BER Analysis for Interfering Visible Light Communication Systems

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***Abstract*—This paper studies the error performance of visible light communication systems consisting of multiple transmitter-receiver pairs, where mutual interference exists. Unlike the traditional method which approximates the interference signal as a Gaussian random process and express the bit error rate (BER) as a function of the signal-to-interference-plus-noise power ratio (SINR), we conduct a detailed analysis that captures the signal structure of the interference in terms of the number of light sources, and derive the BER explicitly. Simulation results show that under a realistic small-room scenario with basic illumination requirement, the BER predicted by the Gaussian model is much higher than the exact BER. To provide an alternative metric for resource allocation purpose, we propose a new approximate expression for the BER. This new approximation is shown to be very accurate under our simulation model.**

I. INTRODUCTION

Visible light communication (VLC) is an emerging technology for indoor optical wireless communication. It makes use of light emitting diodes (LED) as transmitters, and provides illumination and data communication simultaneously [1]. Comparing with traditional radio frequency (RF) communications, VLC has many advantages such as, it can use unregulated and license-free visible electromagnetic (EM) spectrum to transmit data and it does not interfere with existing RF systems. Besides, optical wireless signal provides higher security since it is very difficult for an eavesdropper to pick up the signal from outside the room. The first IEEE standard for VLC was published in [2], which represents a significant milestone in promoting deployment of VLC.

In the literature, there have been some papers working on the error performance analysis of VLC, including [3]–[6]. However, they all focus on the scenario that there is a single transmitter-receiver pair. In this paper, we take a system perspective and consider indoor VLC networks consisting of multiple transmitter-receiver pairs. Due to the broadcast nature of wireless channels, interference arises whenever multiple transmitter-receiver pairs are active concurrently in the same frequency band, and each receiver is only interested in retrieving information from its own transmitter. In RF systems, interference is typically modeled as a Gaussian process [7]. Then the performance of a communication link such as BER and throughput can be expressed as functions of the signal-to-interference-plus-noise power ratio (SINR). This Gaussian interference assumption can be partially justified by the Central Limit Theorem (CLT) if the interference comes from mutually independent, identically distributed interferer signal processes and if the number of such interferers is large. If there are only a few interferers, the CLT is not applicable. References [8], [9] proposed a more accurate interference model in an RF system to deal with this situation and showed that the Gaussian interference assumption did yield inaccurate BER, which could be overestimated or underestimated.

In VLC systems, it is also very common to make Gaussian assumption on the interference and use SINR as the performance metric (see e.g. [10], [11]). However, in an indoor VLC network, the number of transmitter-receiver pairs is typically small due to the limited number of LEDs in a room area, and therefore the CLT may not be applicable either. In this paper, we dispense with the Gaussian assumption on the interference and analyse the BER for VLC systems using on-off keying (OOK), which is one of the most commonly used modulation schemes in VLC (see e.g., [12]–[14]). We derive the BER expressions under an exact analysis and under the Gaussian model, and perform simulations to compare them. It is shown that the Gaussian interference model is generally pessimistic. To facilitate effective resource allocation, we propose a new approximation on BER, which is numerically shown to be close to the exact BER.

The rest of the paper is organized as follows. Section II gives the preliminary. The system model and BER analysis are presented in Sections III and IV, respectively. Section V provides simulation results under a realistic setting. In Section VI, a new method of approximation on BER is discussed. Some concluding remarks are offered in Section VII.

II. PRELIMINARY

In an indoor VLC system, LEDs are used as transmitters and photodiodes are used as receivers. Assume that the LEDs are all in line-of-sight (LOS) of the receivers. In this section, we explain the channel gain and noise variance in VLC systems, which will be used later.

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A. Channel Gain

For Lambertian radiation pattern of the transmitting LED, the LOS link gain can be derived as [3]

$$h = \frac{(m+1)A_r}{2\pi D^2}\cos^m(\phi)T(\psi)g(\psi)\cos(\psi), \quad (1)$$

where the parameters are defined as follows. A_r is the effective area of the receiver photodiode. D is the transmitter-receiver distance. ϕ is the irradiance angle with respect to the normal at the transmitter and ψ is the incident angle with respect to the normal at the receiver (see Fig. 1). $T(\psi)$ and $g(\psi)$ are the filter gain and concentrator gain at the receiver, respectively. In this paper, we assume $T(\psi) = g(\psi) = 1$. m is the Lambertian parameters given by $m = \frac{-\ln 2}{\ln(\cos(\phi_{1/2}))}$, where $\phi_{1/2}$ is the half-power angles of LED. Note that the receivers are not necessary to be horizontal. When a receiver is horizontal, $\psi = \phi$.

