A Game-Theoretic Analysis of Uplink Power Control for a Non-Orthogonal Multiple Access System with Two Interfering Cells

Chi Wan Sung
City University of Hong Kong
Tat Chee Avenue, Kowloon, Hong Kong
Email: albert.sung@cityu.edu.hk

Yaru Fu
City University of Hong Kong
Tat Chee Avenue, Kowloon, Hong Kong
Email: yarufu2-c@my.cityu.edu.hk

***Abstract*—This paper investigates the power control problem for the uplink of a non-orthogonal multiple access (NOMA) system with two cells. The game-theoretic approach is used to study the stability of distributed power control algorithms. It is shown that a unique Nash equilibrium exists if the Perron-Frobenius eigenvalue of a certain link gain matrix is less than one. A distributed power control algorithm is constructed, which is guaranteed to converge to the Nash equilibrium. Furthermore, the optimality property of the Nash equilibrium is studied. It is shown that the equilibrium is globally optimal in minimizing total power consumption, provided that some technical conditions are satisfied. Numerical results show that the power-controlled NOMA system outperforms its orthogonal counterparts.**

***Index Terms*—Non-orthogonal multiple access (NOMA), successive interference cancellation (SIC), distributed power control.**

I. Introduction

Since data traffic for 5G cellular systems is expected to increase by 1000 times by 2020, spectral efficiency becomes one of the key challenges to meet this huge demand. Non-Orthogonal Multiple Access (NOMA) is an access technique that can improve spectral efficiency, and is considered a promising approach for 5G cellular systems [1], [2]. It has attracted a lot of attentions in recent years [3]-[10].

In conventional multiple access schemes, users are allocated to orthogonal resources in either time, frequency, or code domain. These orthogonal methods are simple in handling intra-cell interference, and can be implemented with low-complexity receiver. On the other hand, they only make sub-optimal use of radio resource, as their achievable rate regions, no matter for uplink or downlink, are proper subsets of the corresponding capacity region, as indicated by information theory (e.g. [11]). Due to the advance of signal processing techniques and lowering cost of hardware, future wireless systems can tolerate a higher complexity of receiver design. To maximize spectral efficiency, multiple access has to be non-orthogonal, and it is now the time to consider NOMA with the use of successive interference cancellation (SIC).

This work was partially supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. CityU 121713).

The uplink of a NOMA system has been considered in [3], [4], [9], [10]. Since resources are not used in an orthogonal manner, it is important to properly manage mutual interference among users. Existing works address this issue by means of scheduling, subcarrier allocation, and power allocation. In this work, we consider the uplink of a two-cell NOMA system and focus on distributed power control. (The downlink problem is studied in [12].) We adopt the game-theoretic approach to analytically study the stability issue of distributed power control in NOMA cellular systems. We establish sufficient conditions for the existence and uniqueness of Nash equilibrium, based on results from Perron-Frobenius matrix theory. Distributed power control algorithm based on standard interference function [13] is constructed and its convergence is shown. In general, the Nash equilibrium is not always globally optimal in terms of minimizing total power consumption in the system. Nevertheless, it is indeed globally optimal provided some technical conditions are satisfied.

The rest of this paper is organized as follows. In the next section, we present the basics of the multiple access channel (MAC), which models a NOMA system with a single cell. The result is then used in Section III for modeling the two-cell NOMA system and formulate the problem in game-theoretic terms. In Section IV, we investigates the properties of Nash equilibrium and considers distributed power control algorithm that converges to it. Optimality of the Nash equilibrium is then considered in Section V. System-level performance is then evaluated in Section VI. Finally, in Section VII, we draw a conclusion.

II. Preliminaries: Basics of MAC

We first give a brief review on the capacity region of MAC. A basic property will be established, which is useful for the game theoretic analysis of a two-cell NOMA system.

