Digital Audio Processing and Applications (EE5809)  
Tutorial 1

Answer Q1

First tone is a masking tone at 50dB. It generates a critical response with 60dB/kHz roll off above 1 kHz.
Second tone is 30 dB at 1.5 kHz. The masking threshold at 1.5 kHz as a result of masking is 10 dB. Therefore, second tone is 20 dB above the masking threshold and is audible. To quantize this tone so that the quantization noise is inaudible, we only need 4 bits, i.e., it gives 4x6=24 dB SQNR and the noise is 4 dB below the masking threshold at 1.5 kHz.
Third tone is 20 dB at 1.25 kHz which is below the masking threshold (25dB at 1.2kHz) and is therefore inaudible.

Answer Q2

The inband noise power will be reduced by half or 3 dB for two-time oversampling. If the oversampling ratio is 64/8 = 8, the inband noise power is

1.63/8 = 0.20375 mW

In terms of SNQR improvement, it is

3×log₂8 = 9dB

which corresponds to 1.5 bit precision improvement, so the final bit precision achieved using an 8 bit quantizer is 9.5 bits

Answer Q3(a)

Quantization levels at -0.75, -0.25, 0.25 and 0.75.
Decision levels at -0.5, 0 and 0.5 (i.e., half way between two quantization levels)
Probability distribution is symmetry, so we can work on one side and multiply the result by two afterward.

\[ p(x) = 1 - x, \quad 0 \leq x \leq 1 \]

There are two quantization bins in each side and their quantization errors are

\[ x - 0.25, \quad 0.0 \leq x \leq 0.5 \]
\[ x - 0.75, \quad 0.5 \leq x \leq 1 \]

Total quantization error power (two side) is

\[
\xi = 2 \left[ \int_0^{0.5} (x - 0.25)^2 p(x)dx + \int_{0.5}^{1.0} (x - 0.75)^2 p(x)dx \right]
\]

\[
= 2 \left[ \int_0^{0.5} (x - 0.25)^2 (1 - x)dx + \int_{0.5}^{1.0} (x - 0.75)^2 (1 - x)dx \right]
\]

\[
= 2 \left[ \int_0^{0.5} (-x^3 + 1.5x^2 - 0.5625x + 0.0625)dx + \int_{0.5}^{1.0} (-x^3 + 2.5x^2 - 2.0625x + 0.5625)dx \right]
\]

\[
= 2 \left[ \left(-\frac{1}{4}x^4 + \frac{1.5}{3}x^3 - \frac{0.5625}{2}x^2 + 0.0625x\right)_{0}^{0.5} + \left(-\frac{1}{4}x^4 + \frac{2.5}{3}x^3 - \frac{2.0625}{2}x^2 + 0.5625x\right)_{0.5}^{1.0} \right]
\]

\[
= 2\left[0.0078125 + 0.0026041667\right] = 0.02083333
\]

Answer Q3(b)

Again because of symmetry pdf, we can work on one side and multiply the result by 2.

Assume the quantization level is \( q \), we have the quantization error power (one side) as

\[
\xi = \int_0^{0.5} (x - q)^2 p(x)dx
\]

\[
= \int_0^{0.5} [-x^3 + (1 + 2q)x^2 - (2q + q^2)x + q^2]dx
\]

\[
= -\frac{1}{4}x^4 + \frac{1 + 2q}{3}x^3 - \frac{2q + q^2}{2}x^2 + q^2
\]

Minimizing \( \xi \) with respect to \( q \), we have

\[
\frac{\partial \xi}{\partial q} = \frac{2}{3} - 1 - q + 2q = q - \frac{1}{3}
\]

setting \( \frac{\partial \xi}{\partial q} = 0 \), we have \( q = \frac{1}{3} \). Therefore, the two quantization levels are at \( -\frac{1}{3} \) and \( \frac{1}{3} \).