Diamond Relay Network under Rayleigh Fading: On-Off Power Control and Outage-Capacity Bound

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***Abstract*— The achievable outage probability of the diamond relay network under Rayleigh fading is investigated. Two existing transmission protocols are considered, namely, the Alamouti-Coded Amplify-and-Forward (ACAF) and the Alamouti-Coded Decode-and-Forward (ACDF). For ACAF, a distributed optimal power control rule for the two relays is analytically derived. Simulation results show that with this power control rule, the diversity gain of ACAF increases from one to two, and its performance approaches that of ACDF in the high signal-to-noise ratio (SNR) regime. For ACDF, a performance bound is analytically obtained: for any outage probability ϵ, its SNR offset is bounded above by 3 dB and its ϵ-outage rate is within 1 bit of the ϵ-outage capacity of the diamond relay network.**

Index terms: Alamouti codes, amplify-and-forward (AF), decode-and-forward (DF), distributed space-time codes, diamond relay network, power control, outage capacity.

I. Introduction

Multiple relay nodes, which can act as a virtual antenna array, have a potential to offer high multiplexing and diversity gain. To study such a phenomenon, the parallel relay network, in which a source node and a destination node are connected by a number of parallel relay nodes, was proposed in [1]. It is important to gain fundamental understanding of this channel model, as the network topology is relevant to many real systems including multihop cellular network and WiMAX.

There are many works that assume practical constraints such as half-duplex operation on relay nodes and bandwidth mismatch [2], [3]. The network model is also extended to the case where there is an extra direct link between the source and the destination [4], [5]. The capacity of the parallel relay network, however, remains unknown.

In this paper, we study the diamond relay network, which consists of two relay nodes, in a slow fading environment. Channel state information (CSI) of a communication link is assumed available at its receiver only. For this network setting, distributed space-time codes (DSTC) [6] are promising methods to achieve high diversity-multiplexing tradeoff. Herein, we consider the use of distributed Alamouti code [7], which is a special case of DSTC. Furthermore, for the relaying techniques, we adopt amplify-and-forward (AF) or decode-and-forward (DF). AF can be divided into fixed-gain AF (FGAF) and adaptive AF (AAF). The amplifying factor of the FGAF relay does not depend on the instantaneous CSI of the channel right before it, while AAF is quite the contrary. Indeed, it is more practical to consider AAF since it avoids the transmit amplifier at the relay being driven into saturation, which may occur in FGAF.

There are a lot of variations that combine DSTC with FGAF. In [8], the source and relay nodes form an Alamouti pair. To prove second order diversity, the authors assume severe asymmetric link between the two hops. In [9], the authors generalize the idea to $R > 2$ parallel relay nodes and conjecture a diversity factor around $R/2$ from their simulations. The work in [10] shows that the distributed linear dispersion codes, a wide class of DSTC, when combined with FGAF, achieves full diversity, based on the assumption of large number of relay nodes with optimal power allocation between the source and the relays. Recently, the row-monomial DSTC is combined with FGAF and shown to achieve full diversity [11], with the data rate inversely proportional to the number of parallel relay nodes. AAF in combination with DSTC is considered in [12], which proves full diversity under an approximate analysis.

In this paper, we focus on the combination of Alamouti code and AAF. We call it ACAF. Our simulation result indicates that when both relay nodes transmit with full power, the achieved diversity order is only one. To improve the performance, we analytically derive the optimal power control rule for minimizing the end-to-end outage probability. We show that the optimal rule is on-off in nature. With this rule, full diversity can be obtained. Besides, a most desirable feature of this rule is that it can be implemented in a fully distributed manner. Note that this work differs from [13] in that the objective of power control is to minimize outage probability, whereas in [13], the objective is to maximize the received SNR.

The Alamouti-Coded Decode-and-Forward (ACDF) protocol is a special case of the one considered in [6], where full diversity is proved. However, having full diversity only means that the slope of the outage probability curve is optimal. It is unclear how good it is in terms of power consumption. To address this issue, we define the notion of SNR offset, which measures the gap between the achieved outage probability curve and the optimal one at a certain outage probability. By deriving a new lower bound on the outage probability, we

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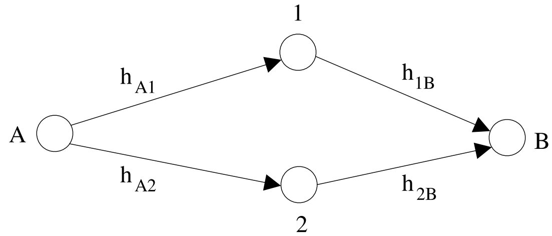

Fig. 1. The diamond relay network model.

prove that the SNR offset is bounded above by 3 dB, for any outage probability. In terms of information rate, we show that ACDF can achieve within one bit of the ϵ-outage capacity. To the best of our knowledge, this is the first time that a bound on the outage capacity gap is derived. We remark that it was shown in [14] that the Shannon capacity of the diamond relay network with additive white Gaussian noise can be achieved within two bits. Our work differs from [14] in that we consider Rayleigh fading channel.

