A Cross-Layer Design of Network Coded Retransmissions in Wireless Relay Channels

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***Abstract*—In this paper we investigate the multiple-access relay channels with feedback available at relay, and we concentrate on designing retransmission protocol at relay. The feedback contains the realized Signal-to-Noise Ratio (SNR) of each packet from different users. The relay utilizes such information from the physical layer and decides what to retransmit with the help of network coding. The objective is to ensure each packet has its SNR larger than some predetermined value so that certain Bit-Error Rate (BER) can be guaranteed. Simulation results show that such a cross-layer design leads to significant throughput improvement.**

I. Introduction

Network coding provides a good solution on improving system throughput [1]. In multiple-access relay channels, the relay overhears messages from the users and may transmit network coded messages to the destination to achieve higher spectral efficiency. Chen, Kishore and Li in [2] discuss wireless networks including multiple-access relay channels with cooperations among users. They demonstrate that by performing network coding at the relay one can have better diversity performance than the case without network coding. On the other hand, Woldegebreal and Karl in [3] consider the outage probability at the destination in multiple-access relay channels with non-ideal source-to-relay channels and show the advantage of performing network coding at relay under some particular settings.

As pointed out in [4], to guarantee the most reliable communication, it is necessary to consider how channel coding can be designed jointly with network coding. In [5], joint channel and network decoding using low-density parity-check codes is investigated, and the results show that joint encoding and decoding performs better than the case without using network coding in terms of Bit-Error Rate (BER) and frame error rate. The joint channel and network coding design using convolutional codes is also considered in [6].

Meanwhile, in designing network layer protocols, the channel models are often assumed to be erasure channels. The authors in [7] consider erasure broadcast channel with feedback and propose some opportunistic methods to construct instantly decodable network coded packets after initial transmission. The feedback in the aforementioned work contains information that tells the source whether a previously transmitted packet is correctly received or simply erased. However, erasure channels cannot precisely describe the actual wireless channel, and direct application of algorithms designed solely for network layer to real world systems may cause significant performance loss.

In this paper, we consider multiple-access relay channels and we concentrate on the relay's retransmission design. The objective is to ensure each packet received by the destination has a BER which is lower than some pre-determined value. The users have different message packets to transmit. After the initial transmission of the users, the destination receives noisy versions of the transmitted packets. Based on the feedback from the destination which describes the Signal-to-Noise Ratio (SNR) of each packet, the relay tries to find a network coded packet that, if correctly received by the destination, can raise the SNR of the packets involved at destination higher than a threshold.

The rest of the paper is organized as follows. We present the system model and explain how the relay can retransmit according to the feedback from destination in Section II. We also give the formal formulation of the relay retransmission problem in the same section. In Section III we develop algorithms for finding network coded packets to retransmit by relay. We finally show the simulation results and conclude the paper in Section IV and V.

II. System Model and Problem Formulation

A. System Model

Consider the relay channel shown in Fig. 1, where there are a total of K users each has an individual packet of message to transmit to the destination. A single relay is placed between the users and the destination. Denote the packet which contains a sequence of bits for user k as P_k, where $k \in \{1, 2, ..., K\}$. Sufficient cyclic redundant checks will be added to a message packet to detect errors, and a message packet will be encoded by some suitable channel codes before transmission. Denote an encoded message packet of user k as $\mathbf{X}_k$. In the initial phase, each user broadcasts its own message packet in different time slots with unit power which can be heard by the relay and

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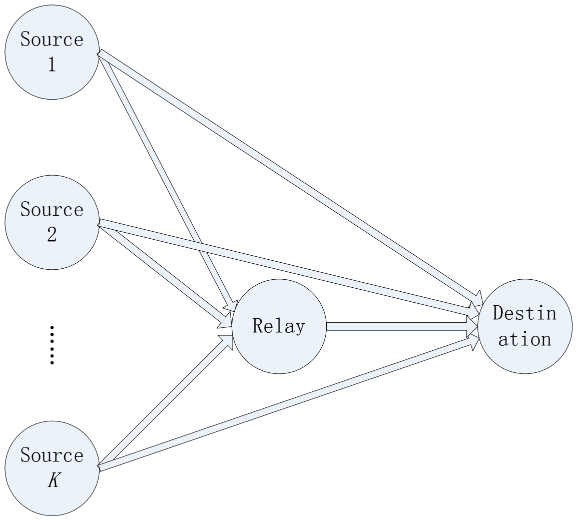