B. Noise

At the receiver, the photodiode current is affected by two noise processes: shot noise and thermal noise. The shot noise is related to the incident optical power and its variance (in A^2) is given by [3]

$$\sigma^2_{shot} = 2qR_pP_rB + 2qI_{bg}I_2B, \quad (2)$$

where P_r (in Watt) is the received optical power and the definitions of other parameters can be found in Table I. The thermal noise variance (in A^2) is independent of the incident power and is given by [3]

$$\sigma^2_{thermal} = \frac{8\pi kT_k}{G}\eta A_r I_2 B^2 + \frac{16\pi^2 kT_k\Gamma}{g_m}\eta^2 A_r^2 I_3 B^3, \quad (3)$$

where the parameters are defined in Table I. The total noise variance is

$$\sigma^2_{noise} = \sigma^2_{shot} + \sigma^2_{thermal}. \quad (4)$$

In reference [15], measurement results show that none of the shot noise and thermal noise can be ignored. We can also get this conclusion by the following calculation. The typical values of the parameters are listed in Table I. By substituting them into (3), we have $\sigma^2_{thermal} = 17.5348 \times 10^{-16}$. In practice, the standard illumination level for most indoor environments (classroom, conference-room, lecture hall, offices, etc.) is between 300 and 500 lux at 0.8 m height from the floor. This is equivalent to a power requirement of $p_r \geq 2.25 \times 10^{-4}$W (assuming $\psi = \phi$). Therefore the shot noise variance should be at least 2.1120×10^{-16} A^2 and is on the same level as that of thermal noise. In the subsequent analysis of BER, we will incorporate both the shot noise and thermal noise, where the model is different from that in RF systems without shot noise issue (e.g. [8]).

III. System Model

Consider a room where there are N LED transmitters being installed on the ceiling and they are denoted by $\{LED_i : i = 1, \ldots, N\}$. Besides the function of lighting, these LEDs are used to transmit data to N receivers $\{rec_i : i = 1, \ldots, N\}$, which could be mobile devices equipped with photodiodes.

R_p	Responsivity	22 nA/lux (2.933×10^{-2} A/W)
q	Electronic charge	1.6×10^{-19} C
A_r	Effective area	15 mm^2
B	Data rate	100 Mb/s
I_{bg}	Background dark current	10 nA
I_2	Noise Bandwidth factor	0.562
I_3		0.0868
k	Boltzmann constant	1.38×10^{-23} m^2 kg s^{-2} K^{-1}
T_k	Absolute temperature	295 K
G	Open-loop voltage gain	10
η	Capacitance	112×10^{-8} F/m^2 (s^4 A^2 m^{-4} kg^{-1})
Γ	FET channel noise factor	1.5
g_m	FET transconductance	0.03 S (kg^{-1} m^{-2} s^3 A^2)
A=ampere	C=coulomb, m=metre,	s=second, K = kelvin
W=watt	kg=kilogram, F=farad,	S=siemens

TABLE I: Parameters [3], [15]

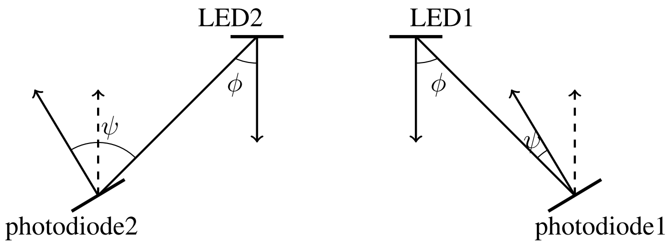

Fig. 1: Irradiance angle and incident angle.

In particular, LED_i intends to transmit data to rec_i and there are N transmitter-receiver pairs. Fig. 1 illustrates an example of a two-pair system. Assume that the LEDs transmit data separately without any central unit controlling, and no cooperation between them is allowed. All transmissions use the same wavelength carrier and thus cause interference to each other.