II. System Model

We consider a single source, node A, communicating to a single destination, node B, in a network over a layer of two parallel relay nodes, relays 1 and 2. Nodes A and B are assumed so far away that the wireless link between them can be neglected. The network model is shown in Fig. 1.

All nodes in the network are assumed to have one single antenna. Node i is subject to power constraint P_i, $i \in \{A, 1, 2\}$. The two relays are operated in full-duplex mode. This assumption is not restrictive, since the results obtained can be directly applied to the half-duplex case if the fraction of time for relay nodes to transmit is the same as the fraction of time for relay nodes to receive. For systems in which these fractions are tunable parameters, an optimization can be performed on top of our developed relaying strategies. In order not to obscure our study, we make the full-duplex assumption.

We consider the slow-fading scenario where the link gains are random but remain constant for all time. It models the situation where the delay requirement is short compared with the channel coherence time. Let h_{Ai} and h_{iB} be, respectively, the link gains from source A to relay i and from relay i to destination B, $i = 1, 2$. We assume that CSI is available at receivers. In other words, relay i knows h_{Ai}, and the destination node knows h_{Ai} and h_{iB} for $i = 1, 2$. Define $\boldsymbol{h}$ as the column vector $(h_{A1}, h_{A2}, h_{1B}, h_{2B})$.

Let $X_i[m]$ be the transmitter symbol from node i at time m, $i \in \{A, 1, 2\}$. We also let $Y_i[m]$ and $w_i[m]$ be, respectively, the received symbol and the thermal noise at node i at time m, $i \in \{1, 2, B\}$. Each channel's input-output relationship is represented by the following formulas:

$$Y_i[m] = h_{Ai}\sqrt{P_A}X_A[m] + w_i[m] \text{ for } i = 1, 2, \quad (1)$$

$$Y_B[m] = \sum_{i=1}^{2} h_{iB}X_i[m] + w_B[m], \quad (2)$$

subject to Var$\{X_A[m]\} \leq 1$ and Var$\{X_i[m]\} \leq P_i$ for $i = 1, 2$. We assume that $P_1 = P_2 \triangleq \kappa P_A$, and $w_i[m], w_B[m] \sim \mathcal{CN}(0, n)$. Define $\Gamma \triangleq P_A/n$. The instantaneous received SNR at relay i is denoted by $\Gamma_i(h_{Ai})$, and is equal to $|h_{Ai}|^2\Gamma$. The instantaneous received SNR at destination B is denoted by $\Gamma_B(\boldsymbol{h})$, whose expression depends on the transmission scheme. Correspondingly, we define $R_A(\boldsymbol{h})$ as the instantaneous end-to-end information rate from source A to destination B, and it is equal to

$$R_A(\boldsymbol{h}) = \log_2\left(1 + \Gamma_B(\boldsymbol{h})\right). \quad (3)$$

An outage event occurs if the instantaneous end-to-end rate $R_A(\boldsymbol{h})$ falls below a certain threshold, R_{thd}. Outage probability is the probability of occurrence of an outage event, i.e., $p_{out} \triangleq \Pr\{R_A(\boldsymbol{h}) < R_{thd}\}$. The ϵ-outage rate R_ϵ for a certain scheme is defined as the value of R_{thd} such that $p_{out} = \epsilon$ holds. It is clearly a function of Γ.

According to (3), there is a one-to-one correspondence between $R_A(\boldsymbol{h})$ and $\Gamma_B(\boldsymbol{h})$. Therefore, the outage probability can also be obtained by comparing $\Gamma_B(\boldsymbol{h})$ with a certain threshold, Γ_{thd}. More specifically, we have $p_{out} = \Pr\{\Gamma_B(\boldsymbol{h}) < \Gamma_{thd}\}$.

For theoretical analysis, we assume all the four links follow i.i.d. Rayleigh fading, in which $h_{Ai}, h_{iB} \sim \mathcal{CN}(0, 1)$ for $i = 1, 2$. Under i.i.d. Rayleigh fading, $|h_{A1}|^2$, $|h_{A2}|^2$, $|h_{1B}|^2$, and $|h_{2B}|^2$ follow the exponential distribution with the same parameter, and the sum of two independent exponential random variables with parameter λ is the same as a gamma random variable with parameter $(2, \lambda)$.

Remark: Throughout this paper, we use $x^\dagger$ to denote the conjugate of the complex number x, and $\boldsymbol{A}^\dagger$ to denote the conjugate transpose of the complex matrix $\boldsymbol{A}$.

III. Alamouti-Coded Amplify-and-Forward (ACAF)

When the two relays receive the transmitted signal from the source node, they can simply amplify the signal without decoding. To forward the amplified signal to the destination, the Alamouti space-time code [7] can be used, hence the name of Alamouti-Coded AF (ACAF). To do that, two consecutive transmissions of symbols are combined into one block. In each block, source A successively transmits two symbols: $X_A[1]$ and $X_A[2]$. Each relay is assumed to have a buffer that can store the received signal of two symbols from source A and forward the signal to the destination with a unit block delay.

