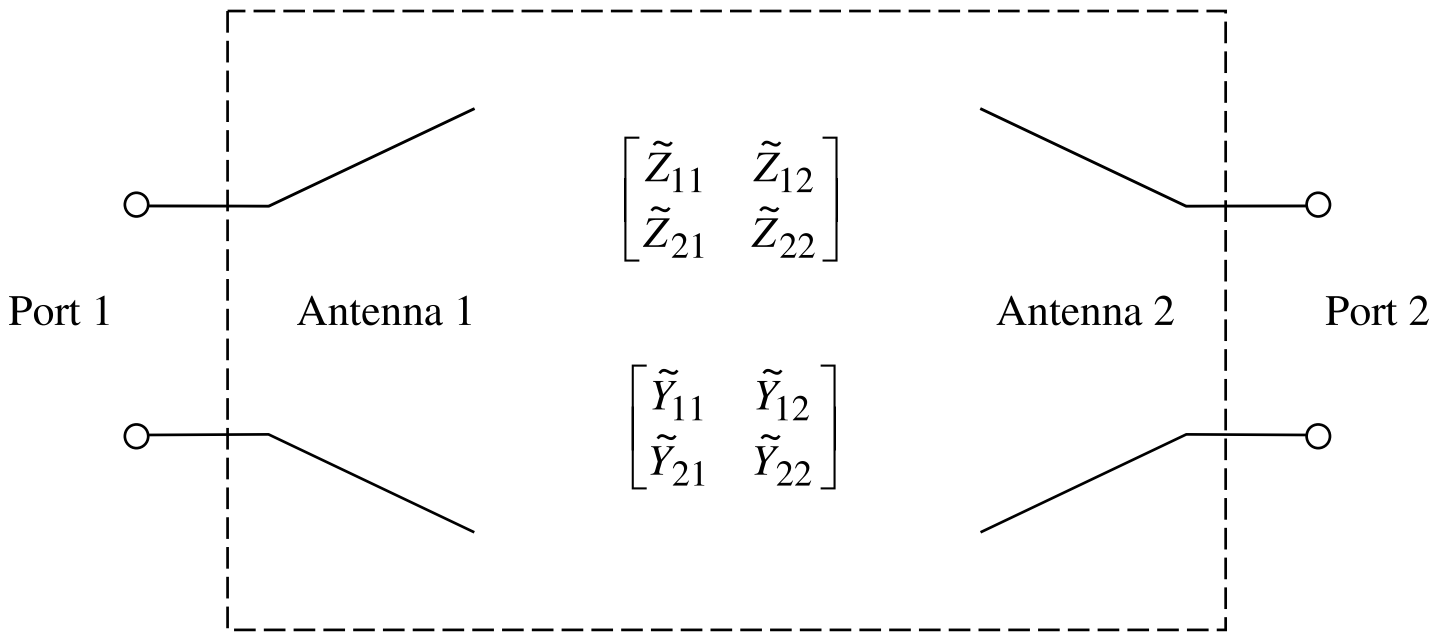
Dimensions are visualized as

$$\begin{bmatrix} [\quad]_{N\times 1} \\ [\quad]_{2\times 1} \end{bmatrix} = \begin{bmatrix} [\qquad\qquad]_{N\times N} & [\qquad]_{N\times 2} \\ [\qquad\qquad]_{2\times N} & [\qquad]_{2\times 2} \end{bmatrix} \begin{bmatrix} [\quad]_{N\times 1} \\ [\quad]_{2\times 1} \end{bmatrix}$$

$\tilde{\tilde{\overleftrightarrow{\Gamma}}}$ and $\Gamma_{i\,j}(t)$: mutual coupling coefficient and impulse response

Proving the Relationship between Transmission and Reception Terms

- Relate power wave expressions to open/short circuit forms using circuit theory.
- Treat two antennas in far field as reciprocal two-port network



