Resource Allocation for Wireless Multi-carrier Network with Receiver Cooperation

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***Abstract*—A receiver-cooperative scheme which divides the four-node cooperative system into two orthogonal sub-channels is considered. In each sub-channel, the system becomes a multiple-access channel with multiple subcarriers. Our goal is to allocate subcarriers, power, and rate in a way that the sum rate is maximized. We propose two resource allocation strategies: time-sharing relaxation and heuristic subcarrier allocation with optimal power. We also give an upper bound for the sum rate achievable by our cooperative scheme. We compare the performance of our two proposed strategies with two other simple benchmarks in terms of sum rate and computation time. Simulation results shows the tradeoff between these two factors.**

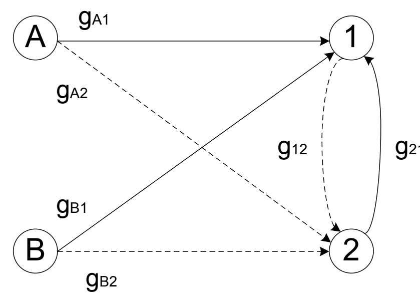

Fig. 1. System Model

I. INTRODUCTION

Cooperative communication is a technique which allows single-antenna mobiles to reap some of the benefits of multiple-input multiple-output (MIMO) systems. How to achieve optimum performance attracts a lot of research interest. For example, cooperative diversity is investigated in [1] [2] in detail. Distributed space-time-coded protocols for exploring cooperative diversity in wireless networks are introduced in [3]. Frequency-division transmitter cooperation is explored in [4]. Receiver cooperative networks are investigated in [5] [6]. Besides, there are also many achievements on resource allocation and system optimization. For example, a new fair resource allocation strategy for Gaussian broadcast channel with inter-symbol interference (ISI) is introduced in [7], and a cooperative diversity aware priority-based resource allocation scheme for relay-enhanced cellular systems is explored in [8].

This paper considers receiver cooperation in wireless multi-carrier systems with two source-destination pairs. We propose a new cooperative scheme, which divides the system into two orthogonal sub-channels. In each sub-channel, the system becomes the Gaussian multiple access channel (MAC), which is well studied in the literature. Our cooperative scheme is based on results from the Gaussian MAC. It has an advantage that any practical code designed for the Gaussian MAC can be applied directly to our scheme. For this cooperative scheme, we consider the resource allocation issue. To optimize the performance of the cooperative scheme, it is important to allocate power, subcarriers, rate in an effective way. In this paper, we propose two new strategies. Simulation results show that one has excellent sum rate performance but high computational complexity, while the other one is fast but has a slight degradation in sum rate.

Our paper is organized as follows: In Section II, we introduce the system model. A cooperative scheme is presented in Section III. In Section IV, we present the time-sharing relaxation strategy which solves the resource allocation problem in Section III. In Section V, a much simpler strategy called heuristic subcarrier allocation with optimal power is introduced. In Section VI, we compare the performance of the two strategies and some counterparts. In Section VII, we draw a short conclusion.

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II. SYSTEM MODEL

We consider a wireless network with two source-destination pairs, denoted by $(A, 1)$ and $(B, 2)$. The two receivers, nodes 1 and 2 can cooperate with each other via a wireless link between them. The wireless link between any two nodes is modeled as a frequency-selective block-fading channel. The whole radio spectrum is divided into N orthogonal subcarriers with the same bandwidth, so that in each subcarrier, the link gain is assumed to be constant. For $i = 1, 2, \ldots, N$, we let $g_{kl}^{(i)}$ be the link gain of subcarrier i from node k to node l (See Fig. 1). We assume that the link gains are known to all nodes. We impose a half-duplex requirement at nodes 1 and 2. The N subcarriers are partitioned into two groups. In the first group, node 1 receives and nodes 2, A and B transmit.