Now we give a formal description of the scheme. As the derivation of the achieved SNR at destination B is the same as that in [13], here we only give an outline.

After receiving the signal from the source, relay i, $i = 1, 2$, recovers a noisy copy of $X_A[m]$, with noise $\tilde{w}_i[m]$:

$$\tilde{Y}_i[m] \triangleq \frac{h_{Ai}^\dagger}{\sqrt{P_A}|h_{Ai}|^2}Y_i[m] = X_A[m] + \tilde{w}_i[m], \quad (4)$$

where $\tilde{w}_i[m] \triangleq \frac{h_{Ai}^\dagger}{\sqrt{P_A}|h_{Ai}|^2}w_i[m]$. Note that $\tilde{w}_i[m] \sim \mathcal{CN}(0, \tilde{n}_i)$ and $\tilde{n}_i \triangleq \frac{1}{|h_{Ai}|^2\Gamma}$.

The transmit symbols of the relays are constructed in the following way:

$$u_{im} \triangleq \tilde{\xi}_i \tilde{Y}_i[m-2] = \tilde{\xi}_i(X_A[m-2] + \tilde{w}_i[m-2]),\ i = 1, 2. \quad (5)$$

To meet the power constraint, we have $\tilde{\xi}_i \triangleq |h_{Ai}|\sqrt{P_A}\xi_i$, where ξ_i satisfies $\xi_i \leq \sqrt{\frac{P_i}{|h_{Ai}|^2 P_A + n}} \triangleq \xi_i^{\max}$.

Under the Alamouti scheme, it suffices to consider only one block of transmission, which consists of four symbols, two from each relay. Without loss of generality, we consider the block that corresponds to $m = 1$ and 2. Each block is divided into two stages. In the first stage, relay 1 transmits u_{11} and relay 2 transmits u_{22}; in the second stage, relay 1 transmits $-u_{12}^\dagger$ and relay 2 transmits $u_{21}^\dagger$. The output symbols at destination B can be written in matrix form as

$$\begin{bmatrix} Y_B[1] \\ Y_B[2]^\dagger \end{bmatrix} = \boldsymbol{H} \begin{bmatrix} X_A[1] \\ X_A[2] \end{bmatrix} + \tilde{\boldsymbol{w}}_R + \tilde{\boldsymbol{w}}_B, \quad (6)$$

where

$$\boldsymbol{H} \triangleq \begin{bmatrix} \tilde{\xi}_1 h_{1B} & \tilde{\xi}_2 h_{2B} \\ \tilde{\xi}_2 h_{2B}^\dagger & -\tilde{\xi}_1 h_{1B}^\dagger \end{bmatrix}, \tilde{\boldsymbol{w}}_B \triangleq \begin{bmatrix} w_B[1] \\ w_B[2]^\dagger \end{bmatrix}, \quad (7)$$

$$\text{and } \tilde{\boldsymbol{w}}_R \triangleq \begin{bmatrix} h_{1B}\tilde{\xi}_1 \tilde{w}_1[1] + h_{2B}\tilde{\xi}_2 \tilde{w}_2[2] \\ -h_{1B}^\dagger \tilde{\xi}_1 \tilde{w}_1[2] + h_{2B}^\dagger \tilde{\xi}_2 \tilde{w}_2[1] \end{bmatrix}. \quad (8)$$

Left multiplying (6) by $\boldsymbol{H}^\dagger$, we obtain

$$\boldsymbol{H}^\dagger \begin{bmatrix} Y_B[1] \\ Y_B[2]^\dagger \end{bmatrix} = \lambda_{\text{AF}} \begin{bmatrix} X_A[1] \\ X_A[2] \end{bmatrix} + \boldsymbol{H}^\dagger \tilde{\boldsymbol{w}}_R + \boldsymbol{H}^\dagger \tilde{\boldsymbol{w}}_B, \quad (9)$$

where, $\lambda_{\text{AF}} \triangleq \{\tilde{\xi}_1^2 |h_{1B}|^2 + \tilde{\xi}_2^2 |h_{2B}|^2\}$. From (9), we can see that the detection problem for $X_A[1]$ and $X_A[2]$ decomposes into two separate, orthogonal, scalar problems. The received SNR's at node B for the detection of $X_A[1]$ and that of $X_A[2]$ are the same. After some tedious but straightforward derivation, we obtain the received SNR at destination B as

$$\Gamma_B^{\text{ACAF}} = \frac{\xi_1^2 |h_{A1}|^2 |h_{1B}|^2 + \xi_2^2 |h_{A2}|^2 |h_{2B}|^2}{\xi_1^2 |h_{1B}|^2 + \xi_2^2 |h_{2B}|^2 + 1}\Gamma. \quad (10)$$

The outage probability is thus given by

$$p_{out}^{\text{ACAF}} = \Pr\{\Gamma_B^{\text{ACAF}} < \Gamma_{thd}\}. \quad (11)$$

