Combinational Code for Channel Estimation in Visible Light Communications and Positioning

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***Abstract*—In visible light communications (VLC) and visible light positioning (VLP), channel gains between receiver and light sources are required to be estimated. Although Time Division Multiple Access (TDMA) is typically used in the channel estimation phase of radio frequency systems, it may not be applicable for VLC and VLP systems due to the maximum power constraint and desired average power constraint that are unique to visible light systems. Recently, combinational code has been proposed as a coding scheme for channel estimation in VLC and VLP. Combinational code can work under the maximum and average power constraints, and it minimises the total and maximum noise variances experienced by the receiver. This paper reports some experimental results to compare combinational code and two schemes based on TDMA. Experimental results show that in terms of noise variance experienced by a receiver, combinational code significantly outperforms other schemes based on TDMA under the same power constraints. Challenges encountered in experiments for channel estimation are discussed and solutions are suggested to overcome these challenges.**

I. Introduction

Visible light communications (VLC) has attracted much attention as a future technology to mitigate the scarcity of the radio frequency spectrum. Building on existing light infrastructure, signals are transmitted from light emitting diodes (LEDs) at transmitters to photo-detectors (PDs) at receivers. A visible light system provides not only illumination but also communications [2] and indoor positioning [3, 4]. The accuracy of positioning systems depends on how accurate the channel gains from multiple transmitters are estimated. Channel gain information is also useful to improve data transmission rates [5, 6]. So it is important to know how channel gains can be precisely estimated.

VLC systems has several properties different from conventional radio frequency (RF) systems. The transmitted signal in VLC must be non-negative real as intensity modulation/direct detection is used. Channel gains are also non-negative real. Since illumination is the primary purpose of LEDs, they must provide sufficient light intensity. Rather than minimizing the average transmitted power as those in RF systems, transmitters in VLC systems must produce a desired average power [6, 7].

A part of this work has been accepted in the IEEE Journal on Selected Areas in Communications [1], that are summarized in Section III. Results from [1] has never been presented in any conference.

While Time Division Multiple Access (TDMA) is typically used for the channel estimation phase of RF systems, this may not be a valid option for VLC systems. The main reason is that each LED needs to satisfy not only the average power constraint but also the maximum power constraint due to its physical limitation. For example, consider a seminar room with $N = 16$ LEDs light sources where each LED produces maximum 16 watts. Suppose TDMA is used. Then the duty cycle of each LED is $\frac{1}{N} = \frac{1}{16}$ and thus each LED actually generates only 1 watt that will certainly make the room too dim. Therefore, if a VLC system has N LEDs where N is not small, TDMA may not be a valid option to avoid interference.

Channel estimation in VLC has been considered [8–10]. In these works, pilot symbols, which are orthogonal in time or frequency, are used. Then least square (LS) based channel estimation is performed to estimate the channel gains before refining the estimates, e.g., by considering adaptive statistical Bayesian minimum mean square error (MMSE) [8]. To the best of our knowledge, how to optimise the system under average and maximum power constraints is still open.

Combinational code has recently been proposed by the authors as a coding scheme for channel estimation in visible light communications and positioning [1]. It is proved that combinational code is an optimal code that simultaneously attains the minimum values of both total noise variance and maximum noise variance experienced at the receiver. In this paper, we will demonstrate through experiments that the combinational code outperforms other coding schemes based on TDMA. Combinational code can achieve smaller total noise variance and maximum noise variance in channel estimation under maximum and average power constraints. Furthermore, we will discuss some challenges encountered in channel estimation experiments and propose some solutions to overcome these challenges.

The remainder of the paper is organised as follows. In Section II, we introduce the system model and problem formulation. Section III presents the combinational code and some of its properties. Numerical and experimental results are given in Sections IV and V, respectively. Finally, some challenges for the experimental results are discussed in Section VI before concluding the paper in Section VII.

