Adaptive Modulation for Two Users in VLC

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***Abstract*—In visible light communications, it is common to have more than one light sources within the line of sight of a receiver. This paper proposes an adaptive modulation scheme for a system with two light sources and two receivers where two different data streams are sent to the two receivers. The transmitted powers of both light sources are optimized to reduce the bit error rate (BER) at the receivers. Although the optimization problem is non-convex, it can be transformed into a set of linear optimization problems. Analytical results show how to reduce the set and reduce the complexity of the optimization problem. Under various settings, our proposed scheme is compared with multi-user maximum likelihood, repetition coding, and linear zero-forcing precoding techniques. Simulation results show that the proposed scheme outperforms the other techniques in terms of BER.**

***Index Terms*—Visible light communications, adaptive modulation, multi-user, multiple input single output, Light-Fidelity.**

I. Introduction

Visible light communications (VLC) has recently attracted a lot of attention as a promising future technology to provide a very high data rate for indoor applications [1], [2] and accurate indoor positioning [3]. It has several appealing properties such as huge license-free spectrum, high security, and green technology. Light-Fidelity (Li-Fi) has been proposed as an alternative technology of Wi-Fi where signals in visible light instead of radio frequency is used to transfer data [4].

VLC however faces some challenges for multi-user case. Built on a lighting system, VLC uses light emitting diodes (LEDs) as transmitters. Receivers are equipped with photodetectors (PDs) and they are usually assumed to be within the line of sight (LoS) of light sources. Since there are usually many light sources in indoor environments, each receiver may receive light from different sources that may cause interference and degrade the performance. Furthermore, the illumination requirements of the lighting system has to be met. So the transmitted signals have to satisfy certain power constraints.

In VLC, multi-user (MU) techniques, which represent an essential need for Li-Fi, have been investigated in literatures. In [5] an optical code division multiple access (CDMA) system based on random optical codes is suggested. A multiple access technique utilizing expurgated PPM (EPPM) is proposed in [6]. This technique transmits the same data from each LED that limits the overall system throughput. In [7], a space of dimensions (W=4 meters, L=4 meters, H=1 meter) is divided into four cells to enhance the overall system throughput. However, the area between cells is not fully covered that limits users' mobility and produces communication-outage areas. In [8], [9], RGB LEDs are proposed for MU systems, where each color can be used for different users. However, interference among LEDs is not considered in both works. Two MU precoding techniques from radio frequency (RF), namely block diagonalization (BD) [10] and linear Zero-forcing (ZF) [11], are adapted for VLC to mitigate MU interference in [12]–[15]. In these works, DC off-set is added to the transmitted signal to fulfill the requirement that only positive signals are allowed in VLC. As a result, the power efficiency degrades. Besides, the bit error rate (BER) performance degrades dramatically when a user is located in the vicinity of another user.

This paper proposes a novel modulation scheme to deal with interference for a system with two LEDs and two users. An optimization problem is formulated to decide the mapping from data bits to LED outputs such that the BERs of both users are reduced. The optimization problem is non-convex but we will analytically show how the complexity of the optimization problem can be significantly reduced. The proposed technique is compared in terms of BER with Repetition Coding (RC) technique [16], Multi-User Maximum Likelihood (MU-ML), and linear ZF precoding technique [15].

This paper is organized as follows. Section II describes the system model and Section III defines our proposed modulation scheme based on an optimization problem. How to reduce the complexity of the optimization problem is shown in Section IV before simulation results are presented in Section V. Finally, discussion and conclusion are given in Sections VI and VII, respectively.

II. System Model

Consider a multi-user VLC system with dimension $W \times H$ depicted in Fig. 1. Two LEDs are fixed in the ceiling and they are controlled by a central unit to transmit independent data streams to the users. Suppose there are two users and each of them has one PD pointing upward for receiving signals from the LEDs. Both users simultaneously receive the superposition of signals from both LEDs. Note that the channel gains from an LED to different users can be different. Assume that the central unit knows the channel gains of both users. This can be done by measuring the channel gains at the users and the users send the measured gains to the central unit through Wi-Fi. Here LoS channel model is assumed. So reflected lights from the surfaces of the indoor environment are not considered,

This work is supported in part by a grant from City University of Hong Kong with project number 7004238.

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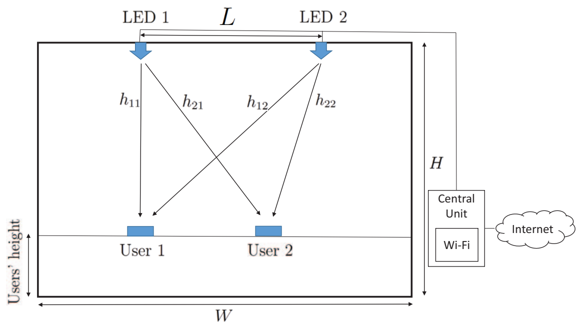


Fig. 1. A model of VLC system with two LEDs and two users.

since their powers are almost negligible compared to the LoS link [16].