In this paper, the OOK modulation scheme is considered. It is possible to use the mathematical technique in this paper to analyze other modulation schemes like pulse amplitude modulation (PAM) but this is reserved for future work due to the limited space. The information bits of LED_i are denoted by $\{b_i^k\}_{k=-\infty}^{\infty}$ where b_i^k is uniformly distributed on $\{0, 1\}$. The LED is on when $b_i^k = 1$ and is off when $b_i^k = 0$. Let $\text{rect}(t)$ be the unit-amplitude rectangular pulse of duration T. The data rate is then $B = 1/T$. The transmitted optical signal $s_i(t)$ of LED_i is

$$s_i(t) = p_i \sum_{k=-\infty}^{\infty} b_i^k \text{rect}(t - kT), \quad (5)$$

where p_i (in Watt) is the peak optical power of the light wave emitter. The average transmitted power is $p_i/2$. Denote $\mathbf{p} = [p_1, \ldots, p_N]$ to be the power vector of the system.

Assume that the LEDs are all in LOS of the receivers. Let h_{ij} and d_{ij} denote the LOS optical channel gain and delay offset from LED_j to rec_i, respectively. The received electric signal at the photodiode of rec_i is

$$r_i(t) = \sum_{j=1}^{N} R_p h_{ij} s_j(t - d_{ij}) + n_i(t), \tag{6}$$

where $n_i(t)$ is the noise. As mentioned in the preliminary, the noise is a combination of shot noise and thermal noise. We model them as the additive white Gaussian noise (AWGN) with two-sided power spectral density N_{s_i} and N_t, respectively.

A receiver demodulates the received signal using a matched filter, followed by a threshold decision. The impulse response $s_i^0(t)$ of the filter of rec_i is a rectangular pulse of amplitude 1 and duration T. Assume $d_{ii} = 0$, that is, the matched filter of rec_i is synchronized to the arrival signal transmitted by LED_i. Consider a bit interval as $[0, T]$ to be demodulated. Label the first bit overlapping with this interval by b_j^0 for all $j = 1, \ldots, N$ and the following bit by b_j^1. We use $\tau_{ij} \in [0, 1)$ to denote the normalized (by T) misalignment of b_j^0 with respect to b_i^0. In calculating the average BER, we assume that the receivers are static over the averaging period so that τ_{ij}'s are all constants.

After matched filtering, the input to the decision device for rec_i is given by

$$\begin{aligned} y_i &= \frac{1}{T}\int_0^T r_i(t) s_i^0(t)\mathrm{d}t \\ &= p_i h_{ii} b_i^0 R_p + \sum_{j \neq i} p_j h_{ij}[\tau_{ij} b_j^0 + (1 - \tau_{ij}) b_j^1] R_p + n_i \\ &= p_i h_{ii} b_i^0 R_p + \sum_{j \neq i} p_j h_{ij} W_{ij} R_p + n_i, \end{aligned} \tag{7}$$

where

$$W_{ij} = \tau_{ij} b_j^0 + (1 - \tau_{ij}) b_j^1 \text{ for } j \neq i \tag{8}$$

is a discrete random variable uniformly distributed over $\{\tau_{ij}, 1 - \tau_{ij}, 1, 0\}$, and

$$n_i = \frac{1}{T}\int_0^T n_i(t) s_i^0(t)\mathrm{d}t \tag{9}$$

is a Gaussian random variable with zero mean and variance $(N_{s_i} B + N_t B)$. The variance of the thermal noise $N_t B$ is decided by (3) and is independent of the incident power. The variance of the shot noise is decided by (2) as

$$N_{s_i} B = 2q(p_i h_{ii} b_i^0 R_p + \sum_{j \neq i} p_j h_{ij} W_{ij} R_p) B + 2q I_{bg} I_2 B,$$

and is dependent on the incident power. For notation simplicity, we use $\sigma_c^2 = 2q I_{bg} I_2 B + N_t B$ to denote the constant part of the variance of the total noise.

IV. Analysis of Bit Error Rate

In this section, we first present an exact analysis of BER. Then, we present an approximate analysis, which makes a simplifying assumption that the interference term is Gaussian distributed.