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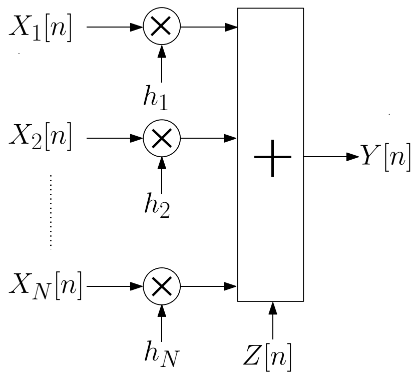

Fig. 1: A model of VLC system with N LEDs.

II. System Model and Problem Formulation

A. System Model

A model of VLC system is depicted in Fig. 1. Assume that N LEDs are installed. LED i transmits $X_i[n] \geq 0$ at time slot n. Slow fading is assumed for the channel gain h_i (i.e., a constant over a short period of time). This is a reasonable assumption as it is common to achieve a transmission rate over 10^6 symbol per second in VLC [2]. If the receiver's displacement and change in orientation are negligible within 10^{-3} seconds, h_i can be seen as invariant for more than 10^3 symbols. The receiver is a mobile device equipped with a photo-detector which measures light intensity. For the n-th time slot, the receiver receives

$$Y[n] = \sum_{i=1}^{N} h_i X_i[n] + Z[n], \tag{1}$$

where $Z[n]$ is an additive Gaussian noise with zero mean and σ^2 variance, i.e., $\mathcal{N}(0, \sigma^2)$. In our model, additive white Gaussian noise is considered which is commonly used in the literature [6, 7, 11].

Let μ be the maximum power per time slot allowed for each LED so that $X_i[n] \leq \mu$ for all i and n. Assume that a receiver can determine h_i for all i by using L time slots with $L \geq N$. A larger L gives larger flexibility in system design but this leads to longer delay for positioning application or shorter time for data transmissions in VLC. To obtain consistent illumination and fulfill the lighting requirement, each LED is required to satisfy an average power constraint, i.e. $\lim_{T\to\infty} \frac{1}{T}\sum_{n=1}^{T} X_i[n] = \Phi$ for all i. Similar to [6, 7], we consider the average power is equal to a desired value. We leave it as a future work to consider a range rather than a specific value in the average power constraint. The system parameters are listed in Table I.

Notations: Matrix theory [12] is used. All matrices and vectors are in boldface. For any matrix $\mathbf{M}$, $\mathbf{M}^T$, $\mathrm{Tr}(\mathbf{M})$, and $\mathrm{diag}(\mathbf{M})$ mean the transpose, trace and diagonal of $\mathbf{M}$, respectively. Identity matrix is denoted by $\mathbf{I}$.

TABLE I: Meaning of notations.

Notation	Meaning
N	Number of LEDs
Φ	Average transmitting optical power per time slot
μ	Maximum instantaneous transmitting optical power
$X_i[n]$	Signal transmitted by LED i in the n-th time slot
$Y[n]$	Signal received by the receiver in the n-th time slot
$Z[n]$	Additive Gaussian noise in the n-th time slot $\sim \mathcal{N}(0, \sigma^2)$

B. Problem Formulation

Consider a codebook represented by an $N \times L$ matrix $\mathbf{A} = [a_{i,k}]$ with

$$0 \leq a_{i,k} \leq \mu \qquad \forall i, k \tag{2}$$

$$\frac{1}{L}\sum_{k=1}^{L} a_{i,k} = \Phi \qquad \forall i, \tag{3}$$

where (2) and (3) are constraints on maximum power and average power for light intensity, respectively. LED i transmits the i-th row in $\mathbf{A}$. The goal is to design $\mathbf{A}$ and an estimation scheme such that the effect of interference and noise can be mitigated in the estimation of h_i for all i. Assume that linear decoding estimator $W_i[n]$ is used to estimate h_i:

$$\hat{h}_i = \sum_{n=1}^{L} W_i[n] Y[n]. \tag{4}$$

This paper considers only Zero-Forcing (ZF) decoders, which can remove interference among LEDs, due to two reasons: 1) In both visible light communications and positioning systems, direct line-of-sight between transmitter and receiver is usually assumed. Therefore, interference power is typically larger than noise power so that it is important to eliminate interference. 2) For positioning systems, the estimated position is a function of $\{h_i\}$. The positioning accuracy is limited by how accurate h_i's are estimated. In other words, if interference cannot be removed, the estimated position does not converge to the true position regardless of how many estimates of $\{h_i\}$ are obtained.