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In the second group, node 2 receives and nodes 1, A and B transmit. Define the following two index sets:

$$\mathcal{S}_1 = \{i = 1, 2, \ldots, N : \text{node 1 receives in subcarrier } i\}$$
$$\mathcal{S}_2 = \{i = 1, 2, \ldots, N : \text{node 2 receives in subcarrier } i\}$$

We note that $\mathcal{S}_1$ and $\mathcal{S}_2$ are disjoint, and $\mathcal{S}_1 \cup \mathcal{S}_2 = \{1, 2, \ldots, N\}$. In Fig. 1, the solid (dashed) arrows indicate the signal flow if the subcarrier is in the first (second) group.

Suppose that each subcarrier has bandwidth W Hz. In a period of T seconds, there are $2WT$ real degrees of freedom if $WT \gg 1$ [9, Section 4.7.2], i.e., we can transmit $2WT$ real channel symbols in each subcarrier. For $t = 1, 2, \ldots, 2WT$, the received channel symbol at subcarrier i in the first group is given by

$$Y_1^{(i)}(t) = g_{A1}^{(i)} X_A(t) + g_{B1}^{(i)} X_B(t) + g_{21}^{(i)} X_2(t) + Z_1^{(i)}(t),$$

where $Y_1^{(i)}(t)$ is the received symbol at node 1, $X_k^{(i)}(t) \in \mathbb{R}$ is the transmitted symbol of node k ($k = A, B, 2$), and $Z_1^{(i)}(t) \in \mathbb{R}$ is independently distributed Gaussian random variable with mean zero and variance n_0. For subchannel i in the second group, the received symbol at node 2 is

$$Y_2^{(i)}(t) = g_{A2}^{(i)} X_A(t) + g_{B2}^{(i)} X_B(t) + g_{12}^{(i)} X_1(t) + Z_2^{(i)}(t),$$

where $X_k^{(i)}(t) \in \mathbb{R}$ is the transmitted symbol of node k ($k = A, B, 1$), and $Z_2^{(i)}(t) \in \mathbb{R}$ is Gaussian noise with variance n_0. We see that in each subcarrier, the channel reduces to a multiple-access channel with three transmitters.

The transmission of node l, where $l = 1, 2$, is subject to the following power constraint:

$$\frac{1}{2WT|\mathcal{S}_l|} \sum_{i \in \mathcal{S}_l} \sum_{t=1}^{2WT} |X_l^{(i)}(t)|^2 \leq P_l.$$

We only sum over $\mathcal{S}_l$, because node l only transmits in the subcarriers indexed by $\mathcal{S}_l$.

For source node k, where $k = A, B$, the power constraint is

$$\frac{1}{2WTN} \sum_{i=1}^{N} \sum_{t=1}^{2WT} |X_k^{(i)}(t)|^2 \leq P_k.$$

We use $C(x) \triangleq 0.5 \log_2(1 + x)$ to denote Shannon's capacity formula and $\Gamma_{kl}^{(i)} \triangleq P_k \cdot (g_{kl}^{(i)})^2 / n_0$, for $i = 1, 2, \ldots, N$ and $k, l = A, B, 1, 2$, to denote the signal-to-noise ratio from node k to node l, in subcarrier i.

III. Cooperative Scheme

Each source node divides its data bits into two streams, called the *direct* stream and the *relay* stream. Data bits in the direct stream are sent directly via the link between the source and the intended destination. Data bits in the relay stream are first sent to the opposite destination node, and then forwarded to the final destination. For example, for source node A, the data in the direct stream is sent to node 1 via the link between node A and node 1, while the data in the relay stream goes through a 2-hop path, from node A to node 2, and then from node 2 to node 1.