Consider a single cell which consists of one base station (BS) and two users. Each user sends an independent message to the BS through a common channel. Let g_1 and g_2 be the link gain from users 1 and 2 to the BS, respectively, and let N be the total power of noise and interference at the BS, assuming that both the noise and the interference are Gaussian

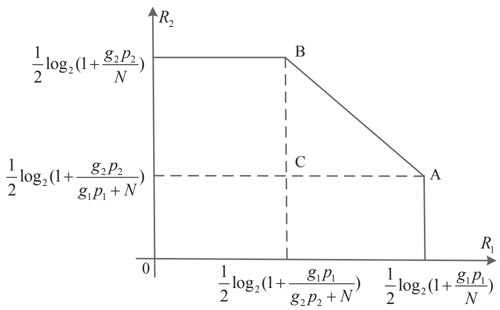

Fig. 1. The capacity region of two-user MAC

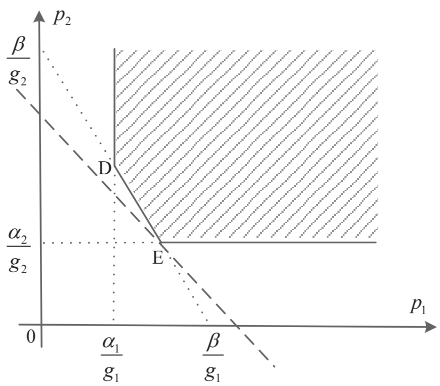

Fig. 2. Transmit power of two users with data rate constraints

distributed. If users 1 and 2 transmit their signals with non-negative powers p_1 and p_2, respectively, then their maximum achievable rates, R_1 and R_2, must be in the capacity region given by

$$R_1 \leq \frac{1}{2}\log_2(1+\frac{g_1p_1}{N}), \tag{1}$$

$$R_2 \leq \frac{1}{2}\log_2(1+\frac{g_2p_2}{N}), \tag{2}$$

$$R_1 + R_2 \leq \frac{1}{2}\log_2(1+\frac{g_1p_1+g_2p_2}{N}). \tag{3}$$

The capacity region can be geometrically represented by the polygon shown in Fig. 1. Point A can be achieved by first decoding the message of user 2, and then subtracting the signal of user 2 before decoding the message of user 1. Point B can be achieved by changing the decoding order of users 1 and 2.

Consider the optimization problem of minimizing the sum power, p_1+p_2, given a rate pair (R_1, R_2) and the constraints in (1)-(3). We have the following basic result on the optimal solution to this problem:

Theorem 1. *Suppose $g_1 > g_2$. Given a positive rate pair (R_1, R_2), the minimum sum power is achieved if and only if equality holds in both* (2) *and* (3).

Proof: Note that the optimization can be re-written as a linear programming problem, with the constraints in the following form:

$$g_1p_1 \geq \alpha_1, \tag{4}$$

$$g_2p_2 \geq \alpha_2, \tag{5}$$

$$g_1p_1 + g_2p_2 \geq \beta, \tag{6}$$

where α_1, α_2 and β are positive constants depending on R_1, R_2 and N, and $\beta > \max\{\alpha_1, \alpha_2\}$.

It is obvious that there is a unique optimal solution. Since $g_1 > g_2$ and we want to minimize $p_1 + p_2$, as can be seen from Fig. 2, the optimal point lies at point E, the intersection of the lines $g_2p_2 = \alpha_2$ and $g_1p_1 + g_2p_2 = \beta$. This corresponds to the case where equality holds in (2) and (3). ■

Note that at the optimal solution, the BS should first decode the message of user 1, treating the signals of user 2 as noise. Next, the signal of user 1 will be subtracted from the received signal. Finally, the BS decodes the message of user 2, whose signal is now interference-free and corrupted only by the noise.

