Uncoordinated Multiple Access Schemes for Visible Light Communications and Positioning

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***Abstract*—In visible light communication (VLC) systems, information are conveyed by visible light instead of radio-frequency electromagnetic waves. Based on received signal strength, accurate indoor positioning systems can also be built. Since a receiver obtains the superposition of signals from all light sources within line of sight together with ambient light, a multiple access scheme is necessary for the receiver to distinguish the received symbol and signal strength from each light source. This paper proposes two multiple access schemes for VLC. The first scheme supports information broadcast and positioning. By using 2^N timeslots, N transmitters transmit $2^N - 1$ symbols in total. The second scheme supports positioning only but places emphasis on minimizing the required timeslots. In each $2N$ timeslots for an odd integer N, the channel gains of $\frac{3N-1}{2}$ transmitters can be estimated.**

I. Introduction

Visible light communications (VLC), which uses visible light instead of radio-frequency (RF) electromagnetic waves for communications, has attracted much attention in recent years. Lighting devices are undergoing a revolution, from fluorescent lamps and tubes to light emitting diodes (LEDs). Most smartphones and tablets today contain built-in sensors such as photodiodes (PDs). VLC systems using LEDs as transmitters and PDs as receivers are energy efficient and have low deployment cost. Since visible light does not interfere RF signals, VLC systems will be compatible with other RF systems, with the potential to form powerful hybrid systems. Some of the promising applications of these systems include information broadcast and indoor positioning.

For information broadcast using VLC, there are different techniques proposed in the literature including expurgated pulse position modulation [1], optical code division multiple access (OCDMA) [2][3] and an adaptive modulation scheme based on power control [4]. Precoding techniques for RF systems are adapted for VLC in [5][6]. Note that all these techniques require a central unit to coordinate the transmissions among the LEDs through an extra infrastructure in a VLC system. The disadvantages of using central unit will be explained after some positioning systems are introduced.

Visible light positioning systems can provide highly accurate indoor positioning [7][8]. Experimental results show that positioning error can be less than 10 centimetres on average. Since the positioning algorithms in these systems are based on the received light intensity or estimated channel gain from each LED, any interference among the signals from different LEDs would severely degrade the accuracy. Furthermore, it is common to have ambient light sources which give extra energy in each measurement. Time division multiple access (TDMA) was used in [7], [8]. To implement TDMA, a central unit is again required to coordinate the transmissions among LEDs.

It is important to know a multiple access scheme for VLC systems without using central unit. Such scheme allows us to build an information broadcast system with positioning capability by simply replacing the existing light sources by stand-alone LED transmitters. Without the need of extra infrastructure to control the LEDs, the installation cost is reduced.

To get rid of central unit, we need to solve a few technical problems. Intensity modulation/direction detection (IM/DD) is used in VLC systems. Therefore, the transmitted signals are always positive real. Multiple receivers may use the system at random time at random locations with arbitrary orientation in a 3-dimensional space. As a result, the transmitted signals suffer random channel gains which are unknown to the receiver. Since the transmitters are also light sources, they may be controlled by different switches and be switched on at different time. So the transmitters and the receiver are asynchronous. It is also preferable to consider simple transmitter design that no communication link presents among transmitters.

We begin with the introduction of channel model and notations in Section II. In Section III, we will propose an efficient scheme in which a receiver obtains $2^N - 1$ symbols from N transmitters and one estimate of ambient light intensity in 2^N timeslots. Section IV explores another scheme which can reduce the required timeslots by supporting only positioning.

II. Channel Model and Notations

Suppose there are N transmitters in the system where a subset of them are active and within the line-of-sight (LoS) of the receiver (see Fig. 1). Let $\mathcal{S}$ be this subset so that $\mathcal{S} \subseteq \{1, \ldots, N\}$. For $i \in \mathcal{S}$, let $a_{ik} \in \{0, 1, \ldots, M-1\}$ be the k-th message that Transmitter i wants to deliver since it is activated. Let $X_i[n] \geq 0$ be the n-th symbol transmitted by Transmitter i. Suppose Transmitter i has transmitted δ_i symbols before the receiver starts to detect signals. Let Φ be the background light intensity which is not negligible. Receiver detects the superposition of the signals from the transmitters together with