The whole transmission time is divided into $\mathfrak{B} + 1$ blocks. The duration of each block is $T_{\mathfrak{B}}$ seconds, and hence we can transmit $2WT_{\mathfrak{B}}$ channel symbols in a block. Each stream of data is also divided into $\mathfrak{B}$ blocks. Each block of data is encoded and transmitted in a time duration of $T_{\mathfrak{B}}$ seconds. To send data through the multi-carrier wireless link, each block of data is further divided into several parts, with the number of parts equal to the number of subcarriers. Each data part is independently encoded in each subcarrier. At the receiver's side, the decoded bits from the subcarriers are merged together.

Since each subcarrier can be modeled as a multiple-access channel, we first review the capacity region of a Gaussian MAC with three senders. For a MAC with users a, b, and c, the channel output is

$$Y = X_a + X_b + X_c + Z,$$

where X_k ($k = a, b, c$) is the symbol transmitted by node k, and Z is independently distributed Gaussian random variable with zero mean and unit variance. Suppose that the power constraint of user k is Q_k. The capacity region of the three-user Gaussian MAC [10, Chapter 8], denoted by $D(Q_a, Q_b, Q_c)$, is the set consisting of rate triple (r_a, r_b, r_c) that satisfies:

$$\begin{aligned} 0 &\leq r_a \leq C(Q_a) \\ 0 &\leq r_b \leq C(Q_b) \\ 0 &\leq r_c \leq C(Q_c) \\ r_a + r_b &\leq C(Q_a + Q_b) \\ r_a + r_c &\leq C(Q_a + Q_c) \\ r_b + r_c &\leq C(Q_b + Q_c) \\ r_a + r_b + r_c &\leq C(Q_a + Q_b + Q_c). \end{aligned}$$

We will devise a cooperative transmission scheme based on the encoding scheme for MAC.

For node $k \in \{A, B\}$, denote the the message bits in the direct stream in the $\mathfrak{b}$-th block by $m_{kd}(\mathfrak{b})$, and denote the message bits in the relay stream in the $\mathfrak{b}$-th block by $m_{kr}(\mathfrak{b})$. We will use subscript "$_d$" to indicate that a variable is associated to the direct path, and "$_r$" for the relay path. The first block is the initialization block. The message $m_{Br}(1)$ is divided into $|\mathcal{S}_1|$ parts, labeled by $m_{Br}^{(i)}(1)$, for $i \in \mathcal{S}_1$. Similarly, $m_{Ar}(1)$ is divided into $m_{Ar}^{(i)}(1)$, for $i \in \mathcal{S}_2$. In the subcarriers indexed by $\mathcal{S}_1$, node 2 transmits $\mathbf{x}_{B1}^{(i)}(m_{Br}^{(i)}(1))$ during the first block, and node 1 transmits $\mathbf{x}_{A2}^{(i)}(m_{Ar}^{(i)}(1))$ in the subcarriers indexed by $\mathcal{S}_2$ during the first block. At the end of the first block, node 1 assemble the bits decoded from the subcarriers indexed by $\mathcal{S}_1$ and recovers $m_{Br}(1)$. Likewise, node 2 recovers $m_{Ar}(1)$ at the end of block 1.

For $\mathfrak{b} = 2, 3, \ldots, \mathfrak{B}$, the codewords transmitted by nodes A, B and 2 are in the subcarriers indexed by $\mathcal{S}_1$ are

$$\mathbf{x}_{A1}^{(i)}(m_{Ad}^{(i)}(\mathfrak{b}-1)),\ \mathbf{x}_{B1}^{(i)}(m_{Br}^{(i)}(\mathfrak{b})),\ \text{and}\ \mathbf{x}_{21}^{(i)}(m_{Ar}^{(i)}(\mathfrak{b}-1))$$

respectively. The superscript $^{(i)}$ signifies the part of the corresponding message that is transmitted in subchannel i, for $i \in \mathcal{S}_1$. We note that the message $m_{Ar}^{(i)}(\mathfrak{b}-1)$ from node A is

