Distributed Power Allocation for the Downlink of a Two-cell MISO-NOMA System

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***Abstract*—In this paper, we investigate the distributed power allocation algorithm for the downlink of a two-cell multiple input and single output non-orthogonal multiple access (MISO-NOMA) system. The problem targets at minimizing the total power consumption of the base stations (BSs) while taking into consideration each user's data rate requirement. A distributed power control algorithm is devised. During each iteration, the BS updates the transmit power of its attached users according to the link gain vector and the inter-cell interference plus noise value at the users. For some special cases, we show that the proposed algorithm is guaranteed to converge to a unique fixed point that could be an optimal solution based on Yate's power control framework. Furthermore, some modifications are made for the iterative algorithm to enhance the convergence performance of the instances with feasible solutions. Simulation results demonstrate that the designed power allocation strategy can significantly improve system performance over conventional orthogonal multiple access (OMA) counterpart in terms of total transmit power and outage probability.**

***Index Terms*—NOMA, multiple-input single-output (MISO), Yate's power control framework, distributed power control.**

I. Introduction

The increasing demand of data traffic (increase by 1000-fold by 2020) and expected new services, such as internet-of-things (IoT) and cloud-based architectural applications, have imposed challenging requirements for the fifth generation (5G) wireless cellular systems [1]. It becomes necessary to seek efficient multiple access techniques as it is the key to meet the enormously increasing bandwidth demand. Non-orthogonal multiple access (NOMA) has been regarded as an enabler for the deployment of 5G cellular networks and been received significant attention. In contrast to conventional orthogonal multiple access (OMA) schemes where each resource block is exclusively used by at most one user, NOMA can support multiple users by performing non-orthogonal resource assignment, which results in a higher system spectral efficiency [1]. Besides, it is worth mentioning that NOMA has been proposed as a radio access technique for downlink scenarios in long-term evolution (LTE) systems by the third generation partnership project (3GPP) [2].

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In downlink NOMA systems, successive interference cancellation (SIC) is adopted at the users to mitigate co-channel interference. Therefore, the transmit power control among multiplexed users becomes essential to ensure that some of the users can correctly decode and subtract the interfering signals from its received signal. Power control for uplink and downlink single-antenna NOMA systems was investigated in [3], [4] and [5]–[7], respectively.

From a technical perspective, multiple-input and multiple-output (MIMO) is another potential technology to meet 5G challenges. The application of MIMO to NOMA system could enhance the performance of NOMA [8], [9]. The concept of MIMO-NOMA was initially proposed in [10], which shows that MIMO-NOMA outperforms conventional MIMO-OMA. This has attracted considerable attentions [11]–[14]. For example, [11] studies the rate maximization problem for MIMO-NOMA networks with total power budget and weak users' data rate requirements taken into account. A NOMA beamforming (NOMA-BF) system is designed in [12], in which a BF vector accommodates two users. To alleviate the effect of interference and improve system capacity, the authors designed a joint user grouping and power control algorithm. In [13], the minimum power beamforming problem with users' required target rates is studied. The author solves the problem through a nonlinear iterative algorithm (Gauss-Seidel algorithm). However, the optimality of such method is not guaranteed. In [14], the minimum power and the optimal precoding vector of a two-user MISO-NOMA system is obtained by using Lagrange duality and Newton's iterative algorithm. All the discussed works [11]–[14], focus on single-cell MISO-NOMA systems.

With aforementioned observations, we investigate the power allocation problem for the downlink of a two-cell MISO-NOMA network in this work. The goal is minimizing the total power consumption of the BSs taking into consideration each user's data rate requirement. An iterative distributed power allocation algorithm is designed. For some special cases, we prove that the algorithm could converge to the unique fixed point that might be an optimal solution based on Yate's power control framework [15]. Additionally, some modifications are made for our iterative algorithm to enhance its convergence performance. Finally, numerical results confirm that the pro-

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posed iterative algorithm can improve the performance of MISO-NOMA over the power controlled OMA scheme significantly.

The rest parts of this work is organized as follows: In Section II, we introduce the MISO-NOMA system model and formulate the minimization problem mathematically. Section III presents a closed-form expression of the minimal power required of a BS satisfying its attached users' data rate requirements under the assumptions of fixed decoding order and inter-cell interference. In addition, the optimal decoding order is defined. The devised iterative power allocation method and its convergence analysis are given in Section IV. In Section V, computer simulations are conducted to compare the performance of the designed power control algorithm and OMA scheme. Finally, we conclude in Section VI.

II. System Model and Problem Formulation

In this section, we first give the MISO-NOMA system description and then formulate the power minimization problem.

A. System Model

Consider a downlink MISO-NOMA network which has two interfering cells. Assume that each cell contains one base station (BS) with an antenna array of N elements for downlink transmission and two single-antenna users. Denote two cells by cells 1 and 2 and assume BS 1 serves users 1 and 2, while users 3 and 4 are associated with BS 2. Define $\boldsymbol{g}_i \in \mathbb{C}^{N\times 1}$ as the link gain vector between user i and its associated BS, where $i \in \{1,2,3,4\}$. In addition, denote by $\boldsymbol{g}_{i,j} \in \mathbb{C}^{N\times 1}$ the channel vector from BS j to user i, where $j = \{1,2\}$. Besides, let $\boldsymbol{w}_i \in \mathbb{C}^{N\times 1}$ be the beamforming vector for user i, where $i \in \{1,2,3,4\}$.

In the NOMA system, each BS transmits independent messages to its associated users through superposition coding. Then, the signal vectors transmitted by BS 1 and BS 2 respectively are given by

$$\boldsymbol{x}_1 = \sqrt{p_1}\boldsymbol{w}_1 s_1 + \sqrt{p_2}\boldsymbol{w}_2 s_2, \tag{1}$$
$$\boldsymbol{x}_2 = \sqrt{p_3}\boldsymbol{w}_3 s_3 + \sqrt{p_4}\boldsymbol{w}_4 s_4, \tag{2}$$

in which p_i indicates the transmit power for user i. s_i represents the desired signal for user i, where $i \in \{1,2,3,4\}$.

We assume a block fading channel is used in this work. Therefore, the received signal of each user can be given as follows:

$$y_i = \boldsymbol{g}_i^H \boldsymbol{x}_1 + \boldsymbol{g}_{i,2}^H \boldsymbol{x}_2 + n_i, \text{ for } i \in \{1,2\}, \tag{3}$$
$$