Suppose we want to send data bits b_1 and b_2 to Users 1 and 2, respectively. For any $(\alpha, \beta) \in \{0,1\}^2$, assume that

$$\Pr\{b_1 = \alpha, b_2 = \beta\} = 0.25. \tag{1}$$

Definition 1 (Transmitter)**.** *For $i = 1, 2$, let $x_i(b_i, b_{3-i})$ be the output power of LED i when b_i and b_{3-i} are transmitted to User i and User $3-i$, respectively. Define*

$$\boldsymbol{x}_i = [x_i(0,0) \quad x_i(0,1) \quad x_i(1,0) \quad x_i(1,1)]$$

to be the mapping used by LED i. We require

$$S = x_i(0,0) + x_i(0,1) + x_i(1,0) + x_i(1,1)$$

which is the power constraint for the desired light intensity from each LED. For simplicity, the power constraint is the same for both LEDs.

Denote the channel gain between the i-th LED and the j-th User by

$$h_{ji} = \frac{(m+1)A}{2\pi d_{ji}^2} \cos^m(\theta_{ij}) \cos^k(\phi_{ij}) \tag{2}$$

for $-\frac{\pi}{2} \leq \theta_{ij} \leq \frac{\pi}{2}$ and $-\frac{\pi}{2} \leq \phi_{ij} \leq \frac{\pi}{2}$. Otherwise, $h_{ji} = 0$. Here, θ_{ij}, ϕ_{ij}, A, and d_{ji} are the irradiance angle at LED i related to PD j, the incident angle at PD j related to LED i, the active area of PD, and the distance between the LED i and PD j, respectively. Meanwhile, k is the field-of-view (FOV) coefficient of the PD. The Lambertian emission order is

$$m = \frac{-\ln 2}{\ln(\cos \phi_{\frac{1}{2}})}, \tag{3}$$

where $\phi_{\frac{1}{2}}$ represents the LED semi-angle at half power [17].

Definition 2 (Receiver)**.** *For $j = 1, 2$, let*

$$y_j(b_j, b_{3-j}) = \sum_{i=1}^{2} h_{ji} \cdot x_i(b_i, b_{3-i}), \tag{4}$$

which can be interpreted as the received power at User j when there is no noise. Define

$$\boldsymbol{y}_j = [y_j(0,0) \quad y_j(0,1) \quad y_j(1,0) \quad y_j(1,1)] \tag{5}$$

TABLE I
NOTATIONS FOR TRANSMITTED AND RECEIVED SYMBOLS.

b_1	b_2	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{y}_1$	$\mathbf{y}_2$
0	0	$x_1(0,0)$	$x_2(0,0)$	$y_1(0,0)$	$y_2(0,0)$
0	1	$x_1(0,1)$	$x_2(1,0)$	$y_1(0,1)$	$y_2(1,0)$
1	1	$x_1(1,1)$	$x_2(1,1)$	$y_1(1,1)$	$y_2(1,1)$
1	0	$x_1(1,0)$	$x_2(0,1)$	$y_1(1,0)$	$y_2(0,1)$

as the vector of possible values received by User j. Define an important parameter

$$\kappa_j = \min_{(\beta,\gamma)\in\{0,1\}^2} \{|y_j(1,\beta) - y_j(0,\gamma)|\}. \tag{6}$$

A summary of these notations is shown in Table I. Note that the orders of bits in the brackets are purposely defined in this way such that the presentation in the rest of the paper is simplified.

The received signals at User j is denoted by

$$Y_j = \sum_{i=1}^{2} h_{ji} x_i + w_j, \tag{7}$$

where w_j is a real value additive white Gaussian noise (AWGN) with zero mean and variance σ_j^2 at User j. The noise variance is given by

$$\sigma_j^2 = \sigma_{j,th}^2 + \sigma_{j,sh}^2, \tag{8}$$

where $\sigma_{j,th}^2$ and $\sigma_{j,sh}^2$ are the thermal noise variance and the shot noise variance at User j, respectively.

III. PROPOSED MODULATION SCHEME

The decoder design of our proposed modulation scheme is described in Section III-A. Then the way to choose the encoding mapping is formulated as an non-convex optimization problem in Section III-B. The optimization problem is transformed in Section III-C.