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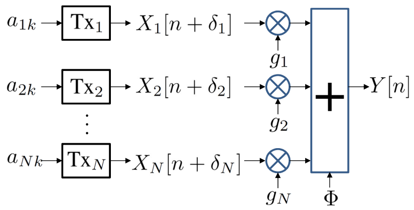

Fig. 1. Transmitters want to broadcast different messages. They are suffering from random delays due to the different activation time. The superposition of ambient light and the transmitted signals is received by the receiver.

the background light. Let $Y[n]$ be the n-th symbol received at the receiver. So

$$Y[n] = \sum_{i \in \mathcal{S}} g_i X_i[n+\delta_i] + \Phi, \tag{1}$$

where g_i is the channel gain which depends on the distance between Transmitter i and the receiver, angle of irradiance of the LED, angle of incidence of the PD, etc. Since these parameters depend on the receiver's location which is random, we assume without lost of generality that the cumulative distribution function (cdf) of g_i is continuous if $i \in \mathcal{S}$. We further assume that the cdf of g_i given g_k with $i \neq k$ is continuous. Here, g_i is considered as invariant over a short period of time like block fading channels. This is justified by that it is common to achieve transmission rate over 10^6 samples per second in VLC [9]. If the receiver's displacement is negligible within 10^{-3} seconds, the gain g_i can be seen as invariant for more than 10^3 symbols.

In general, noise should be considered in (1). This paper considers the noiseless case as the starting point of this problem to illustrate what can be achieved. Note that our proposed scheme has potential to be extended to noisy scenarios.

Notations: $L(S,\delta)$ means the cyclic shift of the sequence S to the left by δ steps and $L(S,\delta)[n]$ means the n-th symbol after shift. For example, if $S = (s_1, s_2, s_3)$, then $L(S,1) = (s_2, s_3, s_1)$ and $L(S,1)[3] = s_1$. The complex conjugate of W is denoted by $\overline{W}$. The imaginary number is $j = \sqrt{-1}$.

III. Positioning and Communications in VLC

Our goal of this section is to design a coding scheme such that if Transmitter i is within LoS, the receiver can determine i) the channel gain g_i for positioning and ii) the broadcasted messages a_{ik}. In this section, a_{ik} is assumed to be independent and uniformly distributed in $\{0, 1, \ldots, M-1\}$.

Scheme for Positioning with Comms. Capability:

1) For $1 \le i \le N$, Transmitter i outputs i consecutive a_{ik} followed by i consecutive 0. To be specific, for $k \ge 0$, the n-th output from Transmitter i is

$$X_i[n] = \begin{cases} a_{ik} & \text{for } \ 1 + k \cdot 2^i \le n < 1 + (2k+1)2^{i-1} \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

2) Let a row vector $S_i(T)$ be the first T symbols from a periodic sequence with a repeating pattern of 2^{i-1} consecutive 1's followed by 2^{i-1} consecutive -1's. So

$$S_i(T) = [1 \ \cdots \ 1 \ \ -1 \ \cdots \ -1 \ \ 1 \ \cdots \ 1 \ \cdots]. \tag{3}$$

Let $\mathbf{Y}(T) = [Y[1] \ Y[2] \ \cdots \ Y[T]]$ be a column vector.

3) Let $\hat{\delta}_i(T) = \text{argmax}_{0 \le \delta < 2^i} L(S_i(T), \delta) \cdot \mathbf{Y}(T)$.
4) Transmitter i is considered as inactive if

$$\lim_{T\to\infty} \frac{1}{T} \sum_{n=1}^{T} Y[n] L(S_i(T), \hat{\delta}_i)[n] = 0. \tag{4}$$