A. Exact Analysis

Let ξ_i be the decision threshold of rec_i. An error occurs if $y_i > \xi_i$ when $b_i = 0$ or if $y_i < \xi_i$ when $b_i = 1$. Conditioned on W_{ij} for all $j \neq i$, the error rates are

$$p_r\{y_i > \xi_i | b_i^0 = 0, W_{ij}, j \neq i\} = Q\left(\frac{\xi_i - \sum_{j \neq i} p_j h_{ij} W_{ij} R_p}{\sqrt{2q \sum_{j \neq i} p_j h_{ij} W_{ij} R_p B + \sigma_c^2}}\right), \tag{10}$$

$$p_r\{y_i < \xi_i | b_i^0 = 1, W_{ij}, j \neq i\} = Q\left(\frac{p_i h_{ii} R_p + \sum_{j \neq i} p_j h_{ij} W_{ij} R_p - \xi_i}{\sqrt{2q(p_i h_{ii} R_p + \sum_{j \neq i} p_j h_{ij} W_{ij} R_p) B + \sigma_c^2}}\right). \tag{11}$$

The average BER of the i-th transmitter-receiver pair is

$$\begin{aligned} P_{e_i} = \frac{1}{4^{N-1}} \sum_{W_{iN} \in \{\tau_{iN}, 1-\tau_{iN}, 1, 0\}} \cdots \sum_{W_{i1} \in \{\tau_{i1}, 1-\tau_{i1}, 1, 0\}} \\ \left[\frac{1}{2} Q\left(\frac{\xi_i - \sum_{j \neq i} p_j h_{ij} W_{ij} R_p}{\sqrt{2q \sum_{j \neq i} p_j h_{ij} W_{ij} R_p B + \sigma_c^2}}\right) + \right. \\ \left. \frac{1}{2} Q\left(\frac{p_i h_{ii} R_p + \sum_{j \neq i} p_j h_{ij} W_{ij} R_p - \xi_i}{\sqrt{2q(p_i h_{ii} R_p + \sum_{j \neq i} p_j h_{ij} W_{ij} R_p) B + \sigma_c^2}}\right)\right]. \end{aligned} \tag{12}$$

B. Gaussian approximation

In the literature, it is common to treat interference as white Gaussian noise. That is, the interference term $\sum_{j \neq i} p_j h_{ij} W_{ij} R_p$ in (7) is approximated by a Gaussian random variable with identical mean and variance. We simply call this model as *Gaussian interference model*.

For the Gaussian interference model, the mean and variance of the interference term $\sum_{j \neq i} p_j h_{ij} W_{ij} R_p$ can be calculated by averaging over W_{ij} for all $j \neq i$. As mentioned before, W_{ij} takes values from $\{\tau_{ij}, 1 - \tau_{ij}, 1, 0\}$ with equal probability. We get that the mean is $\frac{1}{2}\sum_{j \neq i} p_j h_{ij} R_p$ and the variance is $\sigma_I^2 = \sum_{j \neq i} p_j^2 h_{ij}^2 R_p^2 (\frac{1}{2}\tau_{ij}^2 - \frac{1}{2}\tau_{ij} + \frac{1}{4})$. The shot noise variance under the Gaussian interference model becomes

$$N_{s_i} B = 2q p_i h_{ii} b_i^0 R_p B + 2q I_{bg} I_2 B.$$

Hence the BER can be derived as

$$\begin{aligned} P_{e_i}^G = \frac{1}{2} Q\left(\frac{\xi_i - \frac{1}{2}\sum_{j \neq i} p_j h_{ij} R_p}{\sqrt{\sigma_I^2 + \sigma_c^2}}\right) + \\ \frac{1}{2} Q\left(\frac{p_i h_{ii} R_p + \frac{1}{2}\sum_{j \neq i} p_j h_{ij} R_p - \xi_i}{\sqrt{\sigma_I^2 + 2q p_i h_{ii} R_p B + \sigma_c^2}}\right). \end{aligned} \tag{13}$$

The BER analysis in this section works for any number of transmitter-receiver pairs. In the next section, we carry out a simulation study to compare the BER performance under the Gaussian interference model with the exact BER.

V. Simulation Studies

Consider a room of length 5 m, breadth 3 m and height 2.7 m. Four LEDs are installed on the ceiling as transmitters whose coordinates are $LED_1(1.2, 0.5, 2.7)$, $LED_2(3.8, 0.5, 2.7)$, $LED_3(1.2, 2.5, 2.7)$ and $LED_4(3.8, 2.5, 2.7)$, as illustrated in Fig. 2. Each LED is assumed to have a typical luminous flux of 6000 lumens (lm) and a luminous efficacy of 117 lm/W, which is the same as the bridgelux LEDs (BXRA-56C5300-H-00) [16]. A receiver is put on the desk at a height of 0.83 m from the floor. It is equipped with a photodiode, which is used to measure the incident light intensity. The responsivity and effective area of this photodiode are given in Table I. The configurations of the LEDs and photodiodes are the same as those used in [15]. The Lambertian parameter is $m = 1$.