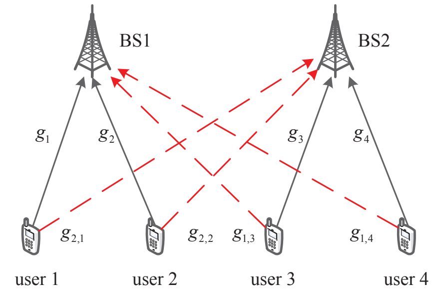

Fig. 3. The two-cell uplink NOMA system model

III. System Model and Game-Theoretic Formulation

Consider the uplink of a NOMA system with two cells, each of which consists of one BS and two users. We label the two cells as cells 1 and 2. Users 1 and 2 are attached to cell 1, while users 3 and 4 are attached to cell 2. For $i = 1, 2, 3, 4$, let r_i be the rate requirement of user i, g_i be the link gain from user i to its attached BS, and $g_{j,i}$ be the link gain from user i to BS j, where BS j is not the BS with which user i is associated. Without loss of generality, assume $g_1 > g_2$ and $g_3 > g_4$. Furthermore, let the noise power at each BS be N. The system model is shown in Fig. 3. We formulate the problem as a two-player non-cooperative game. Each BS performs the role as a player in the game, who wants to minimize the sum power of its two associated users, subject to their rate requirements.

Define $\tilde{\boldsymbol{p}}_1 \triangleq (p_1, p_2)$ and $\tilde{\boldsymbol{p}}_2 \triangleq (p_3, p_4)$. The strategy sets of players 1 and 2 are, respectively,

$$\mathcal{S}_1 \triangleq \{(a_1, \tilde{\boldsymbol{p}}_1) : a_1 \in \{0,1\} \text{ and } \tilde{\boldsymbol{p}}_1 \in \mathbb{R}_+^2\} \tag{7}$$

and

$$\mathcal{S}_2 \triangleq \{(a_2, \tilde{\boldsymbol{p}}_2) : a_2 \in \{0,1\} \text{ and } \tilde{\boldsymbol{p}}_2 \in \mathbb{R}_+^2\}, \tag{8}$$

where a_j is a binary variable indicating the decoding order of SIC used in cell j, for $j = 1, 2$. If $a_j = 0$, then BS j will first decode the message of the user with smaller index, treating the other user as noise. After subtracting the signals of the decoded user, the BS will decode the message of the remaining user. Similarly, if $a_j = 1$, then BS j will first decode the message of the user with larger index. Besides, we assume that inter-cell interference is treated as noise, and define

$$I_1 \triangleq g_{1,3}p_3 + g_{1,4}p_4 \tag{9}$$

and

$$I_2 \triangleq g_{2,1}p_1 + g_{2,2}p_2. \tag{10}$$

Define $\boldsymbol{p} \triangleq (p_1, p_2, p_3, p_4)$. Then the achievable rates of users 1 and 2 are given by

$$R_1(a_1, \boldsymbol{p}) \triangleq \begin{cases} \frac{1}{2}\log_2(1 + \frac{g_1 p_1}{g_2 p_2 + I_1 + N}) & \text{if } a_1 = 0 \\ \frac{1}{2}\log_2(1 + \frac{g_1 p_1}{I_1 + N}) & \text{if } a_1 = 1 \end{cases} \tag{11}$$

$$R_2(a_1, \boldsymbol{p}) \triangleq \begin{cases} \frac{1}{2}\log_2(1 + \frac{g_2 p_2}{I_1 + N}) & \text{if } a_1 = 0 \\ \frac{1}{2}\log_2(1 + \frac{g_2 p_2}{g_1 p_1 + I_1 + N}) & \text{if } a_1 = 1 \end{cases} \tag{12}$$

The achievable rates of users 3 and 4, denoted by $R_3(a_2, \boldsymbol{p})$ and $R_4(a_2, \boldsymbol{p})$, can be defined similarly.