5) According to (4), let $\hat{\mathcal{S}}$ be the estimated set of active transmitters.
6) Set $\hat{\mathcal{S}}' \leftarrow \hat{\mathcal{S}}$. Construct a $(0,1)$–matrix A by concatenating the row vector $\frac{1}{2}(1 + L(S_i(T), \hat{\delta}_i))$ for $i \in \hat{\mathcal{S}}$ in an increasing order of i. The positions of 1's in A represent the timeslots when Transmitter i sends positive $X[n]$.
7) There must exist a zero column vector in A and suppose it is the i-th column. The estimated background intensity is $\hat{\Phi} = Y[i]$. Set $Y'[i] \leftarrow Y[i] - \hat{\Phi}$ for all i.
8) Set $i = \max\{k : k \in \hat{\mathcal{S}}'\}$.
9) There must exist columns in A with the form $[0 \cdots 0 \ 1]^T$. Let Ω be the position of those columns. For $n \in \Omega$, $Y'[n] = g_i X_i[n + \hat{\delta}_i]$ which is kept in a set $\mathcal{M}_i$.
10) For $n \in \Omega$, let $\mathcal{L}_n \ni n$ be the set of consecutive 1's in the last row of A. Update $Y'[\ell] \leftarrow Y'[\ell] - Y'[n]$ for all $n \in \Omega$ and $\ell \in \mathcal{L}_n$.
11) Remove the last row of A and set $\hat{\mathcal{S}}' \leftarrow \hat{\mathcal{S}}' \setminus i$.
12) Go to Step 8 if $\hat{\mathcal{S}}'$ is non-empty.
13) For $i \in \hat{\mathcal{S}}$, the receiver has collected a set $\mathcal{M}_i = \{g_i a_{ik}\}$ where the range of k depends on δ_i and T.
14) If

$$\frac{\max\{\mu : \mu \in \mathcal{M}_i\}}{\min\{\mu > 0 : \mu \in \mathcal{M}_i\}} < M - 1 \tag{5}$$

for any i, set $T \leftarrow 2T$ and go back Step 2.

15) The estimated g_i is given by $\hat{g}_i = \min\{\mu > 0 : \mu \in \mathcal{M}_i\}$. Therefore, a_{ik} can be estimated from $\hat{g}_i$ and $\mathcal{M}_i$. The scheme ends here.

Theorem 1: If a_{ik} for all i and k are independent and uniformly distributed on $\{0, 1, \ldots, M-1\}$, then

$$\lim_{T\to\infty} \Pr\{\hat{\delta}_i(T) = \delta_i\} = 1. \tag{6}$$

Proof: Note that $L(S_i(T), \delta_i)[n] = 1$ if and only if $X_i[n + \delta_i] > 0$. Therefore, from (1)

$$Y[n] = \sum_{i \in \mathcal{S}} g_i X_i[n+\delta_i] L(S_i(T), \delta_i)[n] + \Phi. \tag{7}$$

Now consider $L(S_i(T), \delta_i)[n] L(S_k(T), \delta)[n]$ for $1 \le n \le T$. Firstly, suppose $i < k$. Consecutive 1's with length 2^{k-1} repeatedly appear inside $L(S_k(T), \delta)[n]$. Let $\mathcal{I}^+$ be the positions of those consecutive 1's so that $L(S_k(T), \delta)[n] = 1$ for $n \in \mathcal{I}^+$ and $|\mathcal{I}^+|$ is an integer multiple of 2^{k-1}. On the other hand, consecutive 1's with length 2^{i-1} repeatedly appear inside $L(S_i(T), \delta_i)[n]$. Since 2^{k-1} is divisible by 2^{i-1}, the number of consecutive 1's in $L(S_i(T), \delta_i)[n]$ for $n \in \mathcal{I}^+$ is equal to $\frac{|\mathcal{I}^+|}{2}$. Similarly, let $\mathcal{I}^-$ be the positions of consecutive -1's with length 2^{k-1} in $L(S_k(T),\delta)$. For $n \in \mathcal{I}^-$, the number of consecutive 1's in $L(S_i(T),\delta_i)[n]$ is equal to $\frac{|\mathcal{I}^-|}{2}$. Without lost of generality, we consider that T is an integer multiple of 2^k so that $|\mathcal{I}^+| = |\mathcal{I}^-|$. Thus, for arbitrarily $\epsilon > 0$,

$$\Pr\left\{\frac{1}{T}\left(\sum_{n\in\mathcal{I}^+} g_iX_i[n+\delta_i] - \sum_{n\in\mathcal{I}^-} g_iX_i[n+\delta_i]\right) > \epsilon\right\} < \epsilon \tag{8}$$