Given the coordinates of a transmitter and a receiver as (x_t, y_t, z_t) and (x_r, y_r, z_r), the irradiance angle is given by

$$\phi = \arccos\left(\frac{|z_t - z_r|}{\sqrt{(x_t - x_r)^2 + (y_t - y_r)^2 + (z_t - z_r)^2}}\right). \tag{14}$$

The incident angle also depends on the orientation of the photodiode. If the photodiode's normal is parallel to the normal of the transmitter, then $\psi = \phi$. Otherwise assume that the normal of the photodiode is parallel to the vector $\mathbf{v}_n = [x_n, y_n, z_n]$. Let $\mathbf{v}_{rt} = [x_t - x_r, y_t - y_r, z_t - z_r]$. The incident angle can be calculated by

$$\psi = \arccos\left(\frac{\mathbf{v}_n^T \mathbf{v}_{rt}}{||\mathbf{v}_n|| \cdot ||\mathbf{v}_{rt}||}\right). \tag{15}$$

Let L be the luminous flux of an LED (in lumen) and D is the distance between transmitter and receiver. The illuminance (in lux) at a point (x_r, y_r, z_r) is given by [3]

$$I = L\frac{(m+1)\cos^m(\phi)\cos(\psi)}{2\pi D^2}. \tag{16}$$

Fig. 3 depicts the distribution of illuminance at a height of 0.83 m from the floor (assuming $\psi = \phi$) when the four LEDs are turned on and each has luminous flux of 6000 lm. It can be seen that this LED setting satisfies the typical lighting requirement of 300 lux, which shows that our parameter setting is realistic.

Next, we consider the BER performance for such an indoor VLC network. Consider that there is a receiver at the location $(x_r, y_r, 0.83)$ and the normal of its photodiode is $\mathbf{v}_n = [x_n, y_n, z_n]$. We can figure out the link gains between the receiver and the four LED transmitters by (1). Assume that the receiver's dedicated transmitter is the LED that has the largest link gain, and the remaining three LED transmitters are treated as interferers. For each position of the receiver, the exact BER and the BER under the Gaussian interference model can be calculated by (12) and (13), respectively. Fig. 2 gives the contour plot of BER for levels of $10^{-5}, 10^{-3}$ and 10^{-2} over the (x_r, y_r) plane of the room. In this example, all the transmitters are assumed to be synchronized (i.e., $\tau_{ij} = 0$ for all i, j) and use luminous flux of 6000 lm (i.e., $6000/117 =$ 51.2821 W). The normal of the receiver photodiode is set to be $\mathbf{v}_n = [\tan(30^o)\cos(20^o), \tan(30^o)\sin(20^o), 1]$, and is fixed for all the locations of the receiver. The decision threshold ξ is chosen as the one that minimizes the BER at each particular position, which is numerically obtained by one-dimensional search.

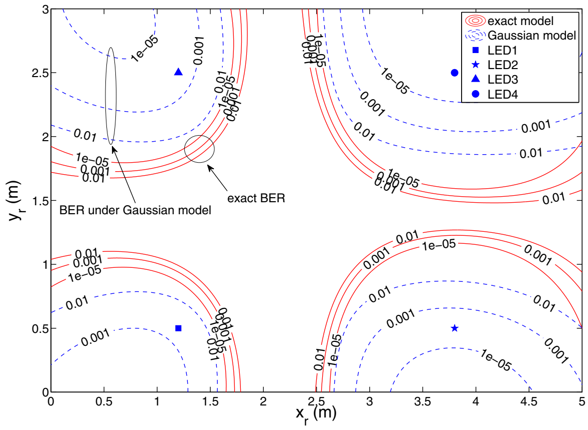

Fig. 2: The contour plot of BER for levels of $10^{-5}, 10^{-3}, 10^{-2}$ over the (x_r, y_r) plane under the exact analysis and the Gaussian interference model.

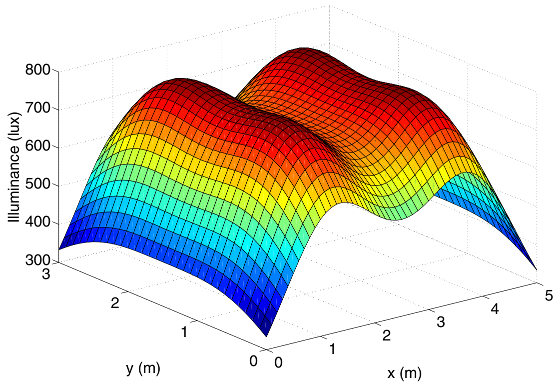

Fig. 3: The distribution of illuminance in a room.