A. Decoder mapping

Let $\hat{b}_j$ be the estimate of b_j at User j. For fixed $\mathbf{x}$ and h_{ji}, the values of y_j in (4) is determined. Maximum-likelihood decoding is used so that y_j in (5) are sorted in ascending order to determine the required thresholds in decoding. For example, if $y_j(0,0) < y_j(0,1) < y_j(1,0) < y_j(1,1)$, then one threshold ξ_1 is needed and $\xi_1 = \frac{y_j(0,1)+y_j(1,0)}{2}$. If $y_j(0,0) < y_j(1,0) < y_j(0,1) < y_j(1,1)$, then three thresholds (ξ_1, ξ_2, ξ_3) are needed and $\xi_1 = \frac{y_j(0,0)+y_j(1,0)}{2}$, $\xi_2 = \frac{y_j(1,0)+y_j(0,1)}{2}$ and $\xi_3 = \frac{y_j(0,1)+y_j(1,1)}{2}$. If the received symbol is within $(-\infty, \xi_1) \cup (\xi_2, \xi_3)$, then the estimate $\hat{b}_j = 0$. Otherwise, $\hat{b}_j = 1$.

The average bit error rate $\Pr\{b_j \neq \hat{b}_j\}$ is depending on the order of y_j. However, it is easy to derive the upper bound

$$\Pr\{b_j \neq \hat{b}_j\} \leq \text{erfc}\left(\frac{\kappa_j}{2\sqrt{2\sigma_j^2}}\right), \tag{9}$$

where κ_j and σ_j^2 are given in (6) and (8), respectively. Therefore, it is important to choose $\mathbf{x}$ such that κ_j is maximized.

B. Optimization Problem 1 (OP1)

The following optimization is used to find the optimal κ_2 for a given κ_1.

$$\begin{aligned}
&\text{maximize} \quad \kappa_2,\\
&\text{subject to}\\
&\qquad |y_1(1,0) - y_1(0,0)| \geq \kappa_1, && (10)\\
&\qquad |y_1(1,0) - y_1(0,1)| \geq \kappa_1, && (11)\\
&\qquad |y_1(1,1) - y_1(0,0)| \geq \kappa_1, && (12)\\
&\qquad |y_1(1,1) - y_1(0,1)| \geq \kappa_1, && (13)\\
&\qquad |y_2(1,0) - y_2(0,0)| \geq \kappa_2, && (14)\\
&\qquad |y_2(1,0) - y_2(0,1)| \geq \kappa_2, && (15)\\
&\qquad |y_2(1,1) - y_2(0,0)| \geq \kappa_2, && (16)\\
&\qquad |y_2(1,1) - y_2(0,1)| \geq \kappa_2, && (17)\\
&\qquad 0 \leq x_1(0,0), x_1(0,1), x_1(1,0), x_1(1,1) \leq x_m, && (18)\\
&\qquad 0 \leq x_2(0,0), x_2(0,1), x_2(1,0), x_2(1,1) \leq x_m, && (19)\\
&\qquad x_1(0,0) + x_1(0,1) + x_1(1,0) + x_1(1,1) = S, && (20)\\
&\text{and } x_2(0,0) + x_2(0,1) + x_2(1,0) + x_2(1,1) = S, && (21)\\
&\text{variables:} \quad \mathbf{x}_1, \mathbf{x}_2,
\end{aligned}$$

where x_m is the maximum allowed instantaneous transmitted optical power, S and $y_j(b_j, b_{3-j})$ are defined in Definition 1 and Definition 2, respectively. The constraints (10)–(17) are due to (6) for $j = 1, 2$. Analytically, we can show that

$$0 \leq \kappa_j \leq \frac{S}{2}(h_{j1} + h_{j2}). \tag{22}$$

Since OP1 is non-convex due to the constraints (10)-(17), it cannot be solved by standard convex optimization tools.

C. Optimization Problem 2 (OP2)

Now, we consider an alternative way to solve the OP1. For a given $\mathbf{E}= [e_1 \ \ e_2 \ \ \cdots \ \ e_8]$ where $e_t = -1$ or 1 for $1 \leq t \leq 8$,