for T sufficiently large. Together with

$$\lim_{T\to\infty}\frac{1}{T}\sum_{n=1}^{T} g_iX_i[n+\delta_i]L(S_i(T),\delta_i)[n]L(S_k(T),\delta)[n] \tag{9}$$

$$\le \lim_{T\to\infty}\frac{1}{T}\left(\sum_{n=1}^{2^k} g_iX_i[n+\delta_i] + \sum_{n\in\mathcal{I}^+} g_iX_i[n+\delta_i]\right. \tag{10}$$

$$\left. - \sum_{n\in\mathcal{I}^-} g_iX_i[n+\delta_i] + \sum_{n=T-2^k+1}^{T} g_iX_i[n+\delta_i]\right) \tag{11}$$

$$= \lim_{T\to\infty}\frac{1}{T}\left(\sum_{n\in\mathcal{I}^+} g_iX_i[n+\delta_i] - \sum_{n\in\mathcal{I}^-} g_iX_i[n+\delta_i]\right), \tag{12}$$

we have

$$\Pr\left\{\frac{1}{T}\sum_{n=1}^{T} g_iX_i[n+\delta_i]L(S_i(T),\delta_i)[n]L(S_k(T),\delta)[n] > \epsilon\right\} < \epsilon \tag{13}$$

for any $\epsilon > 0$ and T sufficiently large.

Secondly, suppose $i > k$. We can still prove (13) by swapping the roles of i and k in the above argument. So we do not repeat the argument here.

Finally, consider $i = k$. It is easy to verify that

$$\sum_{n=1}^{T} g_iX_i[n+\delta_i]L(S_i(T),\delta_i)[n]L(S_i(T),\delta)[n] \tag{14}$$

$$< \sum_{n=1}^{T} g_iX_i[n+\delta_i]L(S_i(T),\delta_i)[n]L(S_i(T),\delta_i)[n] \tag{15}$$

for $\delta \ne \delta_i$. Together with

$$\lim_{T\to\infty}\frac{1}{T}\sum_{n=1}^{T}\Phi L(S_k(T),\delta)[n] = 0, \tag{16}$$

for finite Φ, (6) is verified. ■

Theorem 2: If Transmitter k is inactive, the probability that the receiver considers it as active is arbitrarily small because

$$\Pr\left\{\frac{1}{T}\sum_{n=1}^{T} Y[n]L(S_k(T),\hat{\delta}_k)[n] > \epsilon\right\} < \epsilon \tag{17}$$

for any $\epsilon > 0$ and T sufficiently large.

Proof: If Transmitter k is inactive, $X_k[n] = 0$ for all n. Since (13) holds for $i \ne k$, the theorem is proved. ■

Lemma 1: For $k \ge 1$, if Transmitter i is active for all $i \le k$, there exists $\Delta_k < 2^k$ such that for all integer $\ell \ge 0$,

$$\sum_{i=1}^{k} X_i[\ell\cdot 2^k + \Delta_k] = 0. \tag{18}$$

Proof: We are going to prove the lemma by mathematical induction. When $k = 1$, $\sum_{i=1}^{k} X_i[n] = X_1[n]$. By taking $n = 2\ell + \delta_1$, (18) is verified to be true for $k = 1$. Assume that (18) is true for k. Then

$$\sum_{i=1}^{k+1} X_i[\ell\cdot 2^{k+1} + \Delta_{k+1}] \tag{19}$$

$$= \sum_{i=1}^{k} X_i[(2\ell)\cdot 2^k + \Delta_{k+1}] + X_{k+1}[\ell\cdot 2^{k+1} + \Delta_{k+1}]. \tag{20}$$

If $X_{k+1}[\Delta_k] = 0$, then $X_{k+1}[\ell\cdot 2^{k+1} + \Delta_k] = 0$ for all ℓ due to (2), and hence take $\Delta_{k+1} = \Delta_k$. If $X_{k+1}[\Delta_k] > 0$, then $X_{k+1}[2^k + \Delta_k] = 0$ and hence, $X_{k+1}[\ell\cdot 2^{k+1} + (2^k + \Delta_k)] = 0$ for all ℓ. In this case, $\Delta_{k+1} = 2^k + \Delta_k$. Therefore, the lemma is verified by mathematical induction. ■

Theorem 3: If $\hat{\mathcal{S}} = \mathcal{S}$ and $\hat{\delta}_i = \delta_i$ for all i, then the column vectors described in Steps 7 and 9 must exist.