It is clear that the outage probability depends on the amplifying factors used by the relays. If both relays transmit with full power, we call the resulting strategy ACAF with Full Power (FP), but it may not be optimal. To minimize the outage probability, we first note that the condition $\Gamma_B^{\text{ACAF}} < \Gamma_{thd}$ is equivalent to

$$f(\xi_1, \xi_2) \triangleq \xi_1^2 |h_{1B}|^2 \Gamma_1 + \xi_2^2 |h_{2B}|^2 \Gamma_2 - \Gamma_{thd}(\xi_1^2 |h_{1B}|^2 + \xi_2^2 |h_{2B}|^2 + 1) < 0. \quad (12)$$

Differentiating f with respect to ξ_i for $i = 1, 2$, we get

$$\frac{\partial f}{\partial \xi_i} = 2\xi_i |h_{iB}|^2 (\Gamma_i - \Gamma_{thd}). \quad (13)$$

It can be seen that f is a monotonic function of ξ_i. Whether it is increasing or decreasing depends on the sign of $\Gamma_i - \Gamma_{thd}$. The following result provides a condition under which outage always occurs:

Theorem 1: If $\Gamma_1, \Gamma_2 < \Gamma_{thd}$, then the outage probability under ACAF is equal to one for all ξ_1's and ξ_2's.

Proof: Since f is monotonically decreasing with both ξ_1 and ξ_2, the global maximum of f occurs at $\xi_1 = \xi_2 = 0$. The statement follows from the fact that $f(0, 0) = -\Gamma_{thd} < 0$. ■

Now we consider the case where one or both of the SNR's are greater than the threshold. Denote the optimal amplifying factors that minimize the outage probability by ξ_1^* and ξ_2^*. The following result follows directly from the monotonicity of f:

Theorem 2: Under ACAF,

1) if $\Gamma_1, \Gamma_2 > \Gamma_{thd}$, then $\xi_i^* = \xi_i^{\max}$ for $i = 1, 2$;
2) if $\Gamma_1 > \Gamma_{thd} > \Gamma_2$, then $\xi_1^* = \xi_1^{\max}$ and $\xi_2^* = 0$;
3) if $\Gamma_2 > \Gamma_{thd} > \Gamma_1$, then $\xi_1^* = 0$ and $\xi_2^* = \xi_2^{\max}$;
4) if $\Gamma_i = \Gamma_{thd}$, then $\xi_i^* \in [0, \xi_i^{\max}]$, $i = 1, 2$.

Though Γ_B is continuous with respect to ξ_1, ξ_2, and the link gains, the optimal values, ξ_1^* and ξ_2^*, are *not* continuous with respect to the link gains. Due to the randomly distributed nature of the link gains, the probability that $\xi_i^* \in (0, \xi_i^{\max})$ is zero. Hence, the optimal rule is on-off in nature. Besides, it can be implemented in a distributed way, since the optimal amplifying factor of relay i does not depend on the received SNR at relay j, $j \neq i$. We call this optimal rule as ACAF with Optimal Power Control (OPC).

IV. Alamouti-Coded Decode-and-Forward (ACDF)

This scheme was proposed in [6]. In this strategy, each relay first decodes the received signal. If the decoding is successful, the relay re-encodes the message using the same codebook in source A. If not, it simply keeps silent. Let p_i denote the probability that relay i fails to decode the signal.

$$p_i \triangleq \Pr\{\Gamma_i < \Gamma_{thd}\} = \Pr\{|h_{Ai}|^2 < x\} = 1 - e^{-x}, \quad (14)$$

where $x \triangleq \Gamma_{thd}/\Gamma$.

In the second hop, the Alamouti scheme is also adopted, hence the name of Alamouti-Coded DF (ACDF).

The received SNR at destination B is given by

$$\Gamma_B^{\text{ACDF}}(\phi_1, \phi_2) = (\phi_1 |h_{1B}|^2 + \phi_2 |h_{2B}|^2)\kappa\Gamma, \quad (15)$$

where, for $i = 1, 2$,

$$\phi_i \triangleq \begin{cases} 1, & \text{if } \Gamma_i \geq \Gamma_{thd}, \\ 0, & \text{otherwise.} \end{cases} \quad (16)$$

Correspondingly, we have

$$\Pr[\Gamma_B^{\text{ACDF}}(1,0) < \Gamma_{thd}] = \Pr\{\Gamma_B^{\text{ACDF}}(0,1) < \Gamma_{thd}\} \quad (17)$$
$$= 1 - e^{-x}, \quad (18)$$
$$\Pr[\Gamma_B^{\text{ACDF}}(1,1) < \Gamma_{thd}] = \Pr\{|h_{1B}|^2 + |h_{2B}|^2 < \frac{x}{\kappa}\} \quad (19)$$
$$= 1 - e^{-x} - (x/\kappa)e^{-x/\kappa}. \quad (20)$$

The outage probability can then be obtained by averaging over the four events that can occur at the first hop:

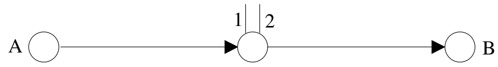

Fig. 2. Model for the derivation of the MIMO bound.