$$\begin{aligned}
&\text{maximize} \quad \kappa_2,\\
&\text{subject to}\\
&\qquad e_1 \cdot (y_1(1,0) - y_1(0,0)) \geq \kappa_1, && (23)\\
&\qquad e_2 \cdot (y_1(1,0) - y_1(0,1)) \geq \kappa_1, && (24)\\
&\qquad e_3 \cdot (y_1(1,1) - y_1(0,0)) \geq \kappa_1, && (25)\\
&\qquad e_4 \cdot (y_1(1,1) - y_1(0,1)) \geq \kappa_1, && (26)\\
&\qquad e_5 \cdot (y_2(1,0) - y_2(0,0)) \geq \kappa_2, && (27)\\
&\qquad e_6 \cdot (y_2(1,0) - y_2(0,1)) \geq \kappa_2, && (28)\\
&\qquad e_7 \cdot (y_2(1,1) - y_2(0,0)) \geq \kappa_2, && (29)\\
&\qquad e_8 \cdot (y_2(1,1) - y_2(0,1)) \geq \kappa_2, && (30)\\
&\qquad 0 \leq x_1(0,0), x_1(0,1), x_1(1,0), x_1(1,1) \leq x_m, && (31)\\
&\qquad 0 \leq x_2(0,0), x_2(0,1), x_2(1,0), x_2(1,1) \leq x_m, && (32)\\
&\qquad x_1(0,0) + x_1(0,1) + x_1(1,0) + x_1(1,1) = S, && (33)\\
&\text{and } x_2(0,0) + x_2(0,1) + x_2(1,0) + x_2(1,1) = S, && (34)\\
&\text{variables:} \quad \mathbf{x}_1, \mathbf{x}_2.
\end{aligned}$$

There are $2^8 = 256$ possible values of $\mathbf{E}$. For each $\mathbf{E}$, the solution of OP2 is also a feasible solution of OP1. On the other hand, the optimal solution of OP1 is a feasible solution of OP2 for a certain $\mathbf{E}$. Now, we define $\tilde{\kappa}_2$ by the following steps:

1) Solve OP2 by linear programming for 256 possible values of $\mathbf{E}$.
2) Let $\tilde{\kappa}_2$ be the maximum values among these 256 solutions.
3) The corresponding $\mathbf{E}$, $\mathbf{x}_1$ and $\mathbf{x}_2$ that yields $\tilde{\kappa}_2$ are denoted by $\mathbf{E}^*$, $\mathbf{x}_1^*$ and $\mathbf{x}_2^*$, respectively.

Theorem 1. *The optimal solution of OP1 is equal to* $\tilde{\kappa}_2$ *achieved by* $\mathbf{x}_1^*$ *and* $\mathbf{x}_2^*$.

Proof. Let κ_2^* be the optimal solution obtained from OP1. For each $\mathbf{E}$, the solution of OP2 is a feasible solution of OP1 so that $\kappa_2^* \geq \tilde{\kappa}_2$. On the other hand, the optimal solution obtained from solving OP1 is a feasible solution of OP2 for one of the possible values of $\mathbf{E}$. Therefore, $\kappa_2^* \leq \tilde{\kappa}_2$ and the theorem follows.. □

IV. Problem simplification

Finding κ_2^* is computationally costly because it requires solving OP2 for 256 possible values of $\mathbf{E}$. In this section, we are going to see that there is a nice underlying structure among $\mathbf{E}$ so that we do not need to check all the possible values of $\mathbf{E}$. As a result, we just need to solve OP2 for 21 values of $\mathbf{E}$. The underlying structure is illustrated through the following lemmas. The proofs of the lemmas are omitted due to space limitations.

Let $\mathbf{F}_1 = [e_1 \ \ e_2 \ \ e_3 \ \ e_4]$ and $\mathbf{F}_2 = [e_5 \ \ e_6 \ \ e_7 \ \ e_8]$ so that

$$\mathbf{E} = [\mathbf{F}_1 \ \ \mathbf{F}_2]. \tag{35}$$

Lemma 1. *Consider a fixed* $\mathbf{F}_j$*. In other words, only half of* $\mathbf{E}$ *is fixed. For any feasible solution of OP2 with* $\mathbf{E}$*, there exists* $(\alpha, \beta, \gamma) \in \{0,1\}^3$ *such that one of the following inequalities is satisfied*

$$y_j(\bar{\alpha}, \bar{\beta}) \leq y_j(\bar{\alpha}, \beta) < y_j(\alpha, \bar{\gamma}) \leq y_j(\alpha, \gamma), \tag{36}$$

$$y_j(\alpha, \bar{\gamma}) < y_j(\bar{\alpha}, \bar{\beta}) \leq y_j(\bar{\alpha}, \beta) < y_j(\alpha, \gamma), \tag{37}$$

$$y_j(\bar{\alpha}, \bar{\beta}) < y_j(\alpha, \bar{\gamma}) < y_j(\bar{\alpha}, \beta) < y_j(\alpha, \gamma), \tag{38}$$

where $\bar{\alpha} = 1 - \alpha$*,* $\bar{\beta} = 1 - \beta$ *and* $\bar{\gamma} = 1 - \gamma$*.*

If $\mathbf{F}_j$ is fixed, Lemma 1 tells that the order of y_j is also fixed. Indeed, User j needs 1, 2 and 3 thresholds for y_j satisfying (36)–(38), respectively. Motivated by Lemma 1, we can classify $\mathbf{F}_j$ into 3 groups according to the number of thresholds required. For $1 \leq \eta \leq 3$ and $1 \leq l \leq 3$, we write

$$\mathbf{E} \in \mathcal{F}_{1,\eta} \times \mathcal{F}_{2,l}, \tag{39}$$

if $\mathbf{F}_1$ and $\mathbf{F}_2$ belong to group η and group l, respectively.