Proof: Let $\mathcal{S}$ be the set of active transmitters and let $k = \max\{i : i \in \mathcal{S}\}$. Note that $L(S_i(T),\delta_i)[n] = 1$ if and only if $X_i[n+\delta_i] > 0$. By Lemma 1, the column vectors described in Step 6 exist. Similarly, applying Lemma 1 with k replaced by $k-1$ shows that for all ℓ,

$$\sum_{i=1}^{k-1} X_i[\ell\cdot 2^{k-1} + \Delta_{k-1}] = 0. \tag{21}$$

Since either $X_k[\ell\cdot 2^{k-1} + \Delta_{k-1}] > 0$ or $X_k[(\ell+1)\cdot 2^{k-1} + \Delta_{k-1}] > 0$, the column vectors described in Step 9 exist. ■

Remarks: 1) From Theorems 1–3, we conclude that the receiver can successfully recover g_i and a_{ik} for $i \in \mathcal{S}$ with arbitrarily small error probability when T is sufficiently large. 2) When all transmitters are active and within the LoS of the receiver, the receiver can obtain an estimate of Φ together with

$$2^N\sum_{i=1}^{N} 2^{-i} = 2^N - 1 \tag{22}$$

symbols in 2^N timeslots. This is better than [10] where only N symbols can be transmitted in 2^N timeslots.

Example 1: Suppose $N = 3$ and $T = 8$. Tables I–III demonstrate some possible transmitted patterns. Receiver receives the summation of each column together with Φ. Assume that the receiver first correctly estimates δ_i by considering $T >> 8$ according to Step 3. The receiver can construct A in Step 6 which is similar to the tables except that all g_ia_{ik}'s are replaced by 1. In Step 7, the background light intensity is estimated by $Y[8], Y[8], Y[6]$ in Tables I–III, respectively, according to the positions of zero column vectors.

Now, we focus on only Table III. After the effect due to Φ is removed from $Y[n]$ in Step 7, g_3a_{31} is obtained from $Y'[2]$. In Step 10, let $\mathcal{L}_2 = \{2,3,4,5\}$ and set $Y'[\ell] \leftarrow Y'[\ell] - Y'[2]$

TABLE I
SCENARIO 1: $\delta_1 = \delta_2 = \delta_3 = 0$

g_1a_{11}	0	g_1a_{12}	0	g_1a_{13}	0	g_1a_{14}	0
g_2a_{21}	g_2a_{21}	0	0	g_2a_{22}	g_2a_{22}	0	0
g_3a_{31}	g_3a_{31}	g_3a_{31}	g_3a_{31}	0	0	0	0

TABLE II
SCENARIO 2: $\delta_1 = 0; \delta_2 = 1; \delta_3 = 0$

g_1a_{11}	0	g_1a_{12}	0	g_1a_{13}	0	g_1a_{14}	0
0	g_2a_{21}	g_2a_{21}	0	0	g_2a_{22}	g_2a_{22}	0
g_3a_{31}	g_3a_{31}	g_3a_{31}	g_3a_{31}	0	0	0	0

TABLE III
SCENARIO 3: $\delta_1 = 0; \delta_2 = 3; \delta_3 = 1$

g_1a_{11}	0	g_1a_{12}	0	g_1a_{13}	0	g_1a_{14}	0
g_2a_{20}	0	0	g_2a_{21}	g_2a_{21}	0	0	g_2a_{22}
0	g_3a_{31}	g_3a_{31}	g_3a_{31}	g_3a_{31}	0	0	0