Fig. 1. The multiple-access relay channel with K users

destination. The received packet of user k at destination can be written as:

$$\mathbf{Y}_k = |h_{kd}|\mathbf{X}_k + \mathbf{n}_{kd}, \quad (1)$$

where h_{kd} denotes the channel coefficient between user k and destination. The channel coefficient is assumed to be complex Gaussian random variables with zero mean and unit variance. It is also assumed that the channel coefficients in different time slots are independent and remain unchanged within the slot, so that the channel can be characterized as block Rayleigh fading channel. $\mathbf{n}_{kd}$ is the Additive White Gaussian Noise (AWGN) vector with each element an i.i.d. zero mean σ^2 variance Gaussian random variable. The SNR for each received packet at destination is denoted as Γ_k. The packet size is sufficiently large such that $\Gamma_k = \frac{|h_{kd}|^2}{\sigma^2}$. As for the user-to-relay channels, we assume the relay is close enough to the users such that after the initial phase, the relay can get all the packets from the K users correctly. For the time being, we also assume the relay-to-destination channel is good enough such that the destination can always decode what the relay transmits correctly.

As mentioned in the previous section, the destination needs to ensure each packet satisfies some BER requirement. This objective can be achieved if we can make every packet's SNR larger or equal to a threshold. Denote Γ_{th} as the SNR threshold such that a packet from user k is considered to be correctly received at destination if $\Gamma_k \geq \Gamma_{th}$.

We assume the destination has the knowledge of channel-state information. After the initial phase, the destination receives K packets each with its SNR Γ_k. Denote the set of SNR values that are lower than Γ_{th} as Φ. The destination will feedback Φ together with the indices of packets that have their SNRs in Φ to the relay and we assume the feedback channel is perfect. The relay then starts the retransmission phase based on the feedback information. The transmit power of relay is set to be 1.

B. Problem Formulation

To illustrate how the relay can retransmit according to the feedback, consider the following example. Assume $\Phi = \{\Gamma_1, \Gamma_2\}$ and $\Gamma_1 + \Gamma_2 > \Gamma_{th}$. The relay can transmit the pairwise combination of P_1 and P_2 given by $P_1 \oplus P_2$, where $\oplus$ means addition modulo 2. The relay encodes $P_1 \oplus P_2$ into $\mathbf{X}_{1,2}$ before transmission. By previous assumptions, the destination can decode $\mathbf{X}_{1,2}$ without error. Assume BPSK modulation, with $\mathbf{Y}_1$ and $\mathbf{X}_{1,2}$, a noisy version of P_2 with SNR Γ_1 can be derived by simply multiplying $\mathbf{Y}_1$ and $\mathbf{X}_{1,2}$ symbol by symbol. Denote the noisy encoded version of P_2 so derived as $\mathbf{Y}_2'$. Using Maximal Ratio Combining (MRC), we can combine $\mathbf{Y}_2$ and $\mathbf{Y}_2'$ to form a new packet with SNR $\Gamma_1 + \Gamma_2$. Since $\Gamma_1 + \Gamma_2 > \Gamma_{th}$, P_2 is rescued by the network coded retransmission from relay. With correct P_2 and $P_1 \oplus P_2$, the correct P_1 can be found. In general, for $\Phi = \{\Gamma_1, \Gamma_2, ..., \Gamma_m\}$, $m \geq 2$, if $\Gamma_1 + \Gamma_2 + ... + \Gamma_m \geq \Gamma_{th}$, the relay can form a m-wise combination in the following way: $P_1 \oplus P_2$, $P_1 \oplus P_3$, ..., $P_1 \oplus P_m$. The relay then transmit each of the m network coded packets. By receiving such a m-wise combination, the destination can first rescue P_1, and then all the other packets can be derived by modulo 2 additions. We term such a m-wise combination as a valid m-wise combination. By doing this the relay transmits one fewer packet compared to the trivial scheme that the relay simply retransmits packets that have SNR values less than Γ_{th} one by one. We define the throughput gain when the relay performs network coding as:

$$G = 1 - \frac{Number\ of\ packets\ after\ network\ coding}{Number\ of\ elements\ in\ \Phi}. \quad (2)$$