From now on, we consider only ZF decoders so that

$$\hat{h}_i = \sum_{n=1}^{L} W_i[n] Y[n] = h_i + \sum_{n=1}^{L} W_i[n] Z[n]. \tag{5}$$

Since the estimate of h_i is affected by a zero-mean Gaussian noise with variance $\sigma^2 \sum_{n=1}^{L} W_i^2[n]$, we consider two metrics to measure the effect of noise:

1. Total noise variance:

$$\sum_{i}\sum_{n=1}^{L} W_i^2[n] \tag{6}$$

2. Maximum noise variance:

$$\max_{i} \sum_{n=1}^{L} W_i^2[n]. \tag{7}$$

Let $\mathbf{A}^\dagger = \mathbf{A}^T(\mathbf{A}\mathbf{A}^T)^{-1}$ be the pseudo-inverse of $\mathbf{A}$ and let $W_i[n]$ be the i-th column in $\mathbf{A}^\dagger$. Then (6) and (7) are equivalent to $\mathrm{Tr}((\mathbf{A}^\dagger)^T\mathbf{A}^\dagger)$ and the maximum element in the $\mathrm{diag}((\mathbf{A}^\dagger)^T\mathbf{A}^\dagger)$, respectively. Note that (6) can be written as

$$\mathrm{Tr}((\mathbf{A}^\dagger)^T\mathbf{A}^\dagger) = \mathrm{Tr}\left(\left((\mathbf{A}\mathbf{A}^T)^{-1}\right)^T\mathbf{A}\mathbf{A}^T(\mathbf{A}\mathbf{A}^T)^{-1}\right) \quad (8)$$

$$= \mathrm{Tr}\left(\left((\mathbf{A}\mathbf{A}^T)^{-1}\right)^T\right) \quad (9)$$

$$= \mathrm{Tr}\left((\mathbf{A}\mathbf{A}^T)^{-1}\right). \quad (10)$$

To minimise (6) is equivalent to solving

$$\min_{\mathbf{A}} \mathrm{Tr}\left((\mathbf{A}\mathbf{A}^T)^{-1}\right) \quad \text{subject to (2) and (3).} \quad (11)$$

Note that (11) is a non-convex optimization problem that can be illustrated by the following example. Let $\mathbf{A}_1$ be a 2×2 identity matrix and $\mathbf{A}_2 = 1 - \mathbf{A}_1$. Let $\mathbf{A} = \frac{1}{4}\mathbf{A}_1 + \frac{3}{4}\mathbf{A}_2$. Then

$$\mathrm{Tr}\left((\mathbf{A}\mathbf{A}^T)^{-1}\right) = 5$$

but

$$\frac{1}{4}\mathrm{Tr}\left((\mathbf{A}_1\mathbf{A}_1^T)^{-1}\right) + \frac{3}{4}\mathrm{Tr}\left((\mathbf{A}_2\mathbf{A}_2^T)^{-1}\right) = 2.$$

Therefore,

$$\mathrm{Tr}\left((\mathbf{A}\mathbf{A}^T)^{-1}\right) > \frac{1}{4}\mathrm{Tr}\left((\mathbf{A}_1\mathbf{A}_1^T)^{-1}\right) + \frac{3}{4}\mathrm{Tr}\left((\mathbf{A}_2\mathbf{A}_2^T)^{-1}\right)$$

and hence, $\mathrm{Tr}\left((\mathbf{A}\mathbf{A}^T)^{-1}\right)$ is not a convex function of $\mathbf{A}$.

