Resource Allocation for Two-way Relay Cellular Networks With and Without Network Coding

Yi Chen and Chi Wan Sung
Department of Electronic Engineering, City University of Hong Kong, Hong Kong
Email: ychen224@cityu.edu.hk, albert.sung@cityu.edu.hk

***Abstract*—This paper investigates the joint optimization of power control and phase time assignment for two-way relay cellular networks. Three two-way relay schemes are considered: 1) Four-phase scheme without network coding, 2) Three-phase scheme with network coding on the network layer, 3) Two-phase scheme with decode-and-forward operation at the relay. Different from conventional work that focused on a single three-node relay network, we consider a multicell scenario consisting of multiple three-node relay networks. These three-node subnetworks share the same channel and thus interfere with each other. Our goal is to study the resource allocation under such scenario and evaluate the benefit of network coding in an environment of interference.**

I. Introduction

In modern cellular networks, relays are employed to improve the system performance, such as coverage extension, power saving and cell-edge throughput enhancement. Two-way relay is one kind of the communication modes, in which the base stations (BS) and the mobile users (MU) exchange data via the half-duplex relay stations (RS). Depending on the number of phases needed for one round communication, there could be three types of two-way relay scheme. The first one is the Four-phase scheme: BS→RS, MU→RS, RS→BS and RS→MU. Network coding is not used and the RS only forwards data from the BS to the MU or from the MU to the BS, one direction at a time. The second scheme is the Three-phase scheme [1]: BS→RS, MU→RS and RS→BS&MU. The RS performs network coding with the data from BS and MU, and multicast the coded data to them. The BS(MU) can then retrieve the data from the MU(BS) by exploiting a priori information of its own data. The third scheme is the Two-phase scheme [2]: BS&MU→RS and RS→BS&MU. It further allows BS and MU to transmit simultaneously in the first phase and physical layer network coding can be applied

In the literature, there have been many theoretically proven results showing that network coding can offer throughput benefits in a three-node relay network, including references [3], [4] for Three-phase scheme and references [2], [5] for Two-phase scheme. Reference [6] compared the achievable rate region of Three-phase scheme and Two-phase scheme with different relaying protocols. In addition to throughput, resource allocation was also investigated in [7]–[10]. References [7] and [8] studied the time assignment and power allocation for a two-way relay network based on Three-phase scheme and Two-phase scheme, respectively. The former aims to minimize the total energy with constraint of queue stability. The latter aims to maximize the sum rate given the bi-directional data rates are symmetric. In [9] and [10], optimal resource allocation was considered for OFDM-based two-way relay cellular systems. Reference [9] investigated an opportunistic resource scheduling algorithm for networks with Three-phase scheme and [10] proposed a convex optimization framework for a hybrid networks with Four-phase and Two-phase scheme.

All the aforementioned works focused on either a single three-node relay network or a single cell scenario where there is no interference. However, since it is an inevitable trend for future cellular networks to use higher channel reuse factor, inter-cell interference will become one of the major elements to decide the system performance. Therefore in this paper, we consider the multi-cell scenario, where there are multiple interfering two-way relays. We study the joint optimization of power control and time assignment for two-way relay multi-cell networks under Four-phase scheme, Three-phase scheme and Two-phase scheme, respectively. The resource allocation problem is to minimize the average total power of the system while guaranteeing the bi-directional data rate of each cell. The benefits of network coding are evaluated in an environment of interference.

Notation: The following notation are used throughout this paper. Vectors are denoted in bold small letter, e.g., $\mathbf{x}$, with their ith entry denoted by x_i. Matrices are denoted by bold capitalized letters, e.g., $\mathbf{X}$, with X_{ij} denoting the $\{i,j\}$th entry. Vector inequalities are component-wise inequalities, i.e., $\mathbf{x} \geq \mathbf{y}$ if $x_i \geq y_i$ for all i. The transpose of a vector $\mathbf{x}$ is denoted as $\mathbf{x}^T$.