for all $\ell \in \mathcal{L}_2$ to remove the effect due to a_{31}. After removing the last row in A, we follow Step 9 and find two positions ($\Omega = \{4, 8\}$) match the condition. Therefore, g_2a_{21} and g_2a_{22} are obtained from $Y'[4]$ and $Y'[8]$, respectively. In Step 10, let $\mathcal{L}_4 = \{4, 5\}$ and $\mathcal{L}_8 = \{8\}$. After setting $Y'[\ell] \leftarrow Y'[\ell] - Y'[4]$ for all $\ell \in \mathcal{L}_4$ and $Y'[\ell] \leftarrow Y'[\ell] - Y'[8]$ for all $\ell \in \mathcal{L}_8$, the last row in A is removed and go back to Step 8. We do not repeat the details here. At the end, the receiver obtains $\{g_1a_{12}, g_1a_{13}, g_1a_{14}\}$, $\{g_2a_{21}, g_2a_{22}\}$ and $\{g_3a_{31}\}$. If $T >> 1$, these sets will be large and contain both $g_i \cdot 1$ and $g_i \cdot (M-1)$. Then Step 15 is reached where g_i and a_{ik} are determined.

IV. VISIBLE LIGHT POSITIONING

This section investigates visible light positioning systems without communication capability. Our focus is to minimise the number of symbols required for finding channel gain g_i for $i \in \mathcal{S}$. Smaller number of symbols takes shorter transmission time; this improves the sensitivity to the changes in channel gains and is important for delicate positioning systems.

Compared with wireless communications using radio frequency, one of the major differences in VLC is that the transmitted signals are required to be positive real. Under this requirement, suppose Transmitter i outputs periodic signals $X_i[n]$ with period T. Here, if the transmitters output signals with different periods, we can take T as the least common multiple of the periods. Since $X_i[n]$ is positive real, the Fourier transform of X_i denoted by W_i must satisfy $W_i[n] = \overline{W_i}[-n]$. As a result, only half of the elements in the Fourier transform of the received signal $Y[n]$ are useful. In order to support N transmitters by using $2N$ timeslots, we can use the following simple scheme to decide the values of $X_i[n]$. Let $\tilde{C}_i[n]$ be the inverse Fourier transform of the i-th row in a $2N \times N$ matrix

$$\mathbf{B} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 & \cdots & 0 & 1 \\ 0 & 0 & 1 & \ddots & \vdots & \iddots & 1 & 0 \\ \vdots & \vdots & \ddots & \ddots & 0 & \iddots & \iddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \end{pmatrix}. \tag{23}$$

For $1 \le i \le N$ and $1 \le n \le 2N$, let

$$C_i[n] = \tilde{C}_i[n] + 2. \tag{24}$$

Let $X_i[n + k \cdot 2N] = C_i[n]$ for $k \ge 0$. By considering the Fourier transform of the received $2N$ symbols at the receiver denoted by $\tilde{Y}[n]$ for $0 \le n \le 2N - 1$, the structure of $\mathbf{B}$ ensures: 1) there is no interference among the transmitters in frequency domain and 2) $C_i[n]$ are positive real for all i and n. According to (1), $\tilde{Y}[i] = g_i e^{\frac{ji\pi}{N}\delta_i}$ where δ_i causes a phase shift in frequency domain. Although δ_i is unknown, g_i can be determined by $|\tilde{Y}[i]|$.

The interesting question is: If the transmitted signal is required to be positive real and the delay δ_i is unknown, can we design a scheme which can support more than N transmitters by using $2N$ timeslots? We are going to reveal that it is possible by using the property that g_i is positive real while $\tilde{Y}[n]$ is complex.

Definition 1: For an odd integer N, let $\mathcal{C} = \{C_i, 1 \le i \le \frac{3N-1}{2}\}$ where C_i is defined in (24) for $1 \le i \le N$. For $N + 1 \le i \le \frac{3N-1}{2}$ and $k = 0$ or 1, let

$$\phi_k(i) = 2(i - N) - 1 + k, \tag{25}$$

$$Z_{i,k} = (4 - k) + \exp\left(-\frac{j\phi_k(i)\pi}{N}\right), \tag{26}$$

$$\theta_{i,k} = \arg(Z_{i,k}), \tag{27}$$

and

$$\begin{aligned} C_i = & |Z_{i,0}|^{-1}(4 \cdot C_{\phi_0(i)} + L(C_{\phi_0(i)}, 1)) + \\ & |Z_{i,1}|^{-1}(3 \cdot C_{\phi_1(i)} + L(C_{\phi_1(i)}, 1)). \end{aligned} \tag{28}$$

Note that if C_i is sent alone, $\tilde{Y}[\phi_k(i)] = \frac{Z_{i,k}}{|Z_{i,k}|}$ for $k = 0, 1$.