III. Combinational Code

In this section, we present the combinational code and a summary of its properties that have been found in [1].

Definition 1 (Combinational Code). *For N LEDs, consider* $\lambda \in [1, N-1]$. *Let*

$$L = \binom{N}{\lambda} = \frac{N!}{\lambda!(N-\lambda)!}. \quad (12)$$

Define $\mathbf{A}$ *as an* $N \times L$ *matrix whose columns store all the possible permutation of* λ μ*'s and* $(N-\lambda)$ 0*'s. The* i*-th row in* $\mathbf{A}$ *is the codeword assigned to the* i*-th LED. The average power of this coding scheme is* $\Phi = \frac{\lambda\mu}{N}$.

Example 1. *Suppose* $N = 4$, *and* $\lambda = 2$. *So* $\Phi = \frac{1}{2}\mu$ *and* $L = 6$. *In this case,*

$$\mathbf{A} = \begin{bmatrix} \mu & \mu & \mu & 0 & 0 & 0 \\ \mu & 0 & 0 & \mu & \mu & 0 \\ 0 & \mu & 0 & \mu & 0 & \mu \\ 0 & 0 & \mu & 0 & \mu & \mu \end{bmatrix}. \quad (13)$$

We now discuss certain important properties of the combinational code [1]. Although minimising (6) requires to solve a non-convex optimization problem, it has been analytically proved that the combinational code achieves the minimum total noise variance in (6) and the smallest maximum noise variance in (7) among other codes having the same requirements on (μ, Φ, N, L). When the combinational code is used, the total noise variance in (6) becomes

$$\frac{(\lambda-1)!(N-\lambda-1)!(\lambda N - 2\lambda + 1)}{\lambda(N-1)!}. \quad (14)$$

For a fixed number of LEDs and given power constraints, the codeword length for a combinational code is given by (12). However, the permissible codeword length, which depends on the delay constraint in applications, can be much longer than the codeword length. For example, consider $N = 5$. If the permissible codeword length is 100, the receiver can accumulate 100 symbols to reduce the noise impact in channel estimation. However, the codeword length of the combinational code is at most 10 where the maximum is attained when $\lambda = 2$. To reduce noise variance at the receiver, we may send the same codeword 10 times before the receiver estimates the channel gain. This is called the concatenation of the *same* combinational code. Interestingly, the resulting code from this concatenation is also an optimal code in the sense that the total noise variance is minimised comparing with all other codes having the same requirements on (μ, Φ, N, L). We are going to further elaborate this in the following numerical and experimental results.

IV. Numerical Results

In this section, numerical results for the tradeoff between total noise variance and codeword length are shown. Consider $N = 6$, $\mu = 1$ and $\Phi = \frac{\mu}{2} = \frac{1}{2}$. In Fig. 2, combinational code is compared with other possible coding schemes which are based on TDMA. The details are explained below.

- *Concatenated Combinational Code:* Let $\mathbf{A}_{\mu,N,\lambda}(c,d,e)$ be $\mathbf{A}$ in Definition 1 with $(\mu, N, \lambda) = (c, d, e)$. The power constraints are satisfied by $\mathbf{A}_{\mu,N,\lambda}(1,6,3)$ with codeword length equal to 20. On the other hand, we can construct another set of 6 codewords by concatenating the codewords from $\mathbf{A}_{\mu,N,\lambda}(1,6,1)$ and $\mathbf{A}_{\mu,N,\lambda}(1,6,5)$. To be specific, LED i transmits the i-row in $\mathbf{A}_{\mu,N,\lambda}(1,6,1)$ and then the i-row in $\mathbf{A}_{\mu,N,\lambda}(1,6,5)$. So the new codewords have length 12 and the power constraints are satisfied. By considering different concatenation of $\mathbf{A}_{\mu,N,\lambda}(1,6,i)$ for $1 \le i \le 5$, the achievable pairs of total noise variance and codeword length are plotted in Fig. 2. We have considered concatenating $\mathbf{A}_{\mu,N,\lambda}(1,6,i)$ by 4 times.
- *Optimal Code:* This is a special case of the above concatenated combinational code with the requirement that codewords from $\mathbf{A}_{\mu,N,\lambda}(c,d,e)$ are concatenated with codewords from the same $\mathbf{A}_{\mu,N,\lambda}(c,d,e)$ by any k times. It is important to notice that this new code is still optimal. So the concatenation of $\mathbf{A}_{\mu,N,\lambda}(1,6,3)$ with itself is indicated as Optimal Code in Fig. 2.
- *TDMA with all LEDs 'On':* Let