Lemma 2. *If* $\boldsymbol{F}_1$ *and/or* $\boldsymbol{F}_2$ *equal to* $[-1 \ 1 \ 1 \ -1]$ *or* $[1 \ -1 \ -1 \ 1]$*, then no feasible solution satisfies* $\boldsymbol{E}$*=[*$\boldsymbol{F}_1 \quad \boldsymbol{F}_2$*].*

TABLE II
OUTLINE OF THE PROOF OF THEOREM 2.

E	The remaining numbers of the values of E after Lemma 2	Lemma 3	Lemma 4
$\mathcal{F}_{1,1} \times \mathcal{F}_{2,1}$	4	1	1
$\mathcal{F}_{1,1} \times \mathcal{F}_{2,2}$	8	2	2
$\mathcal{F}_{1,1} \times \mathcal{F}_{2,3}$	16	4	4
$\mathcal{F}_{1,2} \times \mathcal{F}_{2,1}$	8	2	2
$\mathcal{F}_{1,2} \times \mathcal{F}_{2,2}$	16	4	4
$\mathcal{F}_{1,2} \times \mathcal{F}_{2,3}$	32	8	8
$\mathcal{F}_{1,3} \times \mathcal{F}_{2,1}$	16	4	0
$\mathcal{F}_{1,3} \times \mathcal{F}_{2,2}$	32	8	0
$\mathcal{F}_{1,3} \times \mathcal{F}_{2,3}$	64	16	0
total	**196**	**49**	**21**

Lemma 3. *Let κ_2^* be the optimal solution of OP2 with*

$$\mathbf{E} = [e_1 \;\; e_2 \;\; e_3 \;\; e_4 \;\; e_5 \;\; e_6 \;\; e_7 \;\; e_8] \in \mathcal{F}_{1,\eta} \times \mathcal{F}_{2,l}. \tag{40}$$

If

$$\boldsymbol{E}_1 = [e_4 \;\; e_3 \;\; e_2 \;\; e_1 \;\; -e_5 \;\; -e_7 \;\; -e_6 \;\; -e_8], \tag{41}$$
$$\boldsymbol{E}_2 = [-e_1 \;\; -e_3 \;\; -e_2 \;\; -e_4 \;\; e_8 \;\; e_7 \;\; e_6 \;\; e_5], \tag{42}$$
$$\boldsymbol{E}_3 = [-e_4 \;\; -e_2 \;\; -e_3 \;\; -e_1 \;\; -e_8 \;\; -e_6 \;\; -e_7 \;\; -e_5], \tag{43}$$

then for $i = 1, 2, 3$, $\mathbf{E}_i \in \mathcal{F}_{1,\eta} \times \mathcal{F}_{2,l}$ and the solution of OP2 with $\mathbf{E}_i$ is equal to κ_2^*.

Lemma 4. *For $1 \le l \le 3$ and any $\mathbf{E} \in \mathcal{F}_{1,3} \times \mathcal{F}_{2,l}$, if the solution of OP2 with $\mathbf{E}$ is κ_2^*, then there exists a feasible solution for OP2 with $\mathbf{E}' \in \mathcal{F}_{1,\eta} \times \mathcal{F}_{2,m}$ with $1 \le \eta \le 2$ and $1 \le m \le 2$ and this feasible solution yields κ_2' such that $\kappa_2' \ge \kappa_2^*$.*

Theorem 2. *For $1 \le i \le 21$, if $\kappa_{2,i}$ is the solution of the OP2 with $\mathbf{E}$ equal to the i-th values from:*

$$\begin{aligned}
&[++++++++], [+++++-+-], [++++++--],\\
&[+++++---], [+++++++-], [++++-+--],\\
&[+++++-++], [++--++++], [+-+-++++]\\
&[++--++--], [+-+-++--], [-+-+++--],\\
&[--++++--], [++--+---], [++---+--],\\
&[++----+-], [++-----+], [++--+++-],\\
&[++--++-+], [++--+-++], [++---+++],
\end{aligned} \tag{44}$$

where "+" denotes 1 and "−" denotes -1, then the solution of OP1 is equal to $\max_i \kappa_{2,i}$.