II. System Model

Consider a multi-cell wireless network consisting of N cells. In each cell i for $i = 1, \cdots, N$, the base station BS_i wants to communicate with the mobile user MU_i via the relay station RS_i at a rate φ. All the cells use a common channel, where the bandwidth of the frequency spectrum is W Hz. Therefore interference arises. Let g_{r_j,b_i} denote the channel gain from BS_i to RS_j, and g_{m_j,r_i} denote the channel gain

This work was supported in part by a grant from the University Grants Committee of the Hong Kong Special Administrative Region, China under Project AoE/E-02/08, and in part by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China under Project CityU 121713.

978-1-4799-5832-0/14/$31.00 ©2014 IEEE

from RS_i to MU_j. The channel gains between other transmitters and receivers are defined accordingly. Define nonnegative matrices $\mathbf{G}_{BR} = [G_{ij}]_{N\times N}$ and $\mathbf{D}_{BR} = [D_{ij}]_{N\times N}$ for the channel gains from the BSs to the RSs as

$$\mathbf{G}_{BR} = \begin{bmatrix} 0 & g_{r_1,b_2} & \cdots & g_{r_1,b_N} \\ g_{r_2,b_1} & 0 & \cdots & g_{r_2,b_N} \\ \vdots & \vdots & \ddots & \vdots \\ g_{r_N,b_1} & g_{r_N,b_2} & \cdots & 0 \end{bmatrix}, \quad \mathbf{D}_{BR} = \begin{bmatrix} g_{r_1,b_1} & 0 & \cdots & 0 \\ 0 & g_{r_2,b_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & g_{r_N,b_N} \end{bmatrix}.$$

Matrices of $\mathbf{G}_{MR}$, $\mathbf{D}_{MR}$, $\mathbf{G}_{RB}$, $\mathbf{D}_{RB}$, $\mathbf{G}_{RM}$ and $\mathbf{D}_{RM}$ are defined accordingly.

We assume the phases are synchronized among all the cells. Let τ_k for $k = 1, \cdots, K$ be the fraction of time allocated to the kth phase, where K is the number of phases.The time fractions should satisfy the constraint $\sum_{k=1}^{K} \tau_k = 1$.

Let p_{b_i} and p_{m_i} be the power used for BS_i and MU_i, respectively. Define $\mathbf{p}_b = [p_{b_1}, \cdots, p_{b_N}]^T$ and $\mathbf{p}_m = [p_{m_1}, \cdots, p_{m_N}]^T$. In Four-phase scheme, let p_{rb_i} and p_{rm_i} be the power used for RS_i to BS_i and RS_i to MU_i, respectively. Define $\mathbf{p}_{rb} = [p_{rb_1}, \cdots, p_{rb_N}]^T$ and $\mathbf{p}_{rm} = [p_{rm_1}, \cdots, p_{rm_N}]^T$. In Three-phase and Two-phase scheme, since the RS broadcasts to both the BS and MU, its power is simply denoted by p_{r_i}. Define $\mathbf{p}_r = [p_{r_1}, \cdots, p_{r_N}]^T$.

In each cell, the interfering signals transmitted from other cells are treated as additive white Gaussian noise. Let $\gamma_{i,br}$, $\gamma_{i,mr}$, $\gamma_{i,rb}$ and $\gamma_{i,rm}$ denote the signal-to-interference-plus-noise-ratio (SINR) at RS_i transmitted by BS_i, RS_i transmitted by MU_i, BS_i transmitted by RS_i and MU_i transmitted by RS_i, respectively.

Our aim is to minimize the average total power of the system while guaranteeing the data rate requirements of all cells. The data rate is modeled as a function of the SINR by Shannon capacity formula: $W \log_2(1 + \text{SINR})$. The optimization variables are the time fractions and powers. In the following, we investigate the problem under network with Four-phase, Three-phase and Two-phase schemes, respectively.