$$\mathbf{A} = [\mathbf{I}_{6\times 6} \quad \mathbf{1}_{6\times 4}], \quad (15)$$

where $\mathbf{I}_{6\times 6}$ is a 6×6 identity matrix and $\mathbf{1}_{6\times 4}$ is a 6×4 matrix of ones. Due to the identity matrix, the LEDs use TDMA in the first six time slots to avoids interference. Then all the LEDs are 'On' for four time slots due to the matrix of ones so that the power constraints are satisfied.

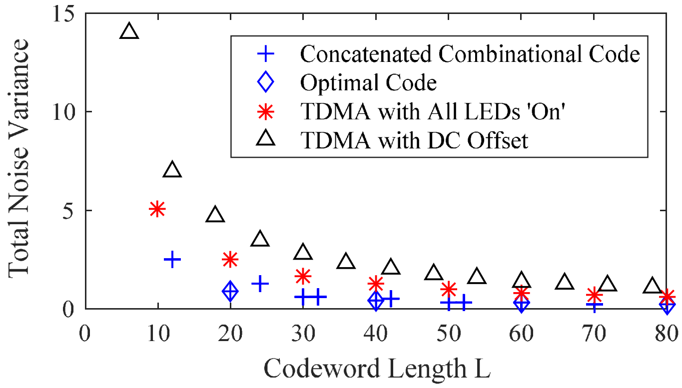

Fig. 2: Comparison among concatenation of different schemes.

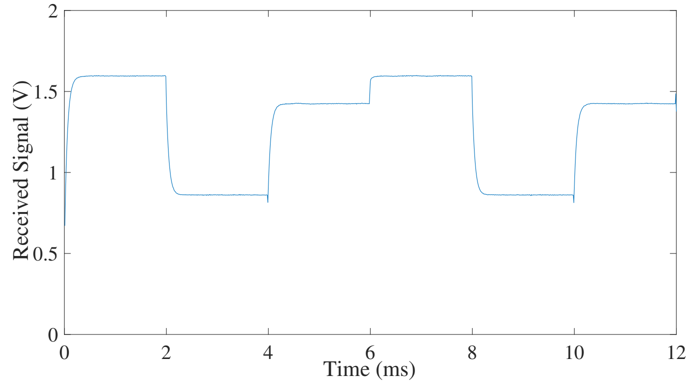

Fig. 3: Received signal of the combinational code.

- *TDMA with DC offset:* Consider TDMA is used and each LED is shifted by a DC offset so that LEDs do not go completely off to satisfy the power constraint. In this case,

$$\mathbf{A} = \frac{3}{5}\mathbf{I}_{6\times 6} + \frac{2}{5}\mathbf{1}_{6\times 6}. \quad (16)$$

In Fig. 2, TDMA with DC offset has the worst performance although it can achieve the shortest codeword length 6. The concatenated combinational code performs better than the two extensions of TDMA. Since combinational code is the optimal code, the fundamental trade-off between total noise variance and codeword length is characterised at $L = 20k$ for integer $k \geq 1$ where the close-form expression of total noise variance is given in (14).