In the power control game (PCG), each player i is required to choose a strategy $\boldsymbol{x}_i \in \mathcal{S}_i$ such that $R_i(a_i, \boldsymbol{p}) \geq r_i$. As each BS wants to minimize the sum power of its two associated users, the payoff functions of players 1 and 2 are defined as $u_1(\boldsymbol{x}_1, \boldsymbol{x}_2) \triangleq -(p_1 + p_2)$ and $u_2(\boldsymbol{x}_2, \boldsymbol{x}_1) \triangleq -(p_3 + p_4)$, respectively. We consider only the pure strategy and use the following solution concept:

Definition 1. A strategy profile $(\boldsymbol{x}_1, \boldsymbol{x}_2) \in \mathcal{S}_1 \times \mathcal{S}_2$ is said to be a *Nash equilibrium* if no unilateral deviation in strategy of player i increases the payoff of user i, i.e.,

$$u_i(\boldsymbol{x}_i^*, \boldsymbol{x}_{-i}^*) \geq u_i(\boldsymbol{x}_i, \boldsymbol{x}_{-i}^*) \ \ \forall i \in \{1, 2\} \text{ and } \boldsymbol{x}_i \in \mathcal{S}_i,$$

where $\boldsymbol{x}_{-i}$ denotes the strategy of the other player whose index is not equal to i.

The following concept is also useful in our analysis:

Definition 2. A strategy $\boldsymbol{x}_i \in \mathcal{S}_i$ is said to be a *best response* of player i, taking the other player's strategy $\boldsymbol{x}_{-i}$ as given, i.e.,

$$u_i(\boldsymbol{x}_i^*, \boldsymbol{x}_{-i}^*) \geq u_i(\boldsymbol{x}_i, \boldsymbol{x}_{-i}^*) \ \ \forall \boldsymbol{x}_i \in \mathcal{S}_i.$$

Using this concept, Nash equilibrium can also be defined as a strategy profile $(\boldsymbol{x}_1^*, \boldsymbol{x}_2^*)$ such that $\boldsymbol{x}_1^*$ is a best response of player 1 given $\boldsymbol{x}_2^*$, and $\boldsymbol{x}_2^*$ is a best response of player 2 given $\boldsymbol{x}_1^*$.

IV. Nash Equilibrium and Distributed Algorithm

In this section, we first derive the best response of each player. Next, we provide a sufficient condition for the existence of Nash equilibrium. Then, we describe a distributed power control algorithm that converges to the Nash equilibrium.

For $i = 1, 2, 3, 4$, define $\gamma_i \triangleq 2^{2r_i} - 1$, which can be interpreted as the target SINR of user i. The following result determines the best response of player 1, given the strategy of player 2. The best response of player 2 can be obtained similarly.

Theorem 2. *Given $\boldsymbol{x}_2 \in \mathcal{S}_2$, the best response of player 1, $\boldsymbol{x}_1^* = (a_1^*, p_1^*, p_2^*)$ is unique and given by*

$$a_1^* = 0 \tag{13}$$

$$p_1^* = \gamma_1(\gamma_2 + 1)(I_1 + N)/g_1 \tag{14}$$

$$p_2^* = \gamma_2(I_1 + N)/g_2, \tag{15}$$

where I_1 is defined in (9).

Proof: Given $\boldsymbol{x}_2$, let $\boldsymbol{x}_1^* = (a_1^*, \tilde{\boldsymbol{p}}_1^*)$ be a best response of user 1. Based on the discussion in Section II, it is clear that we always have $a_1^* = 0$ for any $\boldsymbol{x}_2$. By Theorem 1, equality holds in (2) and (3). From (12), we obtain

$$p_2^* = (2^{2r_2} - 1)(I_1 + N)/g_2,$$

and then by (11),

$$g_1 p_1^* = (2^{2r_1} - 1)(g_2 p_2^* + I_1 + N) = 2^{2r_2}(2^{2r_1} - 1)(I_1 + N).$$