III. Four-phase Scheme

In Four-phase scheme, the BSs and MUs transmit data to the RSs in the first and second phase, respectively. In the third and fourth phase, the RSs transmit data to the BSs and MUs, respectively. For each phase, since only τ_k fraction of time is allocated, the equivalent data rate requirement is φ/τ_k. The optimization problem is formulated as

$$\begin{aligned} \min \quad & \sum_{i=1}^{N} \tau_1 p_{b_i} + \tau_2 p_{m_i} + \tau_3 p_{rb_i} + \tau_4 p_{rm_i} \qquad \text{(OP1)} \\ s.t. \quad & \frac{\varphi}{\tau_1} \leq W \log_2\left(\gamma_{i,br}(\mathbf{p}_b) + 1\right), \quad i = 1, \cdots, N \\ & \frac{\varphi}{\tau_2} \leq W \log_2\left(\gamma_{i,mr}(\mathbf{p}_m) + 1\right), \quad i = 1, \cdots, N \\ & \frac{\varphi}{\tau_3} \leq W \log_2\left(\gamma_{i,rb}(\mathbf{p}_{rb}) + 1\right), \quad i = 1, \cdots, N \\ & \frac{\varphi}{\tau_4} \leq W \log_2\left(\gamma_{i,rm}(\mathbf{p}_{rm}) + 1\right), \quad i = 1, \cdots, N \\ & \tau_1 + \tau_2 + \tau_3 + \tau_4 = 1. \end{aligned}$$

Therein, the SINR is defined as

$$\gamma_{i,br}(\mathbf{p}_b) = \frac{p_{b_i} g_{r_i,b_i}}{\sum_{j\neq i} p_{b_j} g_{r_i,b_j} + \sigma^2},$$

where σ^2 is the variance of the additive white Gaussian noise. $\gamma_{i,mr}(\mathbf{p}_m)$, $\gamma_{i,rb}(\mathbf{p}_{rb})$ and $\gamma_{i,rm}(\mathbf{p}_{rm})$ are defined accordingly.

Theorem 1. *The optimal solution of* (OP1) *if exists, satisfies the inequality constraints with equality.*

Due to page limit, the proof is not provided in this paper but can be found in [11].

A. Feasibility of (OP1)

Given the time fractions, the power control are independent for each phase, and reduces to the classic problem that aims to minimize the total power constrained by fixed SINR target [12]. Let $\lambda(\cdot)$ denote the Perron-Frobenius eigenvalue function of a matrix, i.e., the maximum of the absolute value of the eigenvalues of a matrix. The following lemma from [12] is the fundamental results that characterizes the feasibility.

Lemma 1. [12]. *Suppose* $\mathbf{G}$ *is a non-negative irreducible matrix and* $\mathbf{I}$ *is the identity matrix. The following statements are equivalent:*

1) *There exists a vector* $\mathbf{p} \geq \mathbf{0}$ *such that* $(\mathbf{I} - \mathbf{G})\mathbf{p} \geq \mathbf{0}$.
2) $\lambda(\mathbf{G}) < 1$.
3) $(\mathbf{I} - \mathbf{G})^{-1} = \sum_{k=0}^{\infty} \mathbf{G}^k$ *exists and is positive component-wise, with* $\lim_{k\to\infty} \mathbf{G}^k = 0$.

Take phase one for example. Let $\mathbf{n} = [\sigma^2, \cdots, \sigma^2]^T$ be the noise vector of size N. Define $\mathbf{n}_1 = \mathbf{D}_{BR}^{-1}\mathbf{n}$ and $\mathbf{G}_1 = \mathbf{D}_{BR}^{-1}\mathbf{G}_{BR}$ to be the normalized noise vector and channel gain matrix. Define function $\Gamma(x) := 2^{\frac{\varphi}{Wx}} - 1$, which translates the data rate requirement to an SINR target. By Theorem 1, the inequality constraint in (OP1) can be replaced by equality and written in matrix form as

$$\Gamma(\tau_1)\mathbf{G}_1\mathbf{p}_b + \Gamma(\tau_1)\mathbf{n}_1 = \mathbf{p}_b.$$

By Lemma 1, a nonnegative solution $\mathbf{p}_b$ to the above equation exists for any $\Gamma(\tau_1)\mathbf{n}_1 \geq \mathbf{0}$, if and only if $\lambda(\Gamma(\tau_1)\mathbf{G}_1) < 1$, i.e., $\Gamma(\tau_1) < 1/\lambda(\mathbf{G}_1)$. In that case, the unique solution is given by

$$\mathbf{p}_b(\tau_1) = [\mathbf{I} - \Gamma(\tau_1)\mathbf{G}_1]^{-1}\Gamma(\tau_1)\mathbf{n}_1.$$

Involving all phases, the feasibility of (OP1) is equivalent to the feasibility of the set of the time fractions tuple

$$\mathcal{F}_\tau^4 = \left\{ (\tau_1, \tau_2, \tau_3, \tau_4) : \begin{array}{l} \sum_{i=1}^{4} \tau_i = 1 \\ \tau_i > \frac{\varphi}{W \cdot \log_2(1/\lambda(\mathbf{G}_i)+1)}, \ \forall i \end{array} \right\}.$$

B. Convexity of the problem

Theorem 2. *The optimization problem* (OP1) *is equivalent to a convex optimization problem.*

The proof is provided in [11]. The standard convex optimization algorithms can be used to find the solution and it is globally optimal.