Lemma 2: Both $\theta_{i,k}\frac{2N}{\pi}$ and $|\theta_{i,0} - \theta_{i,1}|\frac{2N}{\pi}$ are not integers.

The proof is omitted due to the limited space. Lemma 2 gives a property which will be frequently used in this section. Instead of combining $C_{\phi_0(i)}$ and $C_{\phi_1(i)}$ as the way shown in (28), it is possible to use other weightings as long as Lemma 2 still holds. Hence, we cannot replace both 4 and 3 in (28) by 1.

Multiple Access Scheme for Visible Light Positioning:

1) For $1 \le i \le \frac{3N-1}{2}$, Transmitter i repeatedly sends C_i.
2) Let $\tilde{\mathbf{Y}} = [\tilde{Y}(0), \tilde{Y}(1), \ldots, \tilde{Y}(2N-1)]$ be the discrete Fourier transform of $\mathbf{Y}$ which is a vector of $2N$ consecutive symbols received at the receiver.
3) The estimate of g_N is given by $\hat{g}_N = \tilde{Y}[N]$.
4) For $i > N$, $g_{\phi_0(i)}, g_{\phi_1(i)}$ and g_i are estimated together from $(\tilde{Y}[\phi_0(i)], \tilde{Y}[\phi_1(i)])$ as follows.
5) For integers $\ell, r \in \{0, 1, \ldots, N-1\}$ and $k \in \{0, 1\}$, let

$$\mu_\ell = \theta_{i,k} - \frac{\ell\pi\phi_k(i)}{N}, \tag{29}$$

and

$$\nu_r = -\frac{r\pi\phi_k(i)}{N}. \tag{30}$$

6) Suppose $\tilde{Y}[\phi_k(i)] = \rho_x + j\rho_y$. Let

$$\gamma_x(i, k, r, \ell) = \frac{\rho_y - \rho_x \tan(\mu_\ell)}{\tan(\nu_r) - \tan(\mu_\ell)} \tag{31}$$

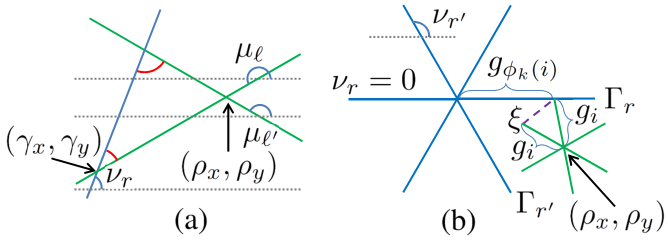

Fig. 2. The complex plane used in Lemma 3. (a) Step 6 finds (γ_x, γ_y) and put the distances between (γ_x, γ_y) and (ρ_x, ρ_y) into $\mathcal{A}(i, k)$. (b) The slopes of the green lines are $\{\mu_\ell\}$ and the slopes of the blue lines are $\{\nu_r\}$. The line ξ connects the ending points of any two green lines with length g_i.

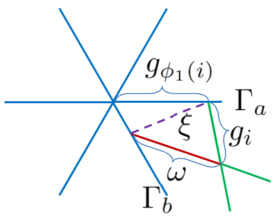

Fig. 3. The slopes of the short green line and short red line are $\{\mu_\ell\}$ and the slopes of the long blue lines are $\{\nu_r\}$ with $k = 1$ in (29) and (30).

and

$$\gamma_y(i, k, r, \ell) = \tan(\nu_r)\gamma_x(i, k, r, \ell); \tag{32}$$

Note that $\tan(\nu_r) \neq \infty$ because N is odd. Define

$$\mathcal{A}(i, k) = \left\{((\gamma_x - \rho_x)^2 + (\gamma_y - \rho_y)^2)^{\frac{1}{2}} \quad \forall r, \ell\right\}. \tag{33}$$