To determine the minimum number of retransmissions by relay given an instance of Φ, we need to find the maximum number of disjoint subsets of Φ, such that the sum SNRs inside each subset is larger or equal to Γ_{th}. Note that this problem is equivalent to the bin covering problem, and it is NP-hard [8]. We therefore consider the following simplified problem and try to find heuristic algorithms for it. Let Φ=$\{\Gamma_1, \Gamma_2, ..., \Gamma_m\}$, $0 \leq m \leq K$. Define $|A|$ as the cardinality of some set A, Φ=$S_1 \cup S_2 \cup ... \cup S_t \cup B$, where S_1, S_2, ..., S_t, and B are disjoint subsets of Φ. The problem can be stated as follows:

$$\begin{aligned} \text{given } & \Phi = \{\Gamma_1, \Gamma_2, ..., \Gamma_m\}, \\ \max\ & t \\ \text{subject to } & S_1 \cup S_2 \cup ... \cup S_t \cup B = \Phi, \\ & S_i \cap S_j = \emptyset, 1 \leq i < j \leq t, \\ & S_k \cap B = \emptyset, 1 \leq k \leq t, \\ & \sum_{\forall i, \Gamma_i \in S_j} \Gamma_i \geq \Gamma_{th}, 1 \leq j \leq t, \\ & 1 < |S_j| \leq U, 1 \leq j \leq t. \end{aligned} \quad (3)$$

In the following section, algorithms for $U = 2$ and $U = 3$ are derived. We call the problem in (3) as cardinality constrained bin covering problem.

III. Algorithms for Cardinality Constrained Bin Covering Problem

In this section, we first present an algorithm which is later proved to be optimum if we fix $|S|$, where S is a subset of Φ that has sum of all its elements larger or equal to Γ_{th}. That is to say, we fix the number of packets involved in a network coding procedure to be all $|S|$-wise combinations. Then we discuss two algorithms where we allow pairwise combinations and 3-wise combinations to take place when doing network coding at relay. Finally we give a very simple upper bound on the number of combinations that can be found given an arbitrary instance Φ, which will be later used as a lower bound on number of retransmissions by relay.

A. Fixed $|S|$ Case

The main idea is that, in order to find the maximum number of subsets when restricted to $|S|$-wise network coded packets, we should avoid combining packets whose sum SNR far exceeds Γ_{th}. This property can be guaranteed if for each time when one searches for a valid $|S|$-wise combination, the sum SNR of this combination is the least one among all valid combinations. To implement such an algorithm, one can first find all possible $|S|$-wise combinations, find a valid one among them with the least sum SNR that is larger than Γ_{th} if exists, remove the elements in Φ involved in the valid $|S|$-wise combination, and then repeat the same process until no valid $|S|$-wise combinations can be found. The details of the algorithm is listed below, where $\mathbf{H}^{a\times b}$ represents a matrix $\mathbf{H}$ with a rows and b columns:

Algorithm 1 For $|S|$-wise combinations only

From destination feedback Φ=$\{\Gamma_1, \Gamma_2, ..., \Gamma_m\}$
Find all possible $|S|$-wise combinations of elements in Φ and store them in $\mathbf{C}^{\binom{m}{|S|}\times|S|}$
Compute the sum of each row in $\mathbf{C}$ and store them in $\mathbf{v}^{\binom{m}{|S|}\times 1}$
$p = 0,\ S_1 = \mathbf{0}$
for $i = 1$ **to** $\binom{m}{|S|}$ **do**
  Find $\mathbf{v}(j)$ = $\min(\mathbf{v}(x)|\mathbf{v}(x) \geq \Gamma_{th}, x \in [1, \binom{m}{|S|}])$
  if $\{$elements in jth row of $C\}\cap S_x = \emptyset, \forall x \in [1, p]$ **then**
    $p = p + 1,\ S_p = \mathbf{v}(j)$
    delete the jth row of $\mathbf{C}$, delete $\mathbf{v}(j)$ from $\mathbf{v}$
  else
    delete the jth row of $\mathbf{C}$, delete $\mathbf{v}(j)$ from $\mathbf{v}$
  end if
end for
$B = \Phi - \{S_1 \cup S_2 \cup ... \cup S_p\}$

After finding $S_1, S_2, ..., S_p$ and B, we may determine the number of retransmissions required by relay: For $S_1, S_2, ..., S_p$, the number of retransmissions required is $(|S_1| - 1)\cdot p$; For B, the number of retransmissions is simply the number of elements in itself. The throughput gain is thus:

$$G = 1 - \frac{(|S_1| - 1)\cdot p + |B|}{|\Phi|}. \quad (4)$$

We next prove that Algorithm 1 is optimum for the cardinality constrained bin covering problem if only $|S|$-wise combinations are considered.