IV. Three-phase Scheme

In the first and second phase, the BSs and MUs transmit data to the RSs, respectively. In the third phase, the RSs perform $\mathbf{XOR}$ operation between the data of BSs and MUs, and multicast the coded data to them. The BSs and MUs can retrieve the data by performing $\mathbf{XOR}$ between the coded data and its own data. The optimization problem is formulated as

$$\begin{aligned} \min \quad & \sum_{i=1}^{N} \tau_1 p_{b_i} + \tau_2 p_{m_i} + \tau_3 p_{r_i} & \text{(OP2)} \\ s.t. \quad & \frac{\varphi}{\tau_1} \leq W \log_2\left(\gamma_{i,br}(\mathbf{p}_b) + 1\right), \quad i = 1, \cdots, N \\ & \frac{\varphi}{\tau_2} \leq W \log_2\left(\gamma_{i,mr}(\mathbf{p}_m) + 1\right), \quad i = 1, \cdots, N \\ & \frac{\varphi}{\tau_3} \leq \min\left\{W \log_2\left(\gamma_{i,rb}(\mathbf{p}_r) + 1\right),\right. \\ & \qquad \left. W \log_2\left(\gamma_{i,rm}(\mathbf{p}_r) + 1\right)\right\} \quad i = 1, \cdots, N \\ & \tau_1 + \tau_2 + \tau_3 = 1, \end{aligned}$$

Theorem 3. *The optimal solution of* (OP2) *if exists, satisfies the inequality constraints with equality.*

The proof follows the same argument as in the proof of Theorem 1.

A. Feasibility of (OP2)

The first and the second phase are the same as those in the Four-phase scheme. So we discuss the third phase. For each cell, it is a multicast transmission with RS being the transmitter and BS and MU being the receivers. Let us rewrite the matrices $\mathbf{G}_3 = \mathbf{D}_{RB}^{-1}\mathbf{G}_{RB}$ and $\mathbf{G}_4 = \mathbf{D}_{RM}^{-1}\mathbf{G}_{RM}$ in the following form

$$\mathbf{G}_3 = \begin{bmatrix} \mathbf{g}_1^3 \\ \mathbf{g}_2^3 \\ \vdots \\ \mathbf{g}_N^3 \end{bmatrix} \qquad \mathbf{G}_4 = \begin{bmatrix} \mathbf{g}_1^4 \\ \mathbf{g}_2^4 \\ \vdots \\ \mathbf{g}_N^4 \end{bmatrix},$$

where $\mathbf{g}_i^3, \mathbf{g}_i^4 \in \mathbb{R}^N$ for $i = 1, \cdots, N$ are row vectors. Define the set

$$\mathcal{H} = \left\{\mathbf{H} \in \mathbb{R}^{N\times N} : \mathbf{H} = \begin{bmatrix} \mathbf{g}_1^{k_1} \\ \mathbf{g}_2^{k_2} \\ \vdots \\ \mathbf{g}_N^{k_N} \end{bmatrix}, k_1, \cdots, k_N \in \{3, 4\}\right\}.$$

It can be seen that any $\mathbf{H} \in \mathcal{H}$ corresponds to a unicast scenario, that is, for each transmitter, only one of its intended receiver is considered. Base on the eigenvalue condition for the feasibility of unicast (see Lemma 1), we have the following necessary and sufficient condition for the feasibility of multicast.