For Fig. 2, first it can be seen that the contour plot is asymmetric. This is due to the fact that the normal of the photodiode is not vertical. Second, the three curves representing different levels of BER under the Gaussian interference model are more separative than those under the exact model. Third, given the same location, that is, the same link gains setting, the BER under the Gaussian interference model overestimates the BER by certain orders. For example, at the locations where the BER is 10^{-5}, the BER under the Gaussian interference model is beyond 10^{-2}.

VI. A New BER Approximation

For resource allocation purpose, the Gaussian interference model is commonly used, since it allows resource allocation to be considered without concerning the details of the structure of the physical signals. This model, however, is not accurate in estimating BER for small values of N, as we have seen in the previous section. To determine the exact BER in (12), it is necessary to determine the optimal decision threshold, which can only be done by one-dimensional search. This makes the expression difficult to be used for resource allocation. To circumvent the need of numerically searching for the optimal threshold, in this section, we provide an approximate expression of the BER for which the optimal threshold can be found.

In the following, we restrict our discussion to the cases of channel gains and powers such that

$$p_i h_{ii} R_p \geq \sum_{j \neq i} p_j h_{ij} R_p \text{ for } i = 1, \ldots, N. \tag{17}$$

The intuition behind this condition is that the intended signal is larger than the aggregated interfering signals, so as to guarantee certain communication quality. To see that this restriction is reasonable, assume on the contrary that $p_i h_{ii} R_p < \sum_{j \neq i} p_j h_{ij} R_p$. Then, we have

$$\begin{aligned} P_{e_i} \geq & \frac{1}{4^{(N-1)}} \frac{1}{2} Q \left(\frac{\xi_i - \sum_{j \neq i} p_j h_{ij} R_p}{\sqrt{2q \sum_{j \neq i} p_j h_{ij} R_p B + \sigma_c^2}} \right) \\ & + \frac{1}{4^{(N-1)}} \frac{1}{2} Q \left(\frac{p_i h_{ii} R_p - \xi_i}{\sqrt{2q p_i h_{ii} R_p B + \sigma_c^2}} \right) \quad (18) \\ \geq & \frac{1}{4^{(N-1)}} \frac{1}{2} Q(0) = \frac{1}{4^N}. \quad (19) \end{aligned}$$

In (18), the first Q-item corresponds to the case when the intended bit is zero while all the interfering bits are one, i.e., $W_{ij} = 1$ for all $j \neq i$, and the second Q-item corresponds to the case when the intended bit is one while all the interfering bits are zero, i.e., $W_{ij} = 0$ for all $j \neq i$. The inequality holds because we have removed some non-negative terms from the exact BER expression in (12). The inequality in (19) holds since no matter what the threshold is chosen, at least one of $\xi_i - \sum_{j \neq i} p_j h_{ij} R_p$ and $p_i h_{ii} R_p - \xi_i$ is negative. In general, the maximum tolerable BER is considered to be 10^{-3} for voice and 10^{-7} for data. When $N = 4$, $P_{e_i} \geq 0.0039$, which is too high for data communications.

Reference [8] gives the condition on the link gains when there exists a power vector $\mathbf{p} \geq 0$ such that $p_i h_{ii} \geq \sum_{j \neq i} p_j h_{ij}$ for all i. Due to page limit, we do not explain here and interested readers are recommended to refer to [8]. If the link gains fail to satisfy the condition, scheduling is required to select candidate subsets of concurrently active transmitter-receiver pairs (that use the same wavelength carrier) and this is out of the scope of this paper.