■

At a Nash equilibrium, both players use their best responses. Therefore, to analyze the properties of Nash equilibrium, by Theorem 2, it suffices to fix the decoding orders of the cells as $a_1 = a_2 = 0$. When the decoding orders are fixed, the interference pattern is also fixed, i.e., user 2 interferes with user 1 but not vice versa and user 4 interferes with user 3 but not vice versa. For this reason, we can define a new set of link gains $h_{i,j}$'s, where $h_{i,j}$ represents the link gain from user j to the receiver associated with user i. For $i = 1, 2, 3, 4$, we have $h_{i,i} = g_i$, which represents the direct link gain of user i. Take the users in cell 1 as an example. For user 1, we have $h_{1,2} = g_2$, $h_{1,j} = g_{1,j}$ for $j = 3, 4$. For user 2, we have $h_{2,1} = 0$, since the signal of user 1 is decoded first and then subtracted, thus causing no interference to user 2. Furthermore, we have $h_{2,j} = g_{1,j}$ for $j = 3, 4$. The link gains for users 3 and 4 can be defined similarly.

After defining this new set of link gains, the problem of determining the Nash equilibrium reduces to the classical power control problem with four users, which can be written in the form

$$(\boldsymbol{I} - \boldsymbol{B})\boldsymbol{p} = \boldsymbol{n}, \tag{16}$$

where $\boldsymbol{I}$ is the 4×4 identity matrix, $\boldsymbol{B}$ is the 4×4 matrix whose (i, j)-th element is defined as

$$B_{ij} = \begin{cases} 0 & i = j \\ \gamma_i h_{i,j}/h_{i,i} & i \neq j \end{cases}$$

and $\boldsymbol{n}$ is the 4×4 vector whose i-th component is given by $\gamma_i N/h_{i,i}$. The powers at the Nash equilibrium can be obtained by solving (16). The following result then follows immediately from the classical power control theory (e.g. [14]):

Theorem 3. *A Nash equilibrium of the PCG exists if and only if the Perron-Frobenius eigenvalue of $\boldsymbol{B}$ is less than 1.*

Since the Nash equilibrium is a solution to the classical power control problem, it can be obtained iteratively in a distributed way by Foschini-Mijlanic algorithm [15], which takes the following form in our model:

$$p_i^{(t+1)} := \frac{\gamma_i}{h_{ii}} \Big(\sum_{j \neq i} h_{ij} p_j^{(t)} + N \Big) \tag{17}$$

By the classical power control theory (e.g. [13], [14], [15]), we have the following result:

Theorem 4. *The distributed algorithm in* (17) *converges to the unique Nash equilibrium of the PCG, provided that the Perron-Frobenius eigenvalue of* $\boldsymbol{B}$ *is less than 1.*

V. Optimality Property

Consider the following optimization problem, called P-OPT, which aims to minimize the sum power, i.e, $p_1 + p_2 + p_3 + p_4$, subject to $R_i(a_1, \boldsymbol{p}) \geq r_i$ for $i = 1, 2$ and $R_i(a_2, \boldsymbol{p}) \geq r_i$ for $i = 3, 4$. It is desirable that the Nash equilibrium of the PCG is an optimal solution to P-OPT. But unfortunately, this is not true in general, as the following example shows:

Example 1. Consider the following parameters: The direct link gains are $g_1 = g_3 = 2$ and $g_2 = g_4 = 1$. The cross link gains are $g_{2,1} = g_{1,3} = 1/2$ and $g_{2,2} = g_{1,4} = 0$. The noise power at each BS is equal to 1. The rate requirement of each user is $r_i = 1/2$, which can be translated as an SINR requirement of $\gamma_i = 1$.

At the Nash equilibrium, we have $a_1^* = a_2^* = 0$, $p_1^* = p_3^* = 2$ and $p_2^* = p_4^* = 3/2$. This can be easily verified by applying Theorem 2. The sum of powers is equal to 7.