Theorem 4. *There exists* $\mathbf{p}_r \geq 0$ *such that* $(\mathbf{I}-\Gamma(\tau_3)\mathbf{G}_3)\mathbf{p}_r \geq \Gamma(\tau_3)\mathbf{n}_3$ *and* $(\mathbf{I} - \Gamma(\tau_3)\mathbf{G}_4)\mathbf{p}_r \geq \Gamma(\tau_3)\mathbf{n}_4$*, if and only if* $\Gamma(\tau_3) < \min_{\mathbf{H}\in\mathcal{H}}\{1/\lambda(\mathbf{H})\}$.

This theorem is a special case of the result in our report [13] and the proof can be found therein. We derive the feasible region of time fractions as follows,

$$\mathcal{F}_\tau^3 = \left\{ (\tau_1, \tau_2, \tau_3) : \begin{array}{c} \tau_i > \frac{\varphi}{W\cdot\log_2(1/\lambda(\mathbf{G}_i))+1)}, \; i = 1, 2 \\ \tau_3 > \frac{\varphi}{W\cdot\log_2(\min_{\mathbf{H}\in\mathcal{H}}\{1/\lambda(\mathbf{H})\}+1)} \\ \sum_{i=1}^{3} \tau_i = 1 \end{array} \right\}.$$

The feasibility of (OP2) is equivalent to the feasibility of $\mathcal{F}_\tau^3$.

B. Find the optimal solution

Since (OP2) is in general non-convex, we use one-dimension search over τ_3 to find the optimal solution. Given τ_3, (OP2) can be divided into two independent subproblems:

$$\begin{aligned} \min \quad & \sum_{i=1}^{N} \tau_1 p_{b_i} + \tau_2 p_{m_i} & \text{(OP2a)} \\ s.t. \quad & \frac{\varphi}{\tau_1} \leq W \log_2\left(\gamma_{i,br}(\mathbf{p}_b) + 1\right), \quad i = 1, \cdots, N \\ & \frac{\varphi}{\tau_2} \leq W \log_2\left(\gamma_{i,mr}(\mathbf{p}_m) + 1\right), \quad i = 1, \cdots, N \\ & \tau_1 + \tau_2 = 1 - \tau_3 \end{aligned}$$

$$\begin{aligned} \min \quad & \sum_{i=1}^{N} p_{r_i} & \text{(OP2b)} \\ s.t. \quad & \Gamma(\tau_3) \leq \gamma_{i,rb}(\mathbf{p}_r) \quad i = 1, \cdots, N \\ & \Gamma(\tau_3) \leq \gamma_{i,rm}(\mathbf{p}_r) \quad i = 1, \cdots, N \end{aligned}$$

The first subproblem (OP2a) has the same form as the optimization problem for Four-phase scheme (OP1), and thus its convexity can be verified similarly. Standard algorithm can find the optimal τ_1, τ_2 and $\mathbf{p}_b, \mathbf{p}_m$, all in terms of τ_3. The second subproblem (OP2b) is a linear programming and can be solved easily. Overall, we search over $\frac{\varphi}{W\cdot\log_2(\min_{\mathbf{H}\in\mathcal{H}}\{1/\lambda(\mathbf{H})\}+1)} \leq \tau_3 \leq 1 - \frac{\varphi}{W\cdot\log_2(1/\lambda(\mathbf{G}_1)+1)} - \frac{\varphi}{W\cdot\log_2(1/\lambda(\mathbf{G}_2)+1)}$ to find the globally optimal solution of (OP2).

V. Two-phase Scheme

In Two-phase scheme, the BSs and MUs transmit data to the RSs in the first phase and the RSs broadcast combined data to both the BSs and MUs in the second phase. We consider the relaying strategy of decode-and-forward (DF), in which the RS first applies multiuser detection to decode the data transmitted from the BS and MU, and then performs $\mathbf{XOR}$ between the data and broadcast the new coded data. The two phases are equivalent to a multi-access (MAC) phase and a broadcast (BC) phase.