We impose the condition that the decision threshold must satisfy $\sum_{j \neq i} p_j h_{ij} R_p \leq \xi_i \leq p_i h_{ii} R_p$ for all $i = 1, \ldots, N$. For the cases when the optimal threshold falls outside this range, the BER is greater than $\frac{1}{4^N}$ and is out of our interest. Under this condition, every Q-item in (12) has nonnegative entry. We note in (12) that the denominators in all Q-items are

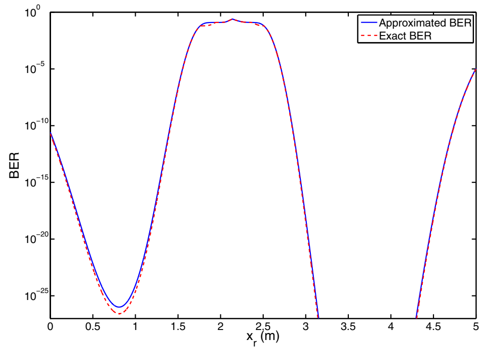

Fig. 4: The exact BER and the approximate BER. The receiver's location is $(x_r, 0.5, 0.83)$, i.e., on the straight line from LED_1 to LED_2.

bounded above by $\sigma_i^2 = 2qB \sum_{j=1}^{N} p_j h_{ij} R_p + \sigma_c^2$. We replace all the denominators with σ_i^2 and approximate the BER by

$$\begin{aligned} \tilde{P}_{e_i} = & \frac{1}{4^{N-1}} \sum_{W_{iN} \in \{\tau_{iN}, 1-\tau_{iN}, 1, 0\}} \cdots \sum_{W_{i1} \in \{\tau_{i1}, 1-\tau_{i1}, 1, 0\}} \\ & \left[\frac{1}{2} Q \left(\frac{\xi_i - \sum_{j \neq i} p_j h_{ij} W_{ij} R_p}{\sigma_i} \right) \right. \\ & \left. + \frac{1}{2} Q \left(\frac{p_i h_{ii} R_p + \sum_{j \neq i} p_j h_{ij} W_{ij} R_p - \xi_i}{\sigma_i} \right) \right]. \quad (20) \end{aligned}$$

Since the function of $Q(x)$ is monotonically decreasing as x is increasing for $x \geq 0$, we always have $P_{e_i} \leq \tilde{P}_{e_i}$. For $\tilde{P}_{e_i}$, the optimal threshold can be found and is given by the following theorem, whose proof is provided in Appendix A.

Theorem 1. *For any $i = 1, \ldots, N$, if $p_i h_{ii} R_p \geq \sum_{j \neq i} p_j h_{ij} R_p$, the optimal threshold that minimizes $\tilde{P}_{e_i}$ and satisfies $\sum_{j \neq i} p_j h_{ij} R_p \leq \xi_i \leq p_i h_{ii} R_p$ is*

$$\tilde{\xi}_i = \frac{1}{2} \big(p_i h_{ii} R_p + \sum_{j \neq i} p_j h_{ij} R_p \big). \tag{21}$$

At $\tilde{\xi}_i$, the bit error rate when $b_i^0 = 0$ is the same as the bit error rate when $b_i^0 = 1$. So $\tilde{P}_{e_i}$ becomes

$$\begin{aligned} \tilde{P}_{e_i} = & \frac{1}{4^{N-1}} \sum_{W_{iN} \in \{\tau_{iN}, 1-\tau_{iN}, 1, 0\}} \cdots \sum_{W_{i1} \in \{\tau_{i1}, 1-\tau_{i1}, 1, 0\}} \\ & Q \left(\frac{\frac{1}{2} p_i h_{ii} R_p + \sum_{j \neq i} p_j h_{ij} (\frac{1}{2} - W_{ij}) R_p}{\sigma_i} \right). \quad (22) \end{aligned}$$

Using the same example in Section V and fixing $y_r = 0.5$, Fig. 4 shows the exact BER P_{e_i} and the approximate BER $\tilde{P}_{e_i}$ versus x_r. The exact BER is obtained by numerically searching for the optimal value of ξ that minimizes (12). We see that the curve of approximate BER is always above the curve of exact BER, as expected. Besides, the two curves are close, and nearly overlap with each other at BER values of concern in the range of 10^{-10} through 10^{-3}. The intuition is that in VLC systems, the noise level is relatively small when compared with the signal level (see in (2), the thermal noise power is the signal power times qB where $q = 1.610^{-19}$ is small). So the BER mainly depends on the sign of $p_i h_{ii} R_p - \sum_{j \neq i} p_j h_{ij} R_p$ and when $p_i h_{ii} R_p \geq \sum_{j \neq i} p_j h_{ij} R_p$, our approximation by enlarging the noise power does not affect the result too much. This intuition also explains why the changes of BER are sharp around $x_r = 1.7$ and $x_r = 2.5$, where the condition (17) fails to be satisfied. We also perform the same simulation for other values of y_r. Similar results are observed but omitted.