Theorem 3: The outage probability under ACDF is

$$\begin{aligned} p_{out}^{\text{ACDF}} = \; & (1-p_1)(1-p_2)\text{Pr}\{\Gamma_B^{\text{ACDF}}(1,1) < \Gamma_{thd}\} \\ & + p_1(1-p_2)\text{Pr}\{\Gamma_B^{\text{ACDF}}(0,1) < \Gamma_{thd}\} \\ & + p_2(1-p_1)\text{Pr}\{\Gamma_B^{\text{ACDF}}(1,0) < \Gamma_{thd}\} \\ & + p_1 p_2 \qquad (21) \\ = \; & 1 - 2e^{-(1+1/\kappa)x} + (1-x/\kappa)e^{-(2+1/\kappa)x}. \qquad (22) \end{aligned}$$

Note that the received SNR at destination B achieved by ACDF in (15) is uniformly greater than that achieved by ACAF in (10), since noise at the relays will not be accumulated in ACDF. Hence, ACDF yields a lower outage probability than ACAF.

V. Lower Bound

Viewing the two relays as one virtual relay node with two antennas, we get a new model as shown in Fig. 2. While assuming that the two antennas can cooperate in transmission and reception, we keep the original power constraint for each antenna. In other words, power sharing between the two antennas is not allowed. Obviously, the optimal outage performance of this system is better than that of the original model, since we have relaxed the constraint that the two relay nodes cannot cooperate. Thus the minimum outage probability can serve as a lower bound of the original system.

To find the minimum outage probability of this new system, we note that $X_A, (Y_1, Y_2)$ and Y_B form a Markov chain. According to the data processing inequality [15, Sec. 2.8], node B cannot decode the message successfully if the relay node cannot. Thus, DF is optimal. Let $p_{out}^{(1)}$ denote the minimum outage probability in the first hop, and $p_{out}^{(2)}$ denote the minimum outage probability in the second hop under the condition that no outage occurs at the relay node.

Theorem 4: The overall outage probability of the MIMO relay system is given by

$$\underline{p}_{out}^{\text{MIMO}} = 1 - (1 + (1+1/\kappa)x + x^2/\kappa)e^{-(1+1/\kappa)x}. \qquad (23)$$

Proof: Since the first hop is equivalent to a 1×2 SIMO channel, according to [16, Sec. 5.4.2], we have

$$p_{out}^{(1)} = \text{Pr}\left\{|h_{A1}|^2 + |h_{A2}|^2 < x\right\} = 1 - e^{-x} - xe^{-x}. \qquad (24)$$

For the second hop, we claim that the Alamouti code has the minimum outage probability among all schemes [16, Sec. 5.4.3]. Hence,

$$\begin{aligned} p_{out}^{(2)} &= \text{Pr}\left\{|h_{1B}|^2 + |h_{2B}|^2 < x/\kappa\right\}, \qquad (25) \\ &= 1 - e^{-x/\kappa} - x/\kappa e^{-x/\kappa}. \qquad (26) \end{aligned}$$

After simplification, the outage probability is given by (23). ■

Note that the expression in (25) is equal to that in (19), implying that the transmission of the second hop under ACDF is optimal, provided that the message can be successfully decoded by both relays in the first hop. In general, as one or both relays may not be able to decode the message, there is a performance gap between ACDF and the MIMO bound.

VI. Performance Bound of the ACDF Scheme

Given a certain outage probability ϵ, the required SNR of a transmission scheme is defined as the infimum of all SNRs such that its outage probability is no less than ϵ. The infimum of the required SNR of all possible transmission schemes is denoted by γ_ϵ^*. We define the SNR offset (in dB) of a transmission scheme as

$$\Delta_\epsilon \triangleq 10\log_{10}(\Gamma_\epsilon/\gamma_\epsilon^*), \qquad (27)$$

where Γ_ϵ is the required SNR of that scheme with outage probability ϵ.

To find an upper bound for the SNR offset of ACDF, we compare its outage probability with that of the MIMO bound. According to (22) and (23), we have

$$p_{out}^{\text{ACDF}}(x) = 1 - 2e^{-(1+\frac{1}{\kappa})x} + (1 - \frac{1}{\kappa}x)e^{-(2+\frac{1}{\kappa})x}, \qquad (28)$$

$$\underline{p}_{out}^{\text{MIMO}}(x) = 1 - (1 + (1+\frac{1}{\kappa})x + \frac{1}{\kappa}x^2)e^{-(1+\frac{1}{\kappa})x}. \qquad (29)$$

The following result is found useful:

Lemma 5: For all $x \geq 0$,

$$p_{out}^{\text{ACDF}}(x) \leq \underline{p}_{out}^{\text{MIMO}}(2x). \qquad (30)$$

Proof: We rewrite (30) as follows:

$$\frac{1 - p_{out}^{\text{ACDF}}(x)}{1 - \underline{p}_{out}^{\text{MIMO}}(2x)} = \frac{2e^{(1+\frac{1}{\kappa})x} - (1 - \frac{1}{\kappa}x)e^{\frac{1}{\kappa}x}}{1 + (2+\frac{2}{\kappa})x + \frac{4}{\kappa}x^2} \geq 1. \qquad (31)$$