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Block 1	Block 2	Block 3
$m_{Ar}(1)$	$m_{Ar}(2)$, $w_{Ad}(1)$	$m_{Ad}(2)$
$m_{Br}(1)$	$m_{Br}(2)$, $w_{Bd}(1)$	$m_{Bd}(2)$
	$m_{Br}(1)$	$m_{Br}(2)$
	$m_{Ar}(1)$	$m_{Ar}(2)$

Fig. 2. The Encoded Messages in Each Block.

relayed by node 2 in the $\mathfrak{b}$-th block. In the subcarriers indexed by $\mathcal{S}_2$, the codewords transmitted by nodes B, A and 1 are

$$\mathbf{x}_{B2}^{(i)}(m_{Bd}^{(i)}(\mathfrak{b}-1)),\ \mathbf{x}_{A2}^{(i)}(m_{Ar}^{(i)}(\mathfrak{b})),\ \text{and}\ \mathbf{x}_{12}^{(i)}(m_{Br}^{(i)}(\mathfrak{b}-1)).$$

The superscript $^{(i)}$ signifies the part of the corresponding message that is transmitted in subchannel i, for $i \in \mathcal{S}_2$. Also, we note that the message $m_{Br}^{(i)}(\mathfrak{b}-1)$ is from node B relayed by node 1 in the $\mathfrak{b}$-th block. At the end of each block, the decoded bits from the subcarriers are put together to form the message $m_{Ad}(\cdot)$, $m_{Ar}(\cdot)$, $m_{Bd}(\cdot)$, and $m_{Br}(\cdot)$.

In the $(\mathfrak{B}+1)$-st block, the codewords sent by nodes A and 2 in the subcarriers indexed by $\mathcal{S}_1$ are $\mathbf{x}_{A1}^{(i)}(m_{Ad}^{(i)}(\mathfrak{B}))$, $\mathbf{x}_{21}^{(i)}(m_{Ar}^{(i)}(\mathfrak{B}))$, respectively, and the codewords sent by nodes B and 1 in the subcarriers indexed by $\mathcal{S}_2$ are $\mathbf{x}_{B2}^{(i)}(m_{Bd}^{(i)}(\mathfrak{B}))$, $\mathbf{x}_{12}^{(i)}(m_{Ar}^{(i)}(\mathfrak{B}))$, respectively.

We illustrate the coding scheme for $\mathfrak{B}=3$ in Fig. 2. The messages in the four rows are the encoded messages sent by node A, B, 1 and 2 respectively. We note that in this transmission scheme, there is loss of data rate by a factor of $\mathfrak{B}/(\mathfrak{B}+1)$. This factor is negligible because it converges to one as $\mathfrak{B}$ tends to infinity.

For $i \in \mathcal{S}_1$, let the code rates of nodes A, B and 2 be $r_{A1}^{(i)}$, $r_{B1}^{(i)}$ and $r_{21}^{(i)}$, respectively. Also, for $i \in \mathcal{S}_2$, let the code rate of nodes$A$, B and 1 be $r_{A2}^{(i)}$, $r_{B2}^{(i)}$ and $r_{12}^{(i)}$, respectively. For $k = A, B, 2$, we let

$$r_{k1} \triangleq \sum_{i \in \mathcal{S}_1} r_{k1}^{(i)}, \tag{1}$$

and for $k = A, B, 1$,

$$r_{k2} \triangleq \sum_{i \in \mathcal{S}_2} r_{k2}^{(i)}. \tag{2}$$

The data rate r_{kl} is the aggregate rate summed over all subcarriers in $\mathcal{S}_l$.

We let $\omega_k^{(i)}$ $(k = A, B, 1, 2)$ be the fraction of the transmission power of node k allocated to subcarrier i. They satisfy the following constraints:

$$\omega_k^{(i)} \geq 0, \quad \forall i, \text{ and } k = A, B, 1, 2, \tag{3}$$

$$\sum_{i=1}^{N} \omega_A^{(i)} = \sum_{i=1}^{N} \omega_B^{(i)} = \sum_{i \in \mathcal{S}_1} \omega_2^{(i)} = \sum_{i \in \mathcal{S}_2} \omega_1^{(i)} = 1 \tag{4}$$

The constraint in (4) show that both source nodes transmit in all N subcarriers, and the two destination nodes transmit only in one of the two groups of subcarriers.