Consider the case where the decoding order in each cell is reversed, i.e., $a_1 = a_2 = 1$. Then it can be verified by (11) and (12) that the rate requirements are met if $p_1 = p_3 = 2/3$ and $p_2 = p_4 = 8/3$. The sum of powers is equal to 20/3, which is less than 7. □

The next result provides a sufficient condition for the Nash equilibrium to be an optimal point:

Theorem 5. *The Nash equilibrium of the PCG is optimal to* P-OPT, *provided that*

$$\frac{g_1}{g_2} \geq \frac{g_{2,1}}{g_{2,2}} \quad \textit{and} \quad \frac{g_3}{g_4} \geq \frac{g_{1,3}}{g_{1,4}}. \tag{18}$$

Proof: Recall that the sum rate of each cell is constrained by the capacity region of MAC. Consider cell 1, and we have

$$r_1 \leq \frac{1}{2} \log_2 (1 + \frac{g_1 p_1}{I_1 + N}), \tag{19}$$

$$r_2 \leq \frac{1}{2} \log_2 (1 + \frac{g_2 p_2}{I_1 + N}), \tag{20}$$

$$r_1 + r_2 \leq \frac{1}{2} \log_2 (1 + \frac{g_1 p_1 + g_2 p_2}{I_1 + N}). \tag{21}$$

At an optimal solution to P-OPT, equality must hold in (21), for otherwise we can scale down (p_1, p_2) to further reduce the sum power, since such a change will only increase the achievable rates of users 3 and 4. Let $(p_1^*, p_2^*, p_3^*, p_4^*)$ be an optimal solution to P-OPT. Then, we have

$$g_1 p_1^* + g_2 p_2^* = \big[(\gamma_1 + 1)(\gamma_2 + 1) - 1\big](I_1 + N).$$

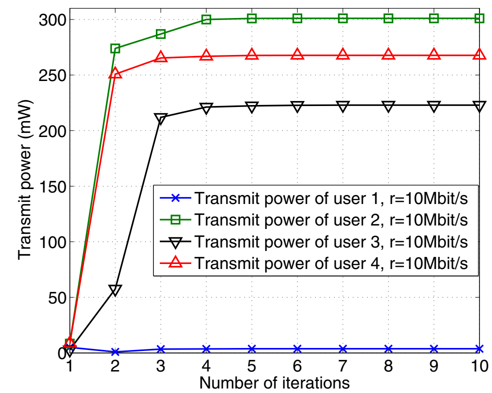

Fig. 4. Convergence of the distributed power control algorithm

Together with (19) and (20), an optimal power pair (p_1^*, p_2^*) must lie on the line segment that corresponds to DE in Fig. 2.

Denote point E in Fig. 2 by (p_1', p_2'). Suppose $(p_1^*, p_2^*) \neq (p_1', p_2')$. Note that the total interference to BS 2 caused by users 1 and 2 is given by $g_{2,1} p_1 + g_{2,2} p_2$. If line DE is steeper than the line corresponding to the total interference, i.e., $g_1/g_2 \geq g_{2,1}/g_{2,2}$, then the total interference to BS 2 at (p_1', p_2') is no greater than that at (p_1^*, p_2^*). Therefore, if the power vector of users 1 and 2 is changed to (p_1', p_2'), the rate constraints of users 3 and 4 can still be satisfied at (p_3^*, p_4^*). By assumption, $g_1 > g_2$. By Theorem 1, we have $p_1' + p_2' < p_1^* + p_2^*$. As a result, $(p_1', p_2', p_3^*, p_4^*)$ yields a lower sum power than $(p_1^*, p_2^*, p_3^*, p_4^*)$. By contradiction, (p_1^*, p_2^*) must correspond to point E.