V. Experimental Results

In this section, we compare the performance of combinational code with two coding schemes based on TDMA through experimental results. In our experiments, three LEDs are placed on the ceiling. The LEDs coordinates (in m) are: (1.1, 0.89, 2.7), (1.12, 1.81, 2.7) and (2.37, 1.81, 2.7). The room dimensions are $5m\times 4m\times 3m$. Bridgelux LEDs (BXRA-56C5300-H-00) are used as transmitters [14]. The receiver has a single Centronic Silicon PD [15] pointing upward that measures light intensity. The responsivity, effective area, Lambertian parameter (experimentally found) and field of view of the PD are 22 nA/lux, 15 mm^2, 1.4 and 1.22 radians, respectively. The received signal is affected by shot and thermal noise as well as background light.

One microcontroller (Arduino Micro) coordinates the transmission of all LEDs. The microcontroller is connected to three current control circuits that drive the LEDs. At the receiver side, the output of the PD is connected to the analog port of the National Instruments Data Acquisition (DAQ) box USB-6341 controlled by LabView.

We compare the following coding schemes:

1) *Combinational Code:*

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}. \quad (17)$$

2) *TDMA with all LEDs 'On':*

$$\mathbf{A}_2 = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}. \quad (18)$$

3) *TDMA with DC offset:*

$$\mathbf{A}_3 = \begin{bmatrix} 1 & 0.5 & 0.5 & 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0.5 & 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 1 & 0.5 & 0.5 & 1 \end{bmatrix}. \quad (19)$$

For a fair comparison, all the schemes in (17)–(19) have the same codeword length 6 and satisfy the same average power $\Phi = \frac{2}{3}$ and maximum power $\mu = 1$.

The performance of the considered coding schemes is evaluated in terms of total noise variance and mean square error (MSE) of the estimated channel gain $\hat{h}_i$ versus receiver position. The receiver is located (in m) at (l_R, 1.44, 0.82) where $l_R = 1.35 + 0.2k$ and k is an integer with $0 \leq k \leq 4$. At each receiver position, we take three measurements, and each measurement contains 1000 time slots.

A. Total Noise Variance

We first evaluate the performance of the coding schemes in terms of the total noise variance. The receiver obtains the received signal vector $Y[n]$ after measuring the output of the PD. A measurement is shown in Fig. 3 when the combinational code in (17) is implemented. The receiver estimates the channel gains $\hat{h}_i$ by applying (5). According to (5), the estimated channel gain is equal to the channel gain h_i plus noise. Since h_i is constant, var$\{\hat{h}_i\}$ equals to the noise variance and thus the total noise variance can be determined by $\sum_i$ var$\{\hat{h}_i\}$.

Figure 4 shows the total noise variance of each measurement and the average total noise variance of all measurements for all schemes in (17)–(19) at different receiver positions. The results reveal that combinational code achieves the smallest total noise variance compared to the other two schemes based on TDMA. This is due to the fact that combinational code can achieve the minimum total noise variance. The figure also shows that TDMA with All LEDs 'On' outperforms TDMA

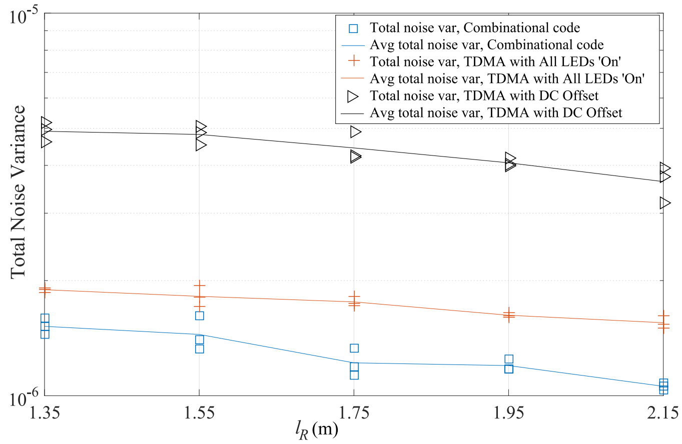

Fig. 4: Total noise variance at different receiver positions.