It suffices to prove that

$$u(x) \triangleq 2e^{(1+\frac{1}{\kappa})x} - (1 - \frac{1}{\kappa}x)e^{\frac{1}{\kappa}x} - [1 + (2+\frac{2}{\kappa})x + \frac{4}{\kappa}x^2] \geq 0. \qquad (32)$$

To do that, we obtain the first two derivatives of $u(x)$ as

$$\begin{aligned} u'(x) = \; & 2(1+\frac{1}{\kappa})e^{(1+\frac{1}{\kappa})x} + \frac{1}{\kappa}e^{\frac{1}{\kappa}x} \\ & - (1 - \frac{1}{\kappa}x)\frac{1}{\kappa}e^{\frac{1}{\kappa}x} - [2(1+\frac{1}{\kappa}) + \frac{8}{\kappa}x], \qquad (33) \end{aligned}$$

$$u''(x) = 2(1+\frac{1}{\kappa})^2 e^{(1+\frac{1}{\kappa})x} + \frac{1}{\kappa^2}e^{\frac{1}{\kappa}x} + \frac{1}{\kappa^3}xe^{\frac{1}{\kappa}x} - \frac{8}{\kappa}. \qquad (34)$$

Applying the inequality $e^x \geq 1 + x$ to the first term of (34), we obtain

$$\begin{aligned} u''(x) \geq \; & (2 + \frac{4}{\kappa} + \frac{2}{\kappa^2})\left[1 + (1+\frac{1}{\kappa})x\right] \\ & + \frac{1}{\kappa^2}e^{\frac{1}{\kappa}x} + \frac{1}{\kappa^3}xe^{\frac{1}{\kappa}x} - \frac{8}{\kappa} \qquad (35) \\ = \; & 2(1 - \frac{1}{\kappa})^2 + (2 + \frac{4}{\kappa} + \frac{2}{\kappa^2})(1+\frac{1}{\kappa})x \\ & + \frac{1}{\kappa^2}e^{\frac{1}{\kappa}x} + \frac{1}{\kappa^3}xe^{\frac{1}{\kappa}x} \qquad (36) \\ \geq \; & 0. \qquad (37) \end{aligned}$$

In addition, since $u'(0) = 0$, we have $u'(x) \geq 0$ for $x \geq 0$, meaning that $u(x)$ is non-decreasing with x. Furthermore, since $u(0) = 0$, we obtain that $u(x) \geq 0$ for $x \geq 0$, which proves (32). ■

Theorem 6: The SNR offset of ACDF, Δ_ϵ, is less than 3 dB for any outage probability, ϵ, and target rate, R_{thd}.

Proof: According to Lemma 5, for any Γ_{thd}, the outage probability of ACDF can be made smaller than the MIMO bound by doubling the SNR for ACDF. Hence, the SNR offset of ACDF is at most $10 \log_{10} 2 \approx 3$ dB. ■

The bound on the SNR offset can be translated into a bound on the ϵ-outage capacity as follows:

Theorem 7: ACDF achieves within one bit of the ϵ-outage capacity of the diamond relay channel for any outage probability, ϵ, and operating SNR, Γ.

Proof: What we need to prove is that

$$R_\epsilon^{\text{MIMO}}(\Gamma) - R_\epsilon^{\text{ACDF}}(\Gamma) \leq 1, \tag{38}$$

for any Γ. From the definition of ϵ-outage rate in Sec. II and the relationship between the ϵ-outage rate and Γ_{thd}, we obtain that (38) is equivalent to

$$\log_2(1 + \Gamma_{thd}^{\text{MIMO}}) - \log_2(1 + \Gamma_{thd}^{\text{ACDF}}) \leq 1. \tag{39}$$

Here we have omitted the dependence of $\Gamma_{thd}^{\text{MIMO}}$ and $\Gamma_{thd}^{\text{ACDF}}$ on ϵ and Γ. Note that in this proof, ϵ and Γ are assumed to be any arbitrary constants. It is clear that the above inequality holds if

$$\Gamma_{thd}^{\text{MIMO}} / \Gamma_{thd}^{\text{ACDF}} \leq 2. \tag{40}$$

Lemma 5 says that

$$\Pr\{\Gamma_B^{\text{MIMO}}(\boldsymbol{h}, \Gamma) < \Gamma_{thd}\} \geq \Pr\{\Gamma_B^{\text{ACDF}}(\boldsymbol{h}, 2\Gamma) < \Gamma_{thd}\} \tag{41}$$

$$= \Pr\{\Gamma_B^{\text{ACDF}}(\boldsymbol{h}, \Gamma) < \frac{\Gamma_{thd}}{2}\}. \tag{42}$$

The last equality holds because scaling Γ and Γ_{thd} by the same amount does not affect the outage probability. This can be verified by observing from (16) that ϕ_1 and ϕ_2 remain constant after the scaling and from (15) that Γ_B^{ACDF} is scaled by the same amount.