By symmetry, the same argument applies to cell 2. Hence, the Nash equilibrium is optimal to P-OPT. ■

VI. Simulation Results

A simulation study is presented in this section. We consider the uplink of a NOMA system with two neighboring hexagonal cells of radius 1 km. There is one BS at the center of each cell. In each cell, there are two users uniformly distributed within its coverage. The system bandwidth is $W = 10$ MHz and the noise density is $N_0 = -174$ dBm/Hz, and the noise power is given by $N = W N_0$. Each user has the same target rate r, i.e., $r_i = r$ for all i. Each link gain is comprised of three factors, namely, the path loss, the shadow fading and the small-scale fading. The path loss component is given by $128.1 + 37.6 \log_{10} d$, where d is the distance between the transmitter and the receiver in km. We assume that there is correlation of shadowing between the two user in the same cell. For $i = 1, 2$, the shadowing fading component of user i is given by $(X_i + X)/\sqrt{2}$, where X_1 and X_2 and X are independent and identical log-normal random variables [16]. The shadow fading components of the users in the second cell is defined similarly. For small-scale fading, each user experiences independent Rayleigh fading with variance 1.

Fig. 4 shows the convergence of our distributed power control algorithm in an arbitrarily generated scenario. Each user

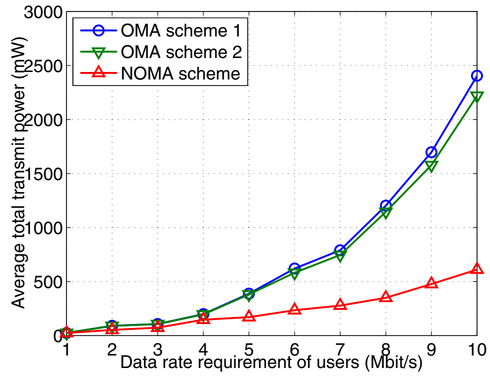

Fig. 5. Total transmit power versus data rate requirements

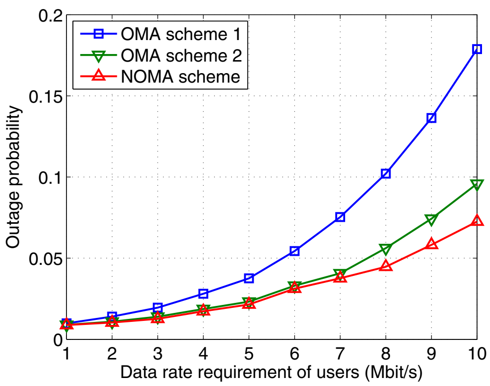

Fig. 6. Outage performance of three multiple access technologies

has a data rate requirement of 10 Mbit/s and its initial power is generated randomly. The x-axis represents the number of iterations, while the y-axis indicates the transmit power of each user. It can be seen that the distributed algorithm takes only a few iterations to converge.

Fig. 5 compares the power consumption of the our power-controlled NOMA scheme with two orthogonal multiple access (OMA) schemes. In both OMA schemes, we divide the spectrum into two channels, each of bandwidth $0.5W$. In OMA scheme 1, we assume that users 1 and 3 share the same channel while users 2 and 4 share the other channel. In OMA scheme 2, we optimize the pairing of users to share the same channel. In other words, user 1 may share the same channel with either user 3 or user 4, depending on which pairing yields less total power consumption. To obtain the data points for each target rate, we generate randomly a large number of problem instances, denoted by N_{sim}, until there are 10,000 of them in which all the three schemes have non-empty feasible regions. Each point in Fig. 5 is then obtained by averaging over the 10,000 instances in which all the three schemes are feasible. NOMA outperforms the other two schemes significantly, especially when the target rate is high. For example, when $r = 10$ Mbit/s, NOMA saves power by 72.4% when compared with OMA scheme 2.

Fig. 6 shows that NOMA has much lower outage probability than the two OMA schemes, where the outage probability is calculated by dividing the number of infeasible problem instances by N_{sim}.