Theorem 1: Algorithm 1 is optimum for any positive integer $|S|$ if we only consider $|S|$-wise combinations.

Proof: For $U = 1$ it is trivial. For $U \geq 2$, let us assume $\Phi = \{\Gamma_1, \Gamma_2, ..., \Gamma_m\}$, and by Algorithm 1 we get Φ=$S_1\cup S_2\cup ...\cup S_t \cup B$, where $|S_q| = U, q \in [1, t]$. Let $h \leq |B|$ and $h < U$. Randomly choose h terms from B, denote these terms as $\Gamma_{1,B}, \Gamma_{2,B}, ..., \Gamma_{h,B}$. Randomly choose h terms from S_q and denote them as $\Gamma_{1,S_q}, \Gamma_{2,S_q}, ..., \Gamma_{h,S_q}$. Swap $\{\Gamma_{1,B}, \Gamma_{2,B}, ..., \Gamma_{h,B}\}$ with $\{\Gamma_{1,S_q}, \Gamma_{2,S_q}, ..., \Gamma_{h,S_q}\}$ so that $\{\Gamma_{1,B}, \Gamma_{2,B}, ..., \Gamma_{h,B}\} \subseteq S_q$, $\{\Gamma_{1,S_q}, \Gamma_{2,S_q}, ..., \Gamma_{h,S_q}\} \subseteq B$. There are two cases in this situation:

1) $\Gamma_{1,B}+\Gamma_{2,B}+...+\Gamma_{h,B} \geq \Gamma_{1,S_q}+\Gamma_{2,S_q}+...+\Gamma_{h,S_q}$. In this case we have $\Gamma_{1,B} + \Gamma_{2,B} + ... + \Gamma_{h,B} + \Gamma_{h+1,S_q} + ... + \Gamma_{U,S_q} \geq \Gamma_{th}$. Since $\Gamma_{1,B} + \Gamma_{2,B} + ... + \Gamma_{h,B} + \Gamma_{h+1,B} + ... + \Gamma_{|B|,B} < \Gamma_{th}$, we have $\Gamma_{1,S_q} + \Gamma_{2,S_q} + ... + \Gamma_{h,S_q} + \Gamma_{h+1,B} + ... + \Gamma_{|B|,B} < \Gamma_{th}$. We end up with a solution that has the same number of packets to retransmit by relay.
2) $\Gamma_{1,B} + \Gamma_{2,B} + ... + \Gamma_{h,B} < \Gamma_{1,S_q} + \Gamma_{2,S_q} + ... + \Gamma_{h,S_q}$. If $\Gamma_{1,B} + \Gamma_{2,B} + ... + \Gamma_{h,B} + \Gamma_{h+1,S_q} + ... + \Gamma_{U,S_q} \geq \Gamma_{th}$, then Algorithm 1 would have already assigned $\{\Gamma_{1,B}, \Gamma_{2,B}, ..., \Gamma_{h,B}\} \subseteq S_q$. Thus, $\Gamma_{1,B} + \Gamma_{2,B} + ... + \Gamma_{h,B} + \Gamma_{h+1,S_q} + ... + \Gamma_{U,S_q} < \Gamma_{th}$, and we end up with a solution that is at most as good as the one where we leave S_q and B unchanged.

■

B. The Pairwise and 3-wise Combinations Case

Now we consider the situation where the relay is not limited to perform network coding with $|S|$-wise combinations. For ease of implementation, we will mainly focus on the case where the relay is capable of performing pairwise and 3-wise combinations.

A straightforward extension of Algorithm 1 may be applied to the case in concern. Intuitively, since there are potentially more valid pairwise combinations than valid 3-wise combinations, it will be straightforward to first consider pairwise combinations and then 3-wise combinations. The relay may follow the steps below:

1) Given $\Phi = \{\Gamma_1, \Gamma_2, ..., \Gamma_m\}$, apply Algorithm 1 with $|S| = 2$ and get Φ=$S_{1,2}\cup S_{2,2}\cup ...\cup S_{t,2}\cup B_2$, where $\{S_{q,2},\ q \in [1, t]\}$ is a subset of Φ that contains a valid pairwise combination, B_2 is a subset of Φ in which no valid pairwise combinations can be found.
2) If $|B_2| \geq 3$, apply Algorithm 1 with $|S| = 3$ to B_2 and get B_2=$S_{1,3}\cup S_{2,3}\cup ...\cup S_{t',3}\cup B_3$, where $\{S_{q,3},\ q \in [1, t']\}$ is a subset of B_2 that contains a valid 3-wise combination, B_3 is a subset of B_2 in which no valid 3-wise combinations can be found. The throughput gain can be calculated similar to (4).