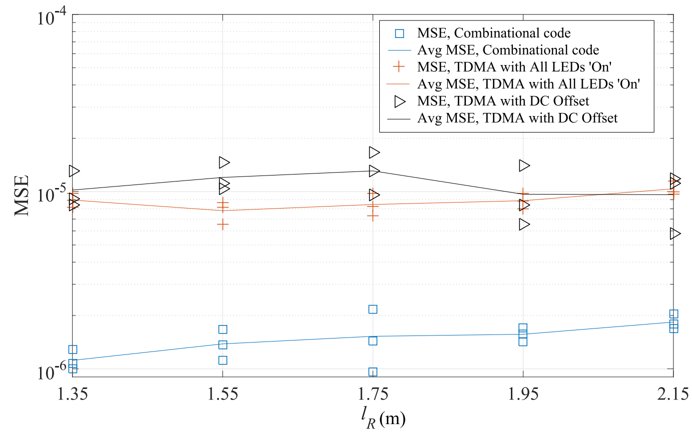

Fig. 5: Experimental results of MSE at different receiver position.

with DC offset. These results agree with the numerical results given in Section IV. It is also noted that with increasing l_R, the total noise variance of all schemes reduces due to the following. With increasing l_R, the receiver moves away from LEDs 1 and 2 and comes closer to LED 3 so that the average received power decreases. As a result, the shot noise, which is a function of the received power, reduces as well.

B. MSE of estimated channel gains

We now compare the performance of the schemes defined in (17)–(19) through the MSE of the estimated channel gains $\hat{h}_i$ at different receiver positions. Define MSE as

$$\text{MSE} = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{h_i - \hat{h}_i}{h_i}\right|^2. \tag{20}$$

One should notice that we cannot use the true channel gain as h_i because it is actually unknown in practice. Therefore, we need to obtain the best estimate of h_i, which is called as a reference channel gain, to replace h_i in (20).

We obtain the reference channel gain for h_i through the following method. Define

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{21}$$

and concatenate it with all the schemes such that (17)–(19) become $[\mathbf{D} \quad \mathbf{A}_1]$, $[\mathbf{D} \quad \mathbf{A}_2]$ and $[\mathbf{D} \quad \mathbf{A}_3]$, respectively. Due to $\mathbf{D}$, only LED i transmits in the i-th time slot for $1 \le i \le 3$ so that the received $Y[n]$ is the summation of the signal received from LED i, background light and noise. Consider $j = 1, 2$ or 3. To obtain a reference channel gain for h_i and the estimated channel gain $\hat{h}_i$ from the scheme $[\mathbf{D} \quad \mathbf{A}_j]$, we need to cancel out the effect of the background light and reduces the noise impact as follows:

1) At each receiver position, we take measurement over 1000 time slots, where 100 samples are obtained at each time slot. Since 10 time slots are used to transmit $[\mathbf{D} \quad \mathbf{A}_j]$, the same codeword $[\mathbf{D} \quad \mathbf{A}_j]$ has been transmitted 100 times in each measurement.
2) We first estimate the background light intensity. When the transmitters transmit the forth time slot in $[\mathbf{D} \quad \mathbf{A}_j]$ (i.e., a column of zeros), no LED transmits and the background light intensity can be measured. We take the average of all the samples obtained during that time slots. At the end, 10000 samples are averaged to estimate the background light intensity.
3) The next step is to obtain a reference channel gain for h_i. When the transmitters transmit the i-th time slot for $1 \le i \le 3$ in $[\mathbf{D} \quad \mathbf{A}_j]$, only LED i transmits. We take the average of all the samples obtained during that time slots and then subtract it by the estimated background light intensity. In this process, 10000 samples are averaged to find the reference channel gain.
4) Finally, we find the estimated channel gain $\hat{h}_i$ as follows. When the transmitters transmit the fifth to tenth time slots in $[\mathbf{D} \quad \mathbf{A}_j]$, the LEDs transmit according to $\mathbf{A}_j$. We take the average of all the samples obtained during these time slots to obtain a vector with six elements. Then each element is subtracted by the estimated background light intensity. Finally, $\hat{h}_i$ is obtained by applying the ZF decoder according to (5) where $W_i[n]$ is the i-th column in $\mathbf{A}_j^{\dagger}$.