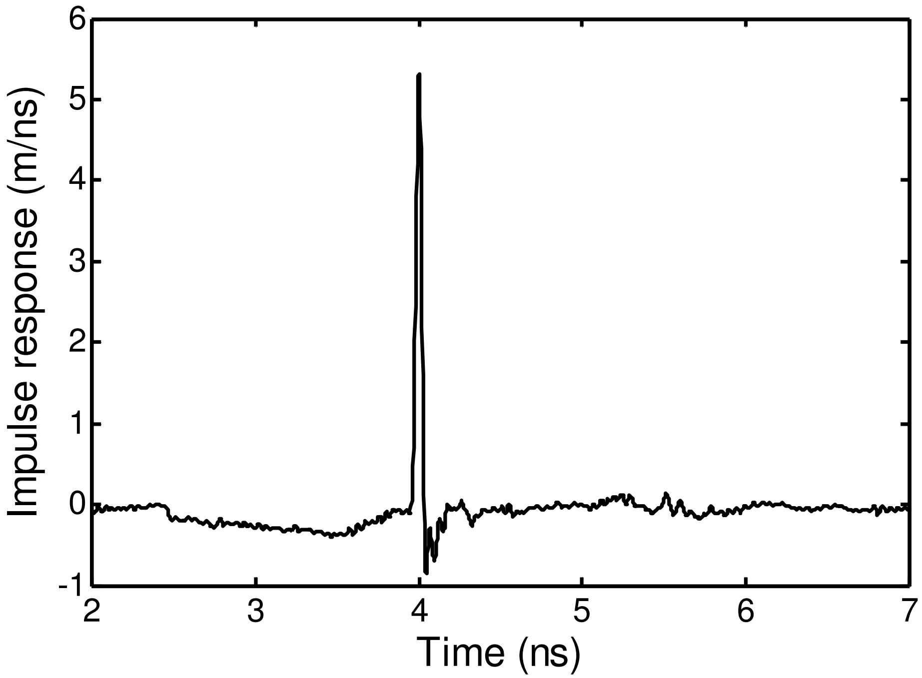
$$\begin{bmatrix} \tilde{Z}_{11} & \tilde{Z}_{12} \\ \tilde{Z}_{21} & \tilde{Z}_{22} \end{bmatrix}$$

$$\begin{bmatrix} \tilde{Y}_{11} & \tilde{Y}_{12} \\ \tilde{Y}_{21} & \tilde{Y}_{22} \end{bmatrix}$$

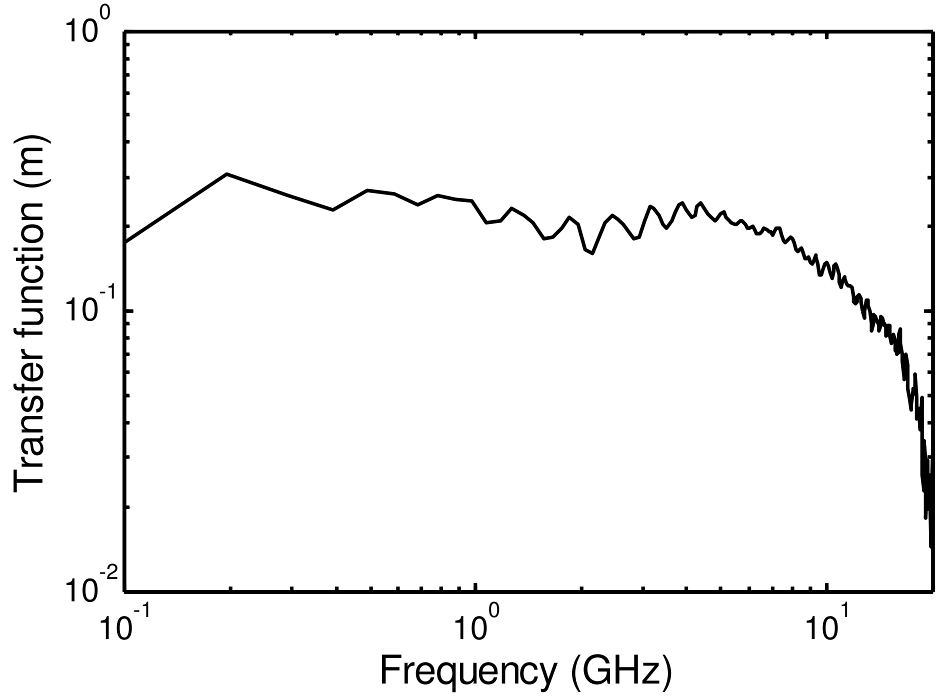
- Assume Antenna 2 is an electrically small electric dipole, whose open/short circuit characteristics are fully known.

Impulse Response Example
IRA-3Q

Impulse Response



Transfer Function



Review of Waveform Norms
(For transient antenna patterns)

Three necessary conditions of norms

$$\| f(t) \| \begin{cases} = 0 \ \text{iff} \ f(t) \equiv 0 \\ > 0 \ \text{otherwise} \end{cases}$$

$$\| \alpha f(t) \| = |\alpha| \, \| f(t) \| \qquad \text{(linearity)}$$

$$\| f(t) + g(t) \| \leq \| f(t) \| + \| g(t) \| \qquad \text{(triangle inequality)}$$

Commonly used: *p*–norms

$$\| f(t) \|_p = \left[\int_{-\infty}^{\infty} |f(t)|^p \, dt \right]^{1/p} , \qquad \| f(t) \|_\infty = \sup_t |f(t)|$$