We show the sub-optimality of the Algorithm 1 extension describe above by an example: Consider Φ =

$\{1.8, 1.8, 0.1, 0.1, 0.1, 0.1\}$ and $\Gamma_{th} = 2$. By following the steps in the Algorithm 1 extension, we have $\Phi = \{1.8, 1.8\} \cup \{0.1, 0.1, 0.1, 0.1\}$, and only one valid pairwise combination is found. The optimum solution for this instance is clearly $\Phi = \{1.8, 0.1, 0.1\} \cup \{1.8, 0.1, 0.1\}$, where there are two valid 3-wise combinations, and the relay can send one fewer packet compared to the application of the Algorithm 1 extension. The problem with the direct extension is that, we may combine two packets with very good SNRs to form a single valid pairwise combination while each of the two packets can be used individually to form a valid 3-wise combination.

Observed the flaw of Algorithm 1 extension, we now present a better algorithm for finding the maximum number of combinations possible if the relay is capable of performing pairwise and 3-wise combinations, given $\Phi = \{\Gamma_1, \Gamma_2, ..., \Gamma_m\}$ and Γ_{th}:

Algorithm 2 For pairwise and 3-wise combinations

Apply Algorithm 1 with $|S| = 2$ to Φ and get Φ=$S_{1,2}$∪$S_{2,2}$∪...∪$S_{t,2}$∪B_2
Apply Algorithm 1 with $|S| = 3$ to B_2 and get B_2=$S_{1,3}$∪$S_{2,3}$∪...∪$S_{t',3}$∪B_3
$i = 1$
while $i \leq t$ and $|B_3| \geq 4$ **do**
  Apply Algorithm 1 with $U = 3$ to $\{S_{i,2} \cup B_3\}$
  if $\{S_{i,2} \cup B_3\} = S_{1,3}^{i,2} \cup S_{2,3}^{i,2} \cup B_3'$, $S_{1,3}^{i,2}$, $S_{2,3}^{i,2}$ and B_3' are disjoint, $\sum_{\forall x, \Gamma_x \in S_{1,3}^{i,2}} \Gamma_x \geq \Gamma_{th}, \sum_{\forall y, \Gamma_y \in S_{2,3}^{i,2}} \Gamma_y \geq \Gamma_{th}$ **then**
    delete $S_{i,2}$ from Φ
    Update B_3 with B_3'
    B_2=$S_{1,3}$∪$S_{2,3}$∪...∪$S_{t',3}$∪$S_{1,3}^{i,2}$∪$S_{2,3}^{i,2}$∪B_3
  end if
  $i = i + 1$
end while

The idea of Algorithm 2 can be explained as follows: After applying Algorithm 1 with $|S| = 2$ to Φ and Algorithm 1 with $|S| = 3$ to B_2, we check each of all the valid pairwise combinations $S_{i,2}$ to see whether we can find two disjoint valid 3-wise combinations in $S_{i,2} \cup B_3$. If so we find one more valid combination and have a better solution compared to Algorithm 1 extension.

C. General Upper bound

We are interested to know how good the two algorithms' performances are. Since the general case of the problem is NP-hard, we give a general upper bound on the number of valid combinations that can be found with out limiting the number of elements involved in a combination. The upper bound can be expressed as:

$$Upper\ bound = \lfloor \frac{\sum_{\forall i, \Gamma_i \in \Phi} \Gamma_i}{\Gamma_{th}} \rfloor, \quad (5)$$

where $\lfloor x \rfloor$ gives the largest integer that is smaller than some number x. It is very easy to see that given an instance Φ, (5) gives the maximum number of possible valid