After that the MSE is calculated according to (20). Figure 5 shows the MSE of each measurement and the average of all measurements for all coding schemes in (17)–(19) at different receiver positions. The figure illustrates that the proposed combinational code achieves the lowest average MSE compared to the other schemes based on TDMA. The achieved average MSE of the combinational code is always less than 2×10^{-6}. The experimental results agree with the theoretical and the numerical results that combinational code can minimize the noise variance experienced at the receiver.

VI. Discussion

We now explain why we need to carefully design the way to obtain the reference channel gains h_i as shown in Section V-B.

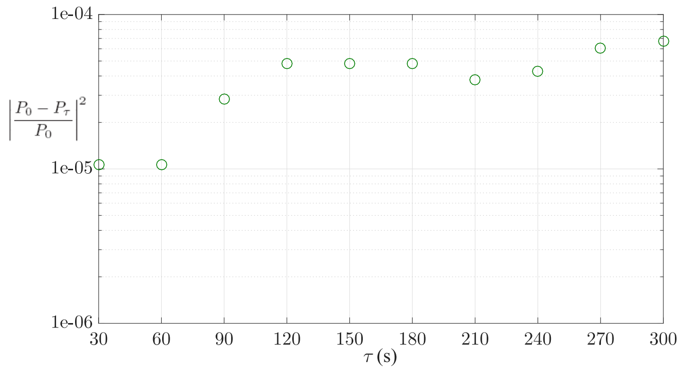

Fig. 6: Experimental results of the difference in received light intensity from one LED over time.

The simplest way is that we turn on only one LED, say LED i, and measure the received light intensity which is used to get an estimate of h_i. We do this for each LED until all the h_i's are obtained. Then we apply the coding schemes in (17)–(19) and decode the received signals by applying the ZF decoder according to (5) and obtain all $\hat{h}_i$ from all schemes one by one. Then we calculate the MSE from (20). The accuracy of this method depends on whether the light intensity emitted from the LEDs are stable or not during the whole time of the experiment. In other words, the light intensity emitted from the LEDs are required to be the same when we estimate h_i and $\hat{h}_i$ from all schemes. Otherwise, the MSE calculation will not be accurate. However, in practice, the light intensity emitted from an LED depends on several factors such as the temperatures of LEDs which may not be a constant over time.

Consider the following experiment which is done in a dark room with only one LED. The measured background light intensity is always zero in this room. Let P_0 and P_τ be the light intensity measured at the beginning and after τ seconds, respectively, where $\tau \geq 30$ seconds. We find that

$$\left|\frac{P_0 - P_\tau}{P_0}\right|^2$$

can have values larger than 10^{-5} as shown in Fig. 6. It means that if it takes more than 30 seconds to obtain $\hat{h}_i$ after h_i is got, the error introduced by the fluctuation in the light intensity has a magnitude comparable to the MSE values obtained in Fig. 5. In this case, the result becomes inaccurate.

In the scheme described in Section V-B, h_i and $\hat{h}_i$ are obtained from measurements taken within 20 ms. This can reduce the effect due to the fluctuation in the LED intensity. The method in Section V-B enables us to demonstrate that combinational code can achieve MSE around 10^{-6} as shown in Fig. 5.

VII. Conclusion

This paper has investigated how to reduce noise variance in channel gain estimation for communications and positioning systems using visible light. For given maximum and average power constraints in VLC systems, combinational code can theoretically attain the minimum values of both total noise variance and maximum noise variance. This paper has reported experimental results to verify this claim. Two schemes based on TDMA have been considered. We have shown that combinational code outperforms theses two schemes in terms of both total noise variance and MSE. The challenges in evaluating the MSE has been explained and a method has been demonstrated to overcome the challenges.

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