VII. Concluding Remarks

In this paper, we derive the exact BER expression for VLC systems where there are multiple transmitter-receiver pairs and interference exists. Unlike the traditional Gaussian interference model which only uses the first and the second moments of the interference, the signal structure of the interference and all of its statistics are preserved in our analysis. Simulation results show that the BER predicted by the Gaussian interference model is not accurate in a small-room setting with four LEDs. Whether the Gaussian interference model is applicable in a large room with many LEDs requires further investigation.

In a VLC system, resource allocation such as power control is important to ensure that the system is operated in an effective manner. For example, as can be seen from Fig. 2, without controlling the emitting power of the LEDs, the BER in part of the area is too high for data communications. To perform power control, an accurate performance metric is important. While the exact BER can be analytically derived, calculating the BER requires numerically determining the optimal decision threshold of the matched filter output. To circumvent this difficulty, we propose a new approximation for the BER, which is shown to be accurate by simulations. We hope that this metric is useful for designing new resource allocation schemes in the future.

Appendix A
Proof of Theorem 1

Proof. For any fixed $i = 1, \ldots, N$, consider $\tilde{P}_{e_i}$ to be a function of ξ_i. Take the first order derivative of (20) with respect to ξ_i and evaluate it at $\tilde{\xi}_i$. We have

$$\left.\frac{\mathrm{d}\tilde{P}_{e_i}}{\mathrm{d}\xi_i}\right|_{\xi_i=\tilde{\xi}_i} = \frac{1}{4^{N-1}} \sum_{W_{iN}\in\{\tau_{iN},1-\tau_{iN},1,0\}} \cdots \sum_{W_{i1}\in\{\tau_{i1},1-\tau_{i1},1,0\}} \left[\frac{-1}{2\sqrt{2\pi}\sigma_i} \exp\left(-\frac{\left(\frac{1}{2}p_i h_{ii} R_p + \sum_{j\neq i} p_j h_{ij}(\frac{1}{2} - W_{ij})R_p\right)^2}{2\sigma_i^2}\right) + \frac{1}{2\sqrt{2\pi}\sigma_i} \exp\left(-\frac{\left(\frac{1}{2}p_i h_{ii} R_p + \sum_{j\neq i} p_j h_{ij}(W_{ij} - \frac{1}{2})R_p\right)^2}{2\sigma_i^2}\right)\right].$$

We know that W_{ij} takes values of $\{\tau_{ij}, 1-\tau_{ij}, 1, 0\}$. So the possible values of $\frac{1}{2} - W_{ij}$ and $W_{ij} - \frac{1}{2}$ are the same. Therefore $\left.\frac{\mathrm{d}\tilde{P}_{e_i}}{\mathrm{d}\xi_i}\right|_{\xi_i=\tilde{\xi}_i} = 0$. The second order derivative of (20) with respect to ξ_i is

$$\frac{\mathrm{d}^2\tilde{P}_{e_i}}{\mathrm{d}\xi_i^2} = \frac{1}{4^{N-1}} \sum_{j\neq i}\sum_{W_{ij}} \left[\frac{1}{2\sqrt{2\pi}\sigma_i} \exp\left(-\frac{\left(\xi_i - \sum_{j\neq i} p_j h_{ij} W_{ij} R_p\right)^2}{2\sigma_i^2}\right) \frac{2\left(\xi_i - \sum_{j\neq i} p_j h_{ij} W_{ij} R_p\right)}{2\sigma_i^2} + \frac{1}{2\sqrt{2\pi}\sigma_i} \exp\left(-\frac{\left(p_i h_{ii} R_p + \sum_{j\neq i} p_j h_{ij} W_{ij} R_p - \xi_i\right)^2}{2\sigma_i^2}\right) \times \frac{2\left(p_i h_{ii} R_p + \sum_{j\neq i} p_j h_{ij} W_{ij} R_p - \xi_i\right)}{2\sigma_i^2}\right].$$

Since $0 \leq W_{ij} \leq 1$ for all j, we have $\frac{\mathrm{d}^2\tilde{P}_{e_i}}{\mathrm{d}\xi_i^2} > 0$ for $\sum_{j\neq i} p_j h_{ij} R_p \leq \xi_i \leq p_i h_{ii} R_p$. It is ready to see that $\tilde{\xi}_i$ is the unique globally optimal solution to minimize $\tilde{P}_{e_i}$. □

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