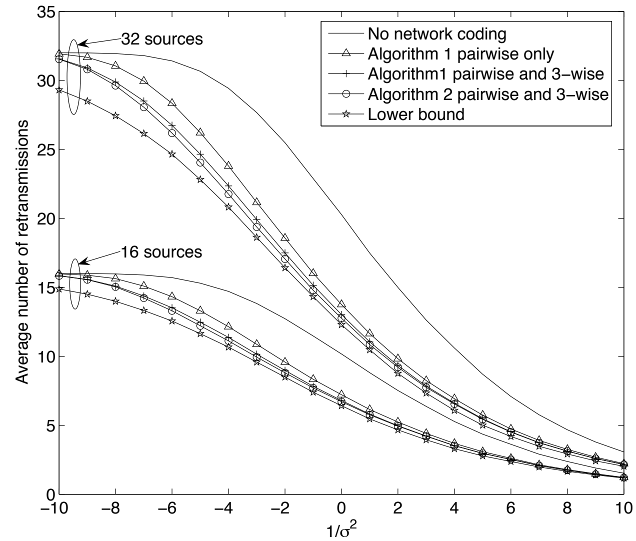

Fig. 2. Ideal relay-to-destination channel, Γ_{th}=3dB, 16 and 32 sources

combinations. At the same time, it may over estimate the number of valid combinations as in a simple case where $\Phi = \{1.8, 1.8, 0.4, 0.1\}, \Gamma_{th} = 2$. So the expression in (5) is indeed an upper bound.

IV. Simulation Results

In this section, we first compare the performance of Algorithm 1, Algorithm 1 extension and Algorithm 2 with the ideal relay assumption. The system where no network coding is applied at relay is served as the reference system for performance comparison. Note that the reference system is equivalent to the setting used to study algorithms in network layer, where the feedback for each packet is assumed to contain only a single bit telling the relay whether the decoding at destination is successful or not. The minimum number of retransmissions by relay in this case is thus the number of elements in Φ. Comparing the number of retransmissions by relay with and without network coding, we can observe the performance gain when cross-layer design is adopted. Then, we consider more realistic situation, where the relay-to-destination channel is modelled as a block Rayleigh fading channel with AWGN same as the user to destination channel. Path loss effect with relay at different positions is also considered.

Fig. 2 plots the result of average number of retransmissions by relay against relay transmission power over noise power when there are 16 and 32 users. Γ_{th} is set to be 2, which implies the SNR requirement at destination is 3dB. The throughput performances when no network coding is adapted and when network coding is applied are compared. For 16 users case, it can be observed that by performing pairwise combinations at relay, a throughput gain of about 30% can be achieved from 0dB to 4dB. The application of Algorithm 1 extended to 3-wise combinations gives about another 3-5% improvement compared to pairwise only case from -6dB to

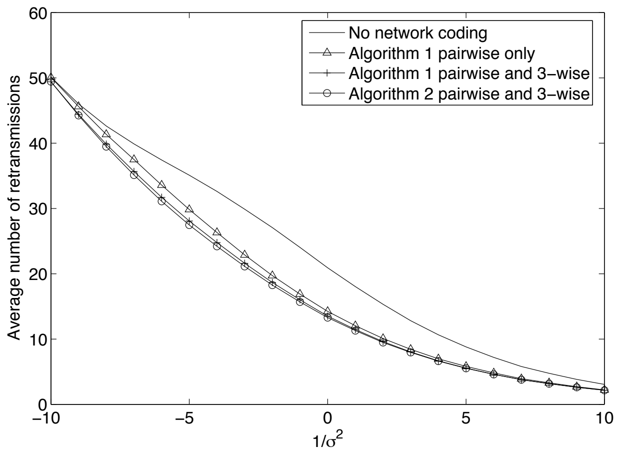

Fig. 3. Relay-to-destination channel is block Rayleigh faded, and relay is at the middle of the users and the destination, Γ_{th}=3dB, 32 users

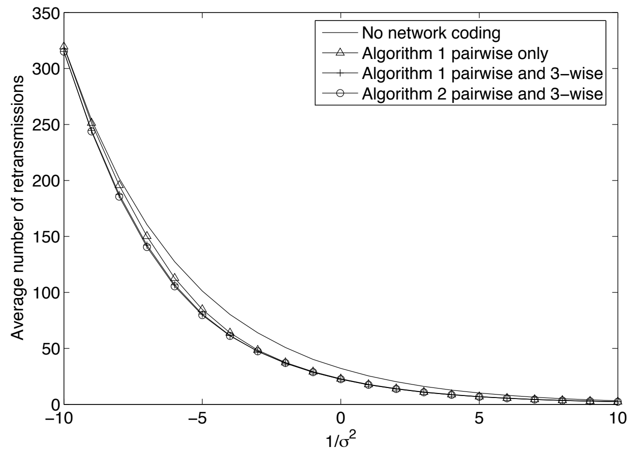

Fig. 4. Relay-to-destination channel is block Rayleigh faded, and the distance from relay to destination is the same as source to destination, Γ_{th}=3dB, 32 users