Transient Antenna Pattern

- Can consider single polarization or total magnitude

$$\left|\vec{h}(\theta,\phi,t)\right| = \sqrt{\left|h_\theta(\theta,\phi,t)\right|^2 + \left|h_\phi(\theta,\phi,t)\right|^2}$$

- Express transient patterns in terms of norms of time domain waveforms

$$\mathrm{P}_\theta(\theta,\phi) = \frac{\left\| h_\theta(\theta,\phi,t) \right\|}{\left\| h_\theta(0,0,t) \right\|}, \qquad \mathrm{P}_\phi(\theta,\phi) = \frac{\left\| h_\phi(\theta,\phi,t) \right\|}{\left\| h_\phi(0,0,t) \right\|}$$

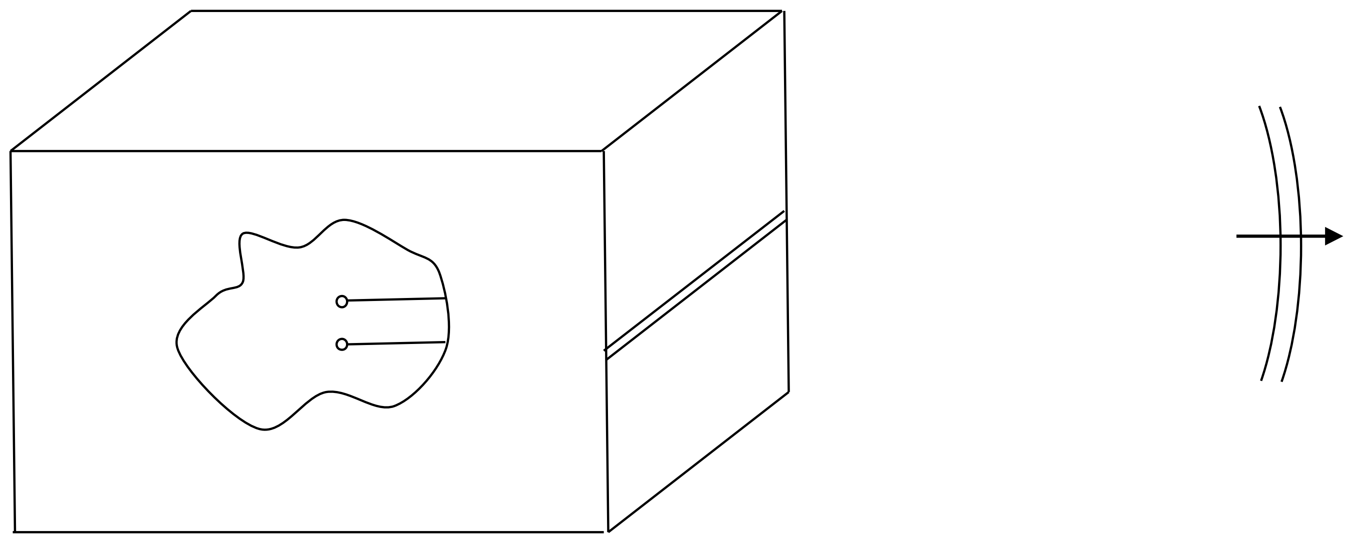
$$\mathrm{P}_t(\theta,\phi) = \frac{\left\| \left|\vec{h}(\theta,\phi,t)\right| \right\|}{\left\| \left|\vec{h}(0,0,t)\right| \right\|}$$

- Normalization to boresight is optional

Radiation from or Coupling into a Complex System

- Complex system looks like a poor antenna

- Antenna parameters should be used
    - Same in TX and RX
    - Works in both frequency and time domains



Conclusion: Effects on Standards

➔ None of the terms in the Antenna Equation have been defined

$$\begin{bmatrix} \tilde{\Pi}_{rec} \\ \tilde{Y}_{rad} \end{bmatrix} = \begin{bmatrix} \tilde{\Gamma} & \tilde{h} \\ s\,\tilde{h}/(2\pi v) & \tilde{\ell} \end{bmatrix} \begin{bmatrix} \tilde{\Pi}_{src} \\ \tilde{\Sigma}_{inc} \end{bmatrix}$$

➔ Closest is scalar versions:

- Impedance mismatch factor, $1-|\tilde{\Gamma}|^2$, instead of $\tilde{\Gamma}$
- Realized gain, G_r, instead of $\tilde{h}$
- RCS, σ, instead of $\tilde{\ell}$

➔ We need to <u>*complexify*</u> the standards to get to the time domain!

Everett Farr
+1(505)410-7722

efarr@farr-research.com
www.farr-research.com

This paper is based on
Sensor and Simulation Note 564, Revision 3
Available at our web site

Soon to appear in FERMAT!
We welcome your online comments!