Therefore, to keep the same outage probability, the SNR threshold for ACDF should be greater than one half of the SNR threshold for the MIMO bound, thus proving (40). ■

VII. Outage Probability Comparison

In this section, we compare the outage probabilities of different schemes under certain fading environments. We have the following setting: $\kappa = 1$, $\Gamma_{thd} = 1$. Each point in the curves is obtained by Monte-Carlo simulations through averaging over 3×10^6 channel realizations.

To improve the performance upon ACDF, we propose that a relay, if fails to decode the message, should adopt compress-and-forward (CF) on its received signal. The adopted relaying scheme at a relay node adaptively switches between DF and CF, hence the name of adaptive decode-and-forward and compress-and-forward (ADFCF). Due to space limitation, we omit the details of this scheme. The lower bound is obtained by combining the MIMO bound and the traditional cut-set bound [15, Sec. 14.10]; an outage event occurs if either an outage occurs in the MIMO relay system mentioned in Section V or the cut-set bound is violated.

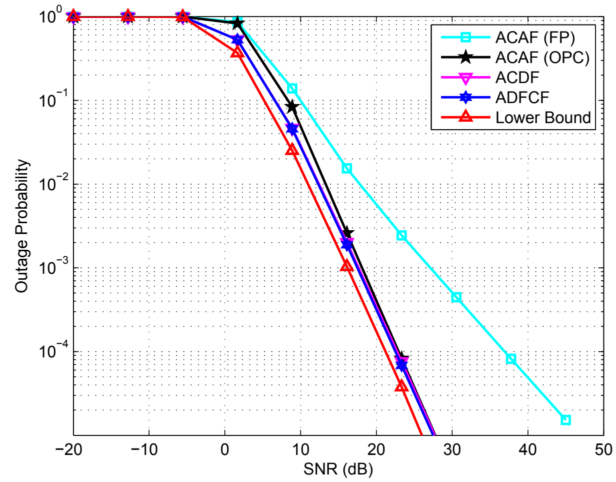

Fig. 3. Comparison of outage probability among different schemes, i.i.d. Rayleigh fading.

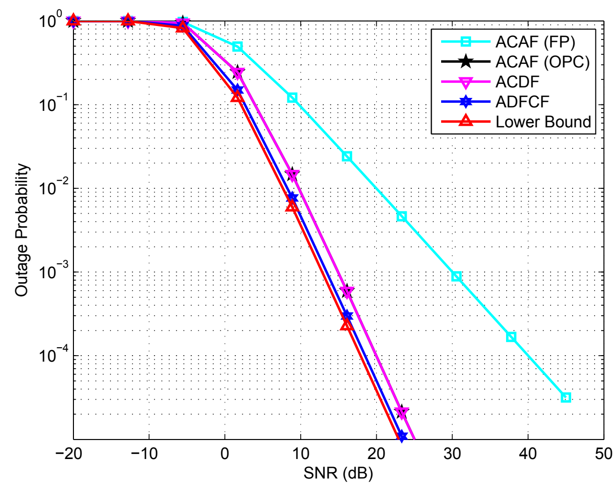

Fig. 4. Comparison of outage probability among different schemes, Rician fading, $\gamma = 100$.

We first consider i.i.d. Rayleigh fading in Fig. 3. We can see that ACAF (FP) does not have any diversity gain. However, when optimal power is used, the outage probability of ACAF (OPC) approaches that of ACDF at high SNR. Indeed, it achieves the maximal diversity gain of two. (Readers may refer to Appendix A for a sketch of the proof). For ACDF, we can see that the offset is about 1.6 dB under this numerical setting.

We next consider the case where the link gain distributions are asymmetric. We assume Rician fading for each link, where $|h_{A1}|, |h_{2B}| \sim \text{Rice}(1, \gamma)$ and $|h_{A2}|, |h_{1B}| \sim \text{Rice}(1, 0)$. The

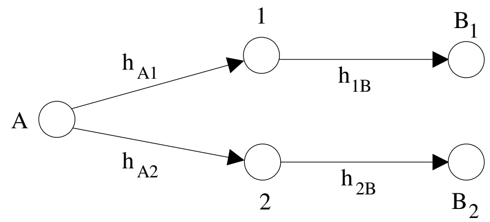

Fig. 5. A model that provides an upper bound on the outage probability of ACAF (OPC).

outage curves for the case where $\gamma = 100$ are plotted in Fig. 4. Contrary to the Rayleigh fading case, ADFCF outperforms ACDF significantly. Indeed, the gap between ADFCF and ACDF is 1.5 dB. The curve for ADFCF is just 0.3 dB away from the lower bound.

VIII. Conclusion

The outage performance of a two-hop diamond relay network is analyzed. An optimal power control rule for ACAF is derived. With this enhancement, ACAF performs as well as ACDF in the high SNR regime. The gap between ACDF and the optimal scheme is analytically derived, with the SNR offset within 3 dB and the ϵ-outage rate gap less than 1 bit.