Let $R_{i,br}$ and $R_{i,mr}$ denote the achievable rates of $BS_i \rightarrow RS_i$ and $MU_i \rightarrow RS_i$ in the MAC phase. The capacity region $\mathcal{C}_{MAC}$ of the Gaussian MAC is [6]

$$\mathcal{C}_{MAC_i}(\mathbf{p}_b, \mathbf{p}_m) = \left\{ (R_{i,br}, R_{i,mr}) : \left\{ \begin{array}{l} R_{i,br} \leq W \log_2\left(1 + \gamma_{i,br}(\mathbf{p}_b, \mathbf{p}_m)\right) \\ R_{i,mr} \leq W \log_2\left(1 + \gamma_{i,mr}(\mathbf{p}_b, \mathbf{p}_m)\right) \\ R_{i,br} + R_{i,mr} \leq W \log_2\left(1 + \right. \\ \qquad \left. \gamma_{i,mr}(\mathbf{p}_b, \mathbf{p}_m) + \gamma_{i,br}(\mathbf{p}_b, \mathbf{p}_m)\right) \end{array} \right. \right\},$$

where

$$\gamma_{i,br}(\mathbf{p}_b,\mathbf{p}_m)=\frac{p_{b_i}g_{r_i,b_i}}{\sum_{j\neq i}p_{b_j}g_{r_i,b_j}+p_{m_j}g_{r_i,m_j}+\sigma^2}, \quad (1)$$

$$\gamma_{i,mr}(\mathbf{p}_b,\mathbf{p}_m)=\frac{p_{m_i}g_{r_i,m_i}}{\sum_{j\neq i}p_{b_j}g_{r_i,b_j}+p_{m_j}g_{r_i,m_j}+\sigma^2}. \quad (2)$$

Fig. 1 illustrates an example of the capacity region. Throughout this section, we use the terms *bound line* of the capacity region to indicate the polyline $CA-AB-BD$, and *hypotenuse* of the bound line to indicate the line segment AB. The optimization problem is formulated as

$$\begin{aligned}\min \quad & \sum_{i=1}^{N}(p_{b_i}+p_{m_i})\tau_1+p_{r_i}\tau_2 \qquad \text{(OP3)}\\ s.t. \quad & \frac{\varphi}{\tau_1}\le \max_{(R_{i,br},R_{i,mr})\in\mathcal{C}_{MAC_i}(\mathbf{p}_b,\mathbf{p}_m)} \min\left\{R_{i,br},R_{i,mr}\right\},\\ & \frac{\varphi}{\tau_2}\le \min\left\{W\log_2\left(\gamma_{i,rb}(\mathbf{p}_r)+1\right),\right.\\ & \qquad \left.W\log_2\left(\gamma_{i,rm}(\mathbf{p}_r)+1\right)\right\} \quad i=1,\cdots,N,\\ & \tau_1+\tau_2=1.\end{aligned}$$

Lemma 2. *Regarding* (OP3) *for the MAC phase, the optimal power* $(\mathbf{p}_b^*,\mathbf{p}_m^*)$ *if exists, should satisfy that for each cell i, the intersection point between the bound line of rate region* $\mathcal{C}_{MAC_i}(\mathbf{p}_b^*,\mathbf{p}_m^*)$ *and the line* $R_{i,br}=R_{i,mr}$ *is on the hypotenuse of the bound line. Moreover, the optimal rate pair of*

$$\left(R_{i,br}^*,R_{i,mr}^*\right)= \arg\max_{(R_{i,br},R_{i,mr})\in\mathcal{C}_{MAC_i}(\mathbf{p}_b^*,\mathbf{p}_m^*)} \min\left\{R_{i,br},R_{i,mr}\right\} \quad (3)$$

is the intersection point.

The proof is provided in [11]. By Lemma 2, the optimal rate pair of the MAC phase should satisfy $R_{i,br}^*=R_{i,mr}^*$, $R_{i,br}^*+R_{i,mr}^*=W\log_2\left(1+\gamma_{i,mr}(\mathbf{p}_b,\mathbf{p}_m)+\gamma_{i,br}(\mathbf{p}_b,\mathbf{p}_m)\right)$, $R_{i,br}^*\le W\log_2\left(1+\gamma_{i,br}(\mathbf{p}_b,\mathbf{p}_m)\right)$ and $R_{i,mr}^*\le W\log_2\left(1+\gamma_{i,mr}(\mathbf{p}_b,\mathbf{p}_m)\right)$. So the optimization can be refined to be