Our goal is to allocate resources in a way that the sum rate

$$R = r_{A1} + r_{A2} + r_{B1} + r_{B2}$$

is maximized, subject to (1), (2), (3), (4), and

$$\mathcal{S}_1 \cap \mathcal{S}_2 = \emptyset, \qquad \mathcal{S}_1 \cup \mathcal{S}_2 = \{1, 2, \ldots, N\} \tag{5}$$

$$(r_{A1}^{(i)}, r_{B1}^{(i)}, r_{21}^{(i)}) \in D(\omega_A^{(i)}\Gamma_{A1}^{(i)}, \omega_B^{(i)}\Gamma_{B1}^{(i)}, \omega_2^{(i)}\Gamma_{21}^{(i)}) \tag{6}$$

$$(r_{A2}^{(i)}, r_{B2}^{(i)}, r_{12}^{(i)}) \in D(\omega_A^{(i)}\Gamma_{A2}^{(i)}, \omega_B^{(i)}\Gamma_{B2}^{(i)}, \omega_1^{(i)}\Gamma_{12}^{(i)}) \tag{7}$$

$$r_{A2} = r_{21},\ r_{B1} = r_{12} \tag{8}$$

where (6) is for $i \in \mathcal{S}_1$, and (7) is for $i \in \mathcal{S}_2$.

We assume that the optimal encoding for three-user Gaussian MAC is used in each subcarrier. We thus have (6) and (7) as the rate constraints. Besides, the two destination nodes only help forward messages, and do not have their own messages, so the incoming rate is equal to the outgoing rate and. This yields the constraint in (8).

In this problem, we have to allocate subcarriers, to allocate power of each node to the subcarriers assigned, and in each subcarrier, to determine the three rates at which the MAC should operate. Therefore, the problem can be regarded as a joint subcarrier, power, and rate allocation problem. It includes non-convex constraints (See (5)). On the other hand, if the subcarrier allocation is determined, the power and rate allocation subproblem is convex and can be solved by standard convex optimization techniques. Based on this observation, in the next two sections, we will present two methods to determine the subcarrier allocation. After that, the power and rate allocation subproblem can be readily solved.

IV. Time-Sharing Relaxation Strategy

In this section, we introduce a time-sharing relaxation method. The idea is to relax the constraint that each subcarrier is allocated exclusively to one of the receiving nodes. In this relaxation method, a fraction of time in a subcarrier is allocated to the MAC with node 1 as the destination, and the rest of time is allocated to the MAC with node 2 as the destination. This relaxed problem is convex and can be solved by standard convex optimization techniques. To find a feasible solution, we quantize the subcarrier allocation and then optimize the transmission power. In subcarrier i, we will call the time duration when node 1 is receiving *phase 1*, and the time duration when node 2 is receiving *phase 2*.

For $0 \leq \tau \leq 1$, we define $V(\Gamma_a, \Gamma_b, \Gamma_c, \tau)$ as the set consisting of rate triple (r_a, r_b, r_c) that satisfies

$$(r_a, r_b, r_c) \in \tau \cdot D(\Gamma_a/\tau, \Gamma_b/\tau, \Gamma_c/\tau).$$

Here, the rate region D is defined in the previous section, and the product of a real number τ and a set S is defined as

$$\tau \cdot S \triangleq \{\tau x : x \in S\}.$$

$V(\Gamma_a, \Gamma_b, \Gamma_c, \tau)$ is the rate region when the the transmission time is only a fraction τ of the total time. Hence, we see that the power is increased by a factor of $1/\tau$, but the data rate is decreased by a factor of τ. When τ is equal to one,

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$V(\Gamma_a, \Gamma_b, \Gamma_c, 1)$ becomes the capacity region of a three-user Gaussian MAC.