Proof. Table II outlines the steps of the proof. Due to Lemma 2, two vectors from $\cup_\eta \mathcal{F}_{1,\eta}$ and two vectors from $\cup_l \mathcal{F}_{2,l}$ can be omitted. Lemma 3 says that any $\mathcal{F}_{1,\eta} \times \mathcal{F}_{2,l}$ can be partitioned into subgroups where each subgroup contains only 4 vectors and these vectors lead to the same solution in solving OP2. Therefore, $\frac{3}{4}$ portion of $\mathcal{F}_{1,\eta} \times \mathcal{F}_{2,l}$ can be omitted. Finally, Lemma 4 says that all the groups $\mathcal{F}_{1,3} \times \mathcal{F}_{2,l}$ with $1 \le l \le 3$ can be omitted. $\square$

Motivated by Theorem 2, Algorithm 1 is proposed to choose $\mathbf{x}_i$ for the transmitters. Simulation results of Algorithm 1 will be shown in next section.

Algorithm 1

1: Given: h_{ij}, S, x_m, κ_1
2: Solve OP2 for 21 times with $\mathbf{E}$ equal to the i-th values in (44) for $i = 1, 2, \cdots, 21$.
3: Among these 21 solutions, choose the maximum one and denote it by κ_2^*. From this particular solution, the optimal vectors of the transmitted powers $\mathbf{x}_1^*$ and $\mathbf{x}_2^*$ are also obtained.
4: Set $\mathbf{x}_1 = \mathbf{x}_1^*$, and $\mathbf{x}_2 = \mathbf{x}_2^*$ in Definition 1.

V. SIMULATION RESULTS

In this section, our proposed scheme is compared in terms of BER with RC, MU-ML, and linear ZF precoding by using Monte Carlo simulation in the environment shown in Fig. 1. Here LoS channel is assumed. To make a fair comparison, the same bandwidth efficiency is set to all techniques so that 2-PAM, 4-PAM and 2-PAM are used with MU-ML, RC and linear ZF precoding, respectively, and 1 bit/symbol/user is assumed in our proposed scheme. At the receiver side, maximum likelihood detector is used by all techniques. Different values of κ_1 are chosen in each round of simulation. For $0 \le i \le 100$, $\kappa_1 = \frac{i}{100} \cdot \frac{S}{2}(h_{11} + h_{12})$ is used in Algorithm 1 to obtain κ_2. Among these 101 pairs of (κ_1, κ_2), the pair achieving the minimum bit error rate is chosen.

TABLE III
PARAMETERS OF THE SIMULATION SETUP.

Parameters	Values	Parameters	Values
A	$10^{-4} m^2$	h_{ji}	from (2)
k	1.4738	S	$0.4W$
m	1	x_m	$\frac{3}{4}S$
$\sigma_{j,th}^2$	5.3784×10^{-16}	$\sigma_{j,sh}^2$	$(641 y_j + 1.84) \times 10^{-14}$

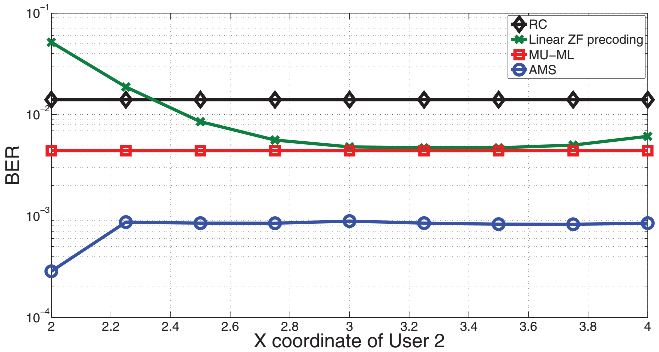
The parameters of the simulation setup are shown in Table III. The values of $\sigma_{j,th}^2$ and $\sigma_{j,sh}^2$ are chosen according to [18] with bandwidth equal to 20 MHz. The comparison is conducted in two different scenarios.