$$\begin{aligned}\min \quad & \sum_{i=1}^{N}(p_{b_i}+p_{m_i})\tau_1+p_{r_i}\tau_2 \qquad \text{(OP4)}\\ s.t. \quad & \frac{\varphi}{\tau_1}\le R_i=\frac{1}{2}W\log_2\left(1+\gamma_{i,mr}(\mathbf{p}_b,\mathbf{p}_m)+\gamma_{i,br}(\mathbf{p}_b,\mathbf{p}_m)\right)\\ & R_i\le \min\left\{W\log_2\left(1+\gamma_{i,br}(\mathbf{p}_b,\mathbf{p}_m)\right),\right.\\ & \qquad \left.W\log_2\left(1+\gamma_{i,mr}(\mathbf{p}_b,\mathbf{p}_m)\right)\right\} \quad i=1,\cdots,N\\ & \frac{\varphi}{\tau_2}\le \min\left\{W\log_2\left(\gamma_{i,rb}(\mathbf{p}_r)+1\right),\right.\\ & \qquad \left.W\log_2\left(\gamma_{i,rm}(\mathbf{p}_r)+1\right)\right\} \quad i=1,\cdots,N\\ & \tau_1+\tau_2=1.\end{aligned}$$

Theorem 5. *The optimal solution of* (OP4) *if exists, satisfies the inequality constraints with equality.*

The proof is provided in [11]. We can use one-dimension search over τ_1 to find the globally optimal solution of (OP4). Given τ_1, (OP4) reduces to two independent power control problems, one for the first phase and the other is for the second phase. They all are linear programming.

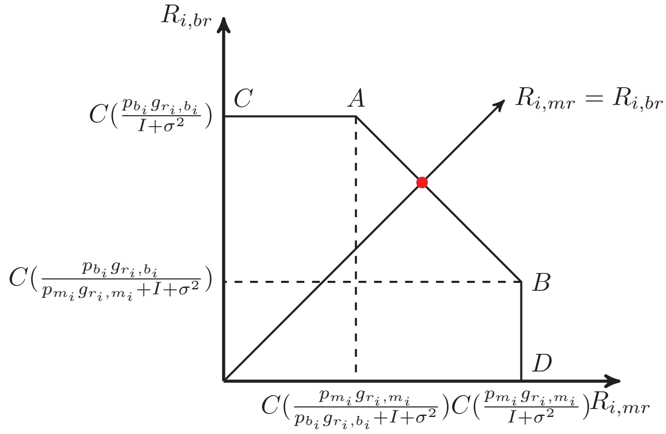

Fig. 1: The capacity region of $\mathcal{C}_{MAC_i}(\mathbf{p}_b,\mathbf{p}_m)$.Therein, $C(x)\triangleq W\log_2(1+x)$ and $I=\sum_{j\neq i}p_{b_j}g_{r_i,b_j}+p_{m_j}g_{r_i,m_j}$.

VI. Numerical Results

In this section, we present the numerical results of the optimization problems designed for the three relaying schemes and compare their performance under multi-cell networks.

We consider a multi-cell network composed of N hexagonal cells with radius 1km, where N is a parameter in our simulation. Within each cell, there is a BS located at the center, a RS and a MU. The location of the RS is generated uniformly on the circle of radius 0.5km centered at the BS. The location of the MU is generated uniformly on the disk of radius 0.2km centered at the point that is 0.2km away from the RS and on the line of BS-RS. This geographic setting models the situation when the direct link between BS and MU is poor and a RS is needed to help their communication. The channel gain is modeled as $g_{r_j,b_i}=A_{r_j,b_i}/d_{r_j,b_i}^4$, where d_{r_j,b_i} is the distance between BS_i and RS_j, and A_{r_j,b_i} is the attenuation factor due to shadowing. We assume A_{r_j,b_i} is lognormal distributed with mean 0dB and standard deviation 4dB.

Fig. 2 plots the average total power versus data rate requirement for a specific network consisting of three cells. It can be seen that Four-phase scheme requires the largest power for all values of φ/W and Two-phase scheme requires the smallest power. The power differences among the three schemes are getting larger as the data rate requirement is increasing. Moreover, When $\varphi/W>1.5$, the Four-phase scheme fails to have a feasible solution.