Let $\tau^{(i)}$ be the fraction of time that subcarrier i is used in phase 1. Let $\bar{\tau}^{(i)} = 1 - \tau^{(i)}$ be the fraction of time subcarrier i is used in phase 2. Let $\varepsilon_{kl}^{(i)}$ be the fraction of node k's power allocated to the link from node k to node l at subcarrier i. The relaxed problem is to maximize $R = r_{A1} + r_{A2} + r_{B1} + r_{B2}$ subject to

$$(r_{A1}^{(i)}, r_{B1}^{(i)}, r_{21}^{(i)}) \in V(\varepsilon_{A1}^{(i)}\Gamma_{A1}^{(i)}, \varepsilon_{B1}^{(i)}\Gamma_{B1}^{(i)}, \varepsilon_{21}^{(i)}\Gamma_{21}^{(i)}, \tau^{(i)}) \; \forall i \quad (9)$$

$$(r_{A2}^{(i)}, r_{B2}^{(i)}, r_{12}^{(i)}) \in V(\varepsilon_{A2}^{(i)}\Gamma_{A2}^{(i)}, \varepsilon_{B2}^{(i)}\Gamma_{B2}^{(i)}, \varepsilon_{12}^{(i)}\Gamma_{12}^{(i)}, \bar{\tau}^{(i)}) \; \forall i \quad (10)$$

$$\sum_{i=1}^{N} (\varepsilon_{A1}^{(i)} + \varepsilon_{A2}^{(i)}) = \sum_{i=1}^{N} (\varepsilon_{B1}^{(i)} + \varepsilon_{B2}^{(i)}) = 1 \quad \forall i \quad (11)$$

$$\sum_{i=1}^{N} \varepsilon_{21}^{(i)} = \sum_{i=1}^{N} \varepsilon_{12}^{(i)} = 1 \quad \forall i \quad (12)$$

$$\varepsilon_{kl}^{(i)} \geq 0, \qquad \forall i \text{ and } k, l = A, B, 1, 2, \quad (13)$$

$$0 \leq \tau^{(i)} \leq 1, \quad \forall i \quad (14)$$

and the constraints in (1), (2), (3), (4), and (8).

Note that the constraint in (9) is for phase 1, and that in (10) is for phase 2. The problem formulated above is convex, and can be solved by convex programming. Besides, any feasible sum rate also satisfies the constraints of the problem above for some suitably chosen parameters. Hence, the resultant solution provides an upper bound to the sum rate maximization problem with constraint (1) to (8).

After solving the relaxed problem, we quantize the subcarrier allocation by assigning subcarrier i exclusively to phase 1 if $\tau^{(i)}$ is larger than $\bar{\tau}^{(i)}$, and phase 2 otherwise. In other words,

$$\mathcal{S}_1 = \{i = 1, 2, \ldots, N : \tau^{(i)} \geq 0.5\},$$
$$\mathcal{S}_2 = \{i = 1, 2, \ldots, N : \tau^{(i)} < 0.5\}.$$

Once the subcarrier allocation is fixed, we can determine powers and rates by solving the original problem with constraints (1) to (4) and (6) to (8). The resulting problem is convex and hence can be solved readily by standard convex optimization software.

V. Heuristic Subcarrier Allocation

Solving the relaxed problem in the previous section is quite time consuming, especially when the number of subcarriers is large. In this section, we propose a much simpler heuristic strategy. In this method, rather than relaxing the problem, we allocate subcarriers based on the channel state information (CSI) of every subcarrier, which is known to every node.