VII. Conclusion

The power control problem for the uplink of a two-cell NOMA system is investigated. Game-theoretic approach is used to study the stability of distributed power control algorithms. It is shown that a unique Nash equilibrium exists if the Perron-Frobenius eigenvalue of a certain link gain matrix is less than one. Then a distributed power control algorithm is constructed, which is guaranteed to converge to the Nash equilibrium. The Nash equilibrium is globally optimal in minimizing total power consumption, provided that some technical conditions are satisfied. Simulation results show the superiority of power-controlled NOMA over its orthogonal counterparts. Future works include the generalization of our results to the case with multiple cells and with multiple antennas.

References

[1] DOCOMO 5G White Paper, "5G radio access: requirement, concept and technologies", NTT DOCOMO , INC, Jul. 2014.

[2] L. Dai, B. Wang, Y. Yuan, S. Han, C.-L. I, and Z. Wang, "Non-orthogonal multiple access for 5G: solutions, challenges, opportunities, and future research trends," *IEEE Commun. Magazine*, pp. 74–81, Sep. 2015.

[3] T. Takeda and K. Higuchi, "Enhanced user fairness using nonorthogonal access with SIC in cellular uplink," in *Proc. IEEE VTC 2011-Fall*, San Francisco, Sep. 2011.

[4] Y. Endo, Y. Kishiyama, and K. Higuchi, "Uplink Non-orthogonal Access with MMSE-SIC in the Presence of Inter-cell Interference", in *Proc. ISWCS*, 2012.

[5] Y. Saito, Y. Kishiyama, A. Benjebbour, T. Nakamura, A. Li, and K. Higuchi, "Non-orthogonal multiple access (NOMA) for future radio access," *Proc., IEEE VTC 2013-Spring*, Jun. 2013, pp. 1-5.

[6] Y. Saito, A. Benjebbour, Y. Kishiyama, and T. Nakamura, "System-level performance evaluation of downlink non-orthogonal multiple access (NOMA)," *Proc., IEEE PIMRC*, Sept. 2013, pp. 611-615.

[7] A. Benjebbour, A. Li, Y. Saito, Y. Kishiyama, A. Harada, and T. Nakamura, "System-level performance of downlink NOMA for future LTE enhancements," *Proc. IEEE Globecom*, Dec. 2013, pp. 66-70.

[8] Z. Ding, Z. Yang, and P. Fan, "On the performance of non-orthogonal multiple access in 5G systems with randomly deployed users," *IEEE Signal Process. Lett.*, vol. 20, no. 12, pp. 1211-1215, Dec. 2013.

[9] X. Chen, A. Benjebbour, A. Li, and A. Harada, "Multi-user proportional fair scheduling for uplink non-orthogonal multiple access (NOMA)," *Proc. IEEE VTC 2014-Spring*, May. 2014, pp. 1-5.

[10] M. Al-Imari, P. Xiao, M. Ali-Imran, and R. Tafazolli, "Uplink non-orthogonal multiple access for 5G wireless networks," *Proc. IEEE ISWCS*, Aug. 2014, pp. 781-785.

[11] D. Tse and P. Viswanath. *Fundamentals of wireless communication*, Cambridge University Press, Cambridge, U.K., 2005.

[12] Y. Fu, Y. Chen and C. W. Sung, "Distributed downlink power control for the non-orthogonal multiple access system with two interfering cells," to appear in *IEEE International Conference on Communications (ICC)*, Kuala Lumpur, Malaysia, May 2016.

[13] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Select. Areas Commun.*, vol. 13, no. 7, pp. 1341-1347, Sep. 1995.

[14] C. W. Sung and W. S. Wong, "Mathematical aspects of the power control problem in mobile communication systems," in *Lectures on Systems, Control and Information: Lectures at the Morningside Center of Mathematics*, AMS/International Press, pp. 127-172, 2000.

[15] G. J. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Trans. Veh. Tech.*, vol. 42, pp. 641-646, Nov. 1993.

[16] GreenTouch, "Mobile communications WG architecture doc2: Reference scenarios," May. 2013.