7) For $i > N$, if $|\mathcal{A}(i, 0) \cap \mathcal{A}(i, 1)| = 1$, let

$$\hat{g}_i = \mathcal{A}(i, 0) \cap \mathcal{A}(i, 1). \tag{34}$$

Let $(r^*_{i,k}, \ell^*_{i,k})$ be the position of $\hat{g}_i$ in $\mathcal{A}(i, k)$ and the estimate of $g_{\phi_k(i)}$ is given by

$$\hat{g}_{\phi_k(i)} = (\gamma_x^2(i, k, r^*_{i,k}, \ell^*_{i,k}) + \gamma_y^2(i, k, r^*_{i,k}, \ell^*_{i,k}))^{\frac{1}{2}}. \tag{35}$$

8) For $i > N$, if $|\mathcal{A}(i, 0) \cap \mathcal{A}(i, 1)| > 1$, let $\hat{g}_i = 0$ and $\hat{g}_{\phi_k(i)} = |\tilde{Y}[\phi_k(i)]|$. The scheme ends here.

Now, we prove the robustness of the proposed scheme and explain the rationale in the proofs.

Lemma 3: We can always find $g_i \in \mathcal{A}(i, k)$. If $g_i > 0$, then g_i appears only once in $\mathcal{A}(i, k)$ almost surely.

Proof: Due to the definition of (ρ_x, ρ_y), (24) and (28),

$$\rho_x + j\rho_y = \tilde{Y}[\phi_k(i)] = g_{\phi_k(i)} \exp(j\nu_r) + g_i \exp(j\mu_\ell), \tag{36}$$

where $r = \delta_{\phi_k(i)}$ and $\ell = \delta_i$. Due to Lemma 2, a non-orthogonal basis on the complex plane can be formed by using the arguments of $\exp(j\nu_r)$ and $\exp(j\mu_\ell)$. Thus projecting $\tilde{Y}[\phi_k(i)]$ on the right basis with correct (r, ℓ) gives g_i. By exhaustively trying all possible values of r and ℓ, g_i must appear in $\mathcal{A}(i, k)$.

We first consider the probability of that two elements at positions (r, ℓ) and (r, ℓ') in $\mathcal{A}(i, k)$ are equal to g_i. This is equivalent to require that the triangle in Fig. 2(a) is an isosceles triangle. By writing the two angles in terms of μ_ℓ, $\mu_{\ell'}$ and ν_r, an isosceles triangle is formed only if

$$\theta_{i,k} = \kappa \cdot \frac{\pi}{2N} \tag{37}$$

for certain integer κ that contradicts Lemma 2.

Now suppose the two elements in $\mathcal{A}(i, k)$ are the same at positions (r, ℓ) and (r', ℓ') where $r \neq r'$ and $\ell \neq \ell'$. This is equivalent to require that the line ξ touches the axes Γ_r and $\Gamma_{r'}$ in Fig. 2(b). Consider a fixed g_i. Then the length and the slope of ξ are fixed for given (ℓ, ℓ'). Therefore, it can be verified that there are only a finite number of $g_{\phi_k(i)}$ such that ξ touches the axes Γ_r and $\Gamma_{r'}$. Since the cdf of $g_{\phi_k(i)}$ given g_i is continuous, the lemma is proved if $g_{\phi_k(i)} > 0$. Finally, if $g_{\phi_k(i)} = 0$, the lemma follows from that ξ cannot be parallel to any $\Gamma_{r'}$ due to Lemma 2. ■

Lemma 4: For $N + 1 \leq i \leq \frac{3N-1}{2}$, if $g_i > 0$, then $|\mathcal{A}(i, 0) \cap \mathcal{A}(i, 1)| = 1$ almost surely.

Proof: Due to the limited space, the detail is omitted here. ■

Theorem 4: The scheme generates $\hat{g}_i = g_i$ almost surely for $1 \leq i \leq \frac{3N-1}{2}$.

Proof: It is easy to verify the case that $g_i = 0$. Suppose $g_i > 0$. From Lemma 4, g_i can be identified by considering $\mathcal{A}(i, 0) \cap \mathcal{A}(i, 1)$. Only one pair $(r^*_{i,k}, \ell^*_{i,k})$ can give g_i due to Lemma 3 so that $g_{\phi_k(i)}$ can be calculated from (35). ■

Therefore, we conclude that the receiver can estimate g_i almost surely for $1 \leq i \leq \frac{3N-1}{2}$ by receiving $2N$ symbols.

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