If the link gains are not subject to i.i.d. Rayleigh fading, the performance of ACDF can be improved by allowing a relay, who fails to decode the message, to compress the received symbol and send it to the destination. This scheme, which we call ADFCF, is shown by simulation that it outperforms ACDF in a Rician fading environment. A more detailed study of this scheme is now under way.

Appendix A
Full Diversity of ACAF (OPC)

Theorem 8: Under i.i.d. Rayleigh fading, ACAF (OPC) achieves full diversity.

Proof: Since an exact diversity analysis for ACAF (OPC) is very complicated, we consider another model, which provides an upper bound on the outage probability of ACAF (OPC). The new model is obtained from the original model by splitting the destination, node B, into two virtual nodes, B_1 and B_2, as shown in Fig. 5. There is no communication or cooperation between these two virtual nodes so that joint decoding is impossible. The source and the relays are assumed to operate in exactly the same way as that in ACAF (OPC). Note that the Alamouti code becomes useless, since signal combining is not possible in the new model. An outage occurs if both B_1 and B_2 fail to decode the message.

It is not difficult to see that the outage probability of the new model provides an upper bound for ACAF (OPC) applied in the original model. If one or both relays encounter an outage in the first hop, i.e., $\Gamma_i < \Gamma_{thd}$, where $i \in \{1, 2\}$, the new model is equivalent to the original model. Otherwise, it can be shown that Γ_B in the original system is greater than both Γ_{B_1} and Γ_{B_2} in the new system, since Γ_B is obtained by substituting $(\xi_1, \xi_2) = (\xi_1^*, \xi_2^*)$ into (10) while Γ_{B_1} and Γ_{B_2} are obtained by substituting $(\xi_1, \xi_2) = (\xi_1^*, 0)$ and $(\xi_1, \xi_2) = (0, \xi_2^*)$ into (10), respectively. Hence, the original model yields a lower outage probability than the new one.

Now we have to obtain the achievable diversity for the new system. The derivation is rather involved, so here we only sketch the idea and omit the mathematical details. Intuitively, it is easy to see that the diversity gain of the new system is twice of that achieved by the following single relay AF scheme applied to a two-hop single-relay system: There is one single relay and no direct link between a source and a destination, the relay scales its received signal if its received SNR is above the threshold and keeps silent otherwise. Moreover, it can be shown that the diversity of the two-hop single relay AF scheme is one. The proof is then completed by combining the above two results. ■

References

[1] B. Schein, "Distributed coordination in network information theory," *PhD dissertation*, MIT, 2001.
[2] F. Xue and S. Sandhu, "Cooperation in a half-duplex Gaussian diamond relay channel," *IEEE Trans. Inf. Theory*, vol. 53, no. 10, pp. 3806-3814, Oct. 2007.
[3] Y. Kochman, A. Khina, R. Erez, and R. Zamir, "Rematch and forward for parallel relay networks," *Proc. International Symposium on Information Theory*, Toronto, Canada, pp. 767-771, Jul. 2008.
[4] I. Maric and R. Yates, "Forwarding strategies for Gaussian parallel-relay networks," *Proc. International Symposium on Information Theory*, Chicago, USA, pp. 270, Jun. 2004.
[5] A. del Coso and C. Ibars, "Distributed antenna channels with regenerative relaying: Relay selection and asymptotic capacity," *EURASIP Journal on Wireless Communications and Networking*, Nov. 2007.
[6] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2415-2425, Oct. 2003.
[7] S. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1451-1458, Oct. 1998.
[8] R. Nabar, H. Bolcskei, and F. W. Kneubohler, "Fading relay channels: Performance limits and space-time signal design," *IEEE J. Select. Areas Commun.*, vol. 22, no. 6, pp. 1099-1109, Aug. 2004.
[9] Y. Hua, Y. Mei, and Y. Chang, "Parallel wireless mobile relays with space time modulations," *IEEE Workshop on Statistical Signal Processing*, Missouri, USA, pp. 375-378, Sep-Oct. 2003.
[10] Y. Jing and B. Hassibi, "Distributed space-time coding in wireless relay networks," *IEEE Trans. Wireless Commun.*, vol. 5, no. 12, pp. 3524-3536, Dec. 2006.
[11] Z. Yi and II.-M. Kim, "Single-symbol ML decodable distributed STBCs for cooperative networks," *IEEE Trans. Inf. Theory*, vol. 53, no. 8, pp. 2977-2985, Aug. 2007.
[12] P. A. Anghel and M. Kaveh, "On the performance of distributed space-time coding systems with one and two non-regenerative relays," *IEEE Trans. Wireless Commun.*, vol. 5, no. 3, pp. 682-692, Mar. 2006.
[13] M. Dai and C. W. Sung, "A distributed on-off amplify-and-forward protocol for the fading parallel relay channel," *IEEE Commun. Letters*, vol. 13, no. 9, pp. 643-645, Sep. 2009.
[14] S. Avestimehr, S. Diggavi, and D. Tse, "A deterministic approach to wireless relay networks," *Proc. Allerton Conference on Communication, Control and Computing*, Illinois, USA, Sep. 2007.
[15] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. John Wiley & Sons, 1991.
[16] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2004.