In Fig. 3 and 4, we further evaluate the cumulative distribution function (CDF) of the normalized power difference between Four-phase and Two-phase schemes, and between Three-phase and Two-phase schemes, respectively. Each curve is obtained over 10000 samples of the network (channel gains) whose feasible region is non-empty. As seen in Fig. 3, Two-phase scheme can save much more power than Four-phase scheme. When the number of cells is seven and $\varphi/W=1.3$, in 95% of the cases, the power needed by Four-phase scheme is three times larger than the power needed by Two-phase scheme. In Fig. 4, we can see that Two-phase scheme also

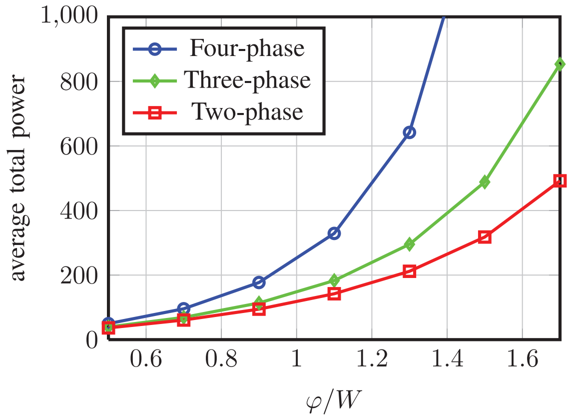

Fig. 2: The average total power versus data rate requirement for a network consisting of three cells.

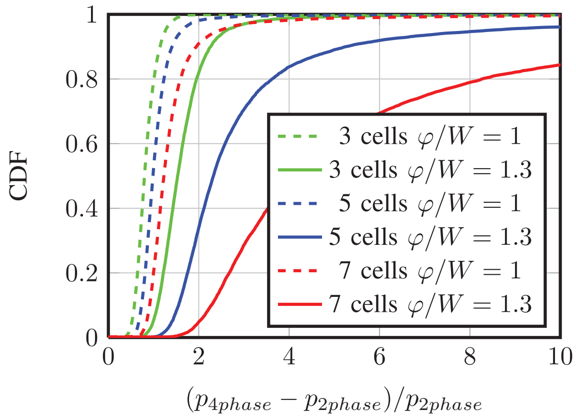

Fig. 3: The CDF of the normalized power difference.

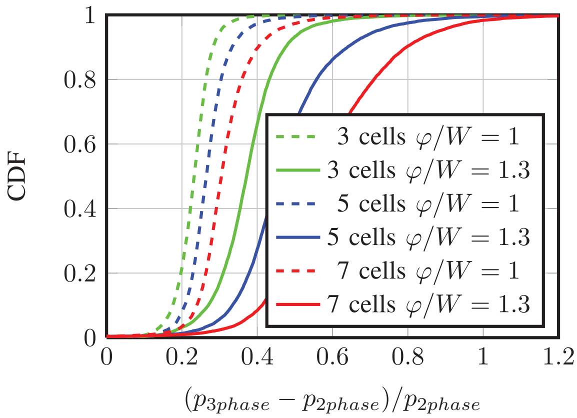

Fig. 4: The CDF of the normalized power difference.

has better performance than Three-phase scheme, but the improvement is not as significant as that over Four-phase scheme.

Fig. 3 and 4 also show that the performance differences among the three schemes are enlarging as the number of cells is increasing. In Fig. 3, when $\varphi/W = 1.3$, the ratio of normalized power difference being less than 2 is around 82% for 3-cells, 32% for 5-cells and 5% for 7-cells.

Overall, we see that Two-phase scheme has the best performance, and Three-phase scheme performs better than Four-phase scheme. The advantage of Two-phase scheme over Three-phase and Four-phase scheme mainly because it uses a smaller number of phases, and thus each phase can be allocated with larger time fraction, which results in smaller SINR target and therefore less power. These results demonstrate that the benefit of network coding also applies to multi-cell networks with interference.

VII. Conclusion

In this paper, the joint optimization of power control and phase time assignment for two-way relay cellular networks is studied under three two-way relay schemes. We prove some properties of the optimization problems and discuss the algorithm to solve them. Simulation results show that Two-phase scheme outperforms Three-phase scheme and Four-phase scheme, and Three-phase scheme outperforms Four-phase scheme. These results demonstrate that network coding can bring benefit even in an environment of interference.

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