A. Scenario 1 (User 2 at different locations)

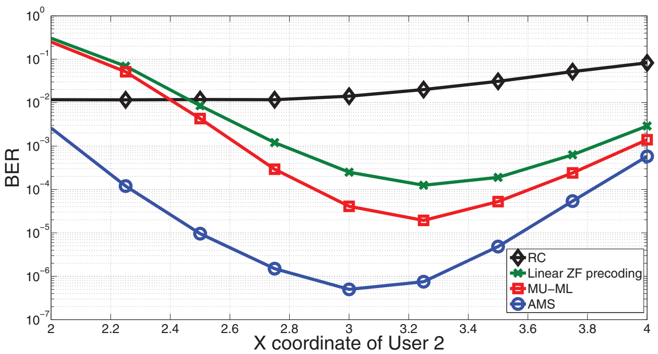
The coordination of the LEDs and users are listed in Table IV. The numerical BER performance of User 1 and User 2 are illustrated in Fig. 2(a) and Fig. 2(b), respectively.

TABLE IV
COORDINATION OF THE LEDS AND USERS FOR SCENARIO 1.

Parameters	Values (in meters)
Office size (W, H)	(4, 3)
LED 1 position	(1, 3)
LED 2 position	(3, 3)
User 1 position	(1.5, 0.85)
User 2 position	from (2, 0.85) to (4, 0.85)
LEDs separation L	2



(a) User 1



(b) User 2

Fig. 2. Simulation results of the BER performance of Users 1 and 2 for MU-ML, RC, linear ZF precoding and the proposed adaptive modulation scheme (AMS) for different positions of User 2 in Scenario 1.

When User 2 is located at (2 meters, 0.85 meters), it has equal channel gains from both LEDs. At this location, MU-ML and linear ZF precoding fail severely with BER around 0.25 because MU-ML cannot distinguish between the two received signals from both LEDs. Similarly, linear ZF precoding fails dramatically to eliminate the interference due to the same channel gains from both LEDs. When User 2 moves towards LED 2, the difference between the channel gains of User 2 with the two LEDs increases. Therefore, the BER performance of MU-ML and linear ZF precoding improve. However, when User 2 moves from (3.25 meters, 0.85 meters) to (4 meters, 0.85 meters), the channel gains with both LEDs reduce and hence the received signal power decreases as well. As a result, the BER performance of User 2 degrades.

Our proposed scheme outperforms MU-ML because it can adaptively change the transmitted constellation $\mathbf{x}$ to reduce the BER of both users while MU-ML transmits fixed signals. Although RC avoids interference entirely, its performance is the worst among the other techniques for most of the cases in Fig. 2. This is due to a higher modulation scheme 4-PAM is employed to compensate the low bandwidth efficiency.

B. Scenario 2 (Different LED separation)

The coordinate of the LEDs and users are listed in Table V. Users' locations are fixed but separation distance between the LEDs from 1 meter to 3 meters are considered. The BER performance of User 1 and User 2 is illustrated in Fig. 3.

TABLE V
COORDINATION OF THE LEDS AND USERS FOR SCENARIO 2.

Parameters	**Values (in meters)**
Office size (W, H)	(4, 3)
LED 1 position	$(2 - L/2, 3)$
LED 2 position	$(2 + L/2, 3)$
User 1 position	(1, 0.85)
User 2 position	(3, 0.85)
LEDs separation L	from 1 to 3

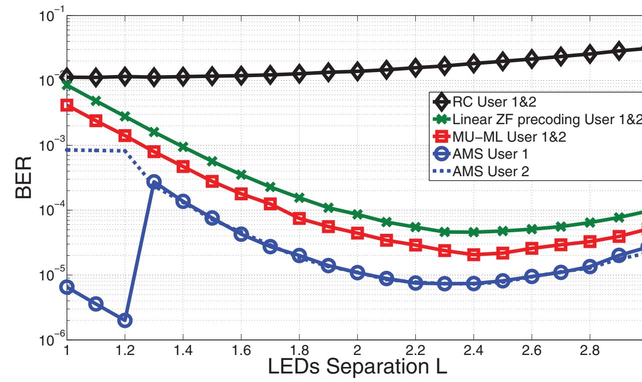


Fig. 3. Simulation results of the BER performance of Users 1 and 2 for MU-ML, RC, and linear ZF precoding and the proposed adaptive modulation scheme (AMS) for different L in Scenario 2.

When $L \geq 1.3$ metres, all techniques perform identically for both users because users' locations and LEDs' locations are symmetrical to the office center, i.e., $h_{11} = h_{22}$ and $h_{12} = h_{21}$. In this case, the solution of the OP2 with $\mathbf{E} = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]$ is chosen in Algorithm 1. When $L < 1.3$ metres, the solution of the OP2 with $\mathbf{E} = [1\ 1\ 1\ 1\ 1\ {-1}\ 1\ {-1}]$ is chosen which is different from the $\mathbf{E}$ for $L \geq 1.3$. This explains why there is a discrete change for the BER of User 1.