4dB. The performance improvement of using Algorithm 2 up to 3-wise combinations is very little, where 2% improvement is observed at -4dB. For 32 users case, a throughput gain of about 34% is obtained from 3-6dB if only pairwise combinations are considered. About 4% of improvement can be gained if we apply Algorithm 1 extended to 3-wise combinations, and further gain is negligible by applying Algorithm 2. The lower bound curves show the average number of retransmissions when the relay finds the maximum possible number of combinations, which is upper bounded by (5). The number of retransmissions for the lower bound given an instance of Φ is thus $|\Phi| - \lfloor \frac{\sum_{\forall i, \Gamma_i \in \Phi} \Gamma_i}{\Gamma_{th}} \rfloor$. For the 32 users case, the lower bound is 7% away from the Algorithm 2 at -10dB, and it is 2-4% away from -6dB and afterwards. Similar figures can be found in 16 users case. By the data summary above, we see that the throughput gain of pairwise only case is about 30%, and it is 7% away from the lower bound in most of the SNR region. We may conclude that by applying Algorithm 1 with pairwise combinations, most of the throughput gain can be achieved.

Now we discard the ideal assumption of the relay-to-destination channel. We assume the relay-to-destination channel is block Rayleigh faded, where the channel gain coefficient and additive noise vector can be described same as h_{kd} and $\mathbf{n}_{kd}$ in (1). We also assume the distance from users to relay is the same as the distance from relay-to-destination. By setting the path loss exponent to be 4, it is equivalent that the relay's transmission power has a 12dB gain compared to a user's. The relay performs network coding same as before. For each packet transmitted by relay, if the realized SNR is less than Γ_{th}, the relay will retransmit the same packet and all the copies are combined using MRC principle until the accumulated SNR of that packet at destination is larger or equal to Γ_{th}. We assume the destination can decode packets from the relay error free by such mechanism. Fig. 3 shows the average number of retransmissions by relay for 32 users case under the above settings when different algorithms are applied. We can see that more than 30% of throughput gain is achieved from 0dB to 6dB by using Algorithm 1 restricted to pairwise combinations. Algorithm 1 extension and Algorithm 2 for pairwise and 3-wise combinations perform almost the same, and they outperform Algorithm 1 with pairwise combinations by about 6% at around -6dB.

It is also interesting to see how the cross-layer design works when the relay is as far as the users to the destination. Fig. 4 shows the performance of the system in such assumption, and significant throughput gain can also be observed in a large range of received SNR (about 21% gain at -5dB and about 31% gain at 0dB). Note that in this setting, the system model is actually equivalent to a point to point communication channel with block Rayleigh fading and the different numbers of retransmitted packets are compared if the source performs network coding or the source does not perform network coding.

V. Conclusion

In this paper, we have considered the issue of designing retransmission protocol at relay in multiple-access relay channels. The source-to-destination channels are assumed to be block Rayleigh fading channels and the source-to-relay channels are good enough such that the relay can decode packets from the sources error free. The destination provides feedback to the relay about the realized SNR value of each packet. The relay then applies network coding across the packets so that the packets involved in a network coding process have their sum SNRs at destination larger than a threshold. By correctly decoding the network coded packets, the destination can derive all individual packets with a guaranteed BER. To find the minimum number of retransmissions by the relay required, it is equivalent to solve a bin covering problem, which is NP-hard. We then reduce the hardness of the problem by limiting the number of packets involved in a network coding procedure.

This is equivalent to limit the number of items inside a bin, and items that cannot find themselves a normal bin will be placed in a trash bin. An algorithm that is optimum when fixing the number of packets in a network coding procedure is proposed. Two other algorithms that dealing with the case when the maximum number of packets involved in a network coding procedure is fixed to three. We compare the number of retransmissions by the relay when network coding is applied and is not applied. Simulation results for the cases when the relay-to-destination channel is perfect and when the channel is block Rayleigh faded are provided. The results show that by doing pairwise network coding, where only two packets are involved in a single network coding procedure, most of the throughput gain can be achieved.

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