Our heuristic is to compare the sums of the three link gains associated with node 1, and the sums of the three link gains associated with node 2, in each subcarrier. A subcarrier is assigned to node 1 if node 1 has the larger sum of link gains. In other words, we perform the subcarrier allocation by

$$\mathcal{S}_1 = \{i = 1, \ldots, N : g_{A1}^{(i)} + g_{B1}^{(i)} + g_{21}^{(i)} \geq g_{A2}^{(i)} + g_{B2}^{(i)} + g_{12}^{(i)}\},$$
$$\mathcal{S}_2 = \{i = 1, \ldots, N : g_{A1}^{(i)} + g_{B1}^{(i)} + g_{21}^{(i)} < g_{A2}^{(i)} + g_{B2}^{(i)} + g_{12}^{(i)}\}.$$

After allocating the subcarriers, power and rate optimization is performed to obtain a complete solution.

VI. Performance Comparisons

For comparison purpose, we consider two other simple strategies. The first one is called random subcarrier allocation with optimal power allocation. It randomly allocates each subcarrier independently with probability 1/2 to phase 1 and probability 1/2 to phase 2. The sum rate is then maximized by allocating power and rate optimally, subject to the constraints (1) to (4), and (6) to (8).

The second one is called random subcarrier allocation with equal power allocation. It allocates each subcarrier also independently with probability 1/2 to phase 1 and probability 1/2 to phase 2. Then, each node allocates its power equally among all the subcarriers it gets. Suppose there are N_1 elements in $\mathcal{S}_1$, and N_2 elements in $\mathcal{S}_2$ ($N_1 + N_2 = N$). Now we have to maximize $R = r_{A1} + r_{A2} + r_{B1} + r_{B2}$, subject to:

$$\omega_A^{(i)} = \omega_B^{(i)} = \frac{1}{N}, \; i = 1, 2, \ldots, N,$$
$$\omega_2^{(i)} = \frac{1}{N_1}, \; i \in \mathcal{S}_1, \quad \omega_1^{(j)} = \frac{1}{N_2}, \; j \in \mathcal{S}_2,$$

and (1), (2), and (6) to (8). We remark that this rate allocation subproblem is a linear programming problem, and can thus be easily solved.

We select several typical examples and compare the corresponding performances. The computation is done by CVX, a package for solving convex programs [11]. The noise power in every case is normalized to $n_0 = 1$. The link gains for every case are independently distributed random variables, and the mean of their squared magnitude is 1. In Fig. 3, we consider the case with high SNR, in which the power at each node is ten times of the noise power. In Fig. 4, we consider the case with low SNR, in which the power at each node is equal to the value of the noise power. In either case, we select the number of subcarriers $N = 4, 8, 16, 32, 64$ for comparisons. We have in total 4 resource allocation strategies to compare:

1) Random subcarrier allocation with equal power;
2) Random subcarrier allocation with optimal power;
3) Heuristic subcarrier allocation with optimal power;
4) Time-sharing relaxation.

We also show the upper bound obtained from the time-sharing relaxed problem. Recall that without doing the quantization, the resultant solution becomes an upper bound.

From the figures, we can see that in both SNR regimes, the sum rate performance increases from Strategy 1 to Strategy 4. Besides, the heuristic subcarrier allocation with optimal power is close to the time-sharing relaxation strategy in both cases. Strategy 4 performs the worst, yielding a sum rate which is roughly only half of that obtained by the time-sharing relaxation strategy. Moreover, the time-sharing relaxation strategy, which performs the best, is very close to the upper bound. Nevertheless, the computational complexity also increases from Strategy 1 to Strategy 4. We compare their average computation times in Fig. 5. We find that although Strategy 1

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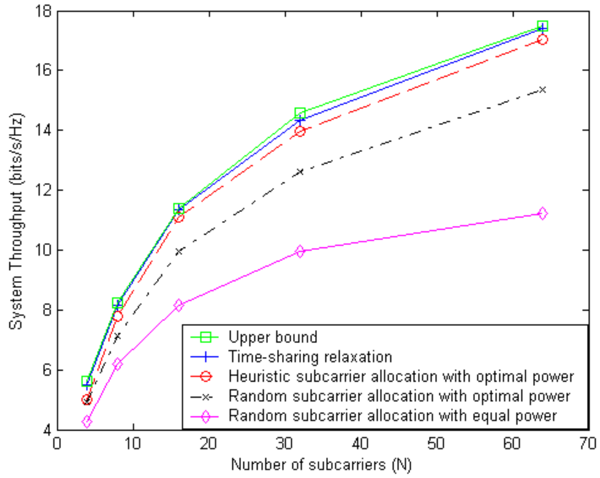