In Fig. 3, the performance of MU-ML and linear ZF precoding degrade severely for smaller L because the channel gains of the two LEDs are similar. However, the performance of RC improves with reducing L because it avoids interference totally. Among all LED separation, our scheme outperforms all the other techniques because it adapts the transmitted symbols $\mathbf{x}$ to reduce the BER for both users.

VI. DISCUSSION

Several observations from the simulation results are discussed in this section. These observations will lead to the further research work of this paper.

A. Number of possible $\mathbf{E}$

For the simulations results depicted in Figs. 2 and 3, we find that the optimal solution in Algorithm 1 is always achieved

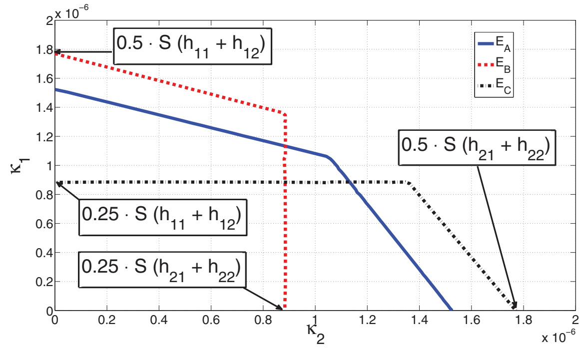


Fig. 4. The values of κ_1 verses κ_2 for $\mathbf{E}_A$, $\mathbf{E}_B$ and $\mathbf{E}_C$ in (45)–(47).

by solving OP2 with $\mathbf{E}$ equal to one of the following vectors:

$$\mathbf{E}_A = [1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1] \tag{45}$$

$$\mathbf{E}_B = [1\ \ 1\ \ 1\ \ 1\ \ 1\ \ -1\ \ 1\ \ -1] \tag{46}$$

$$\mathbf{E}_C = [1\ \ -1\ \ 1\ \ -1\ \ 1\ \ 1\ \ 1\ \ 1]. \tag{47}$$

We have also tried to generate h_{ji} for $i, j = 1, 2$ according to a uniform distribution rather than using (2). Interestingly, we get the same conclusion. Therefore, it is possible to further extend the result in Theorem 2 to show that we just need to check a small number of $\mathbf{E}$.

B. Selection of κ_1

Consider LEDs 1 and 2 are located at (1.2 , 3) and (2.8, 3), respectively. Suppose Users 1 and 2 are located at (1, 0.85) and (3, 0.85), respectively. For this particular configuration, we now take a closer look of Algorithm 1. The optimal κ_2 obtained from solving OP2 for different κ_1 and $\mathbf{E}$ in (45)-(47) is depicted in Fig. 4. If κ_1 is chosen to be larger than 1.15, $\mathbf{E}_B$ is chosen in Algorithm 1. Moreover, Algorithm 1 chooses $\mathbf{E}_A$ and $\mathbf{E}_C$ for $0.9 \le \kappa_1 \le 1.15$ and $\kappa_1 < 0.9$, respectively. To get a fair BER for both users, the turning point of the curve regarding $\mathbf{E}_A$ (i.e., $\kappa_1 = \kappa_2 = 1.05$) is used to obtain Fig. 3.

It is reasonable to ask: what will happen if we change OP1 from $\max \kappa_2$ to $\max(\kappa_1 + \kappa_2)$? For the configuration considered in Fig. 4, the turning points of curves regarding $\mathbf{E}_B$ and $\mathbf{E}_C$ both give the same optimal value. For these two pairs of (κ_1, κ_2), the BERs of users are quite different as κ_1 and κ_2 are different. In general, the result depends on the slopes of the curves regarding $\mathbf{E}_B$ and $\mathbf{E}_C$. For certain channel gain h_{ji}, it is possible that solving $\max(\kappa_1 + \kappa_2)$ always gives an optimal solution with either κ_1 or κ_2 equal to 0. When this happens, there is a fairness issue among the two users.

C. Future work

It is interesting to extend OP1 by adding the fairness constraint in future work. It is also important to prove that it is always sufficient to consider only $\mathbf{E}$ in (45)–(47). This can give a better picture about the relationship between κ_1 and κ_2. Furthermore, this will lead us to the extension of the proposed modulation scheme to support more modulation levels and more users.

VII. Conclusion

We have presented an adaptive modulation scheme for VLC. The proposed technique supports two LEDs and two users where each user has one PD. Our technique optimizes the transmitted symbols of both LEDs to reduce the BER at both users. Although the original optimization problem is non-convex, we have converted the problem into a set of linear optimization problems and then simplified the set significantly. Simulation results showed that our proposed modulation scheme outperforms MU-ML, RC, and linear ZF precoding in terms of BER under different users' positions and different LED separation distances.

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