Fig. 3. Performance comparisons ($P_A = P_B = P_1 = P_2 = 10$)

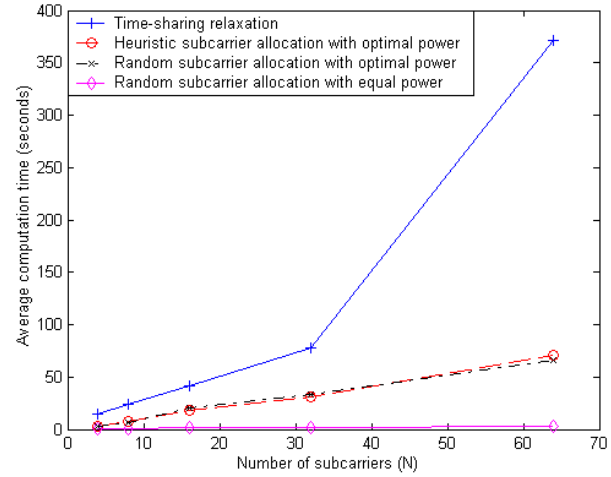

Fig. 5. Average execution time of the two proposed strategies

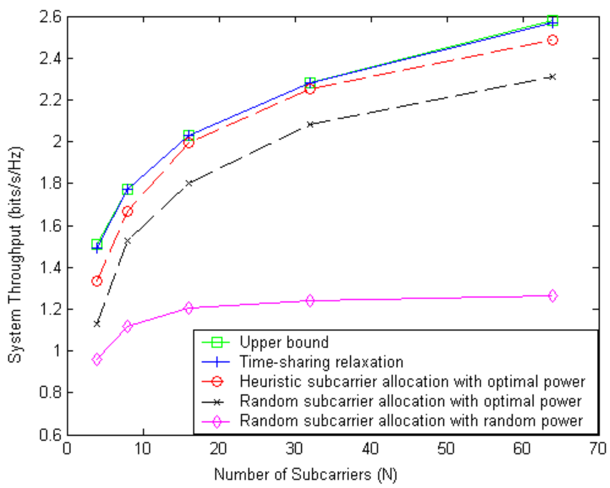

Fig. 4. Performance comparisons ($P_A = P_B = P_1 = P_2 = 1$)

performs the worst among all the strategies, it needs the least average computation time. Particularly, it only needs roughly 0.6% of the computation time of the time-sharing relaxation strategy when there are 64 subcarriers. Its execution time is low, since it only needs to solve a linear programming problem. The computation times of Strategy 2 and Strategy 3 are similar, because both of them need to solve the power and rate allocation subproblem. The heuristic rule in Strategy 3 is so simple that it yields an improvement in sum rate over Strategy 2 without any computational cost. Strategy 4 is complex, and its computation time grows exponentially with the number of subcarriers.

VII. Conclusion

In this paper, we decompose the receiver cooperative channel into two MACs by allocating subcarriers between them. Based on this cooperative scheme, we propose several resource allocation strategies and compare them in terms of sum rate and average computation time. We show that there is a tradeoff between sum rate performance and computational complexity. For example, the time-sharing relaxation strategy is nearly optimal in terms of sum rate, but requires long computation time. Its growth rate is very fast when the number of subcarriers increases. We have also proposed a simple heuristic algorithm. Its performance is close to the time-sharing relaxation strategy but it requires much shorter computation time. We believe that it is very suitable for practical use in systems with large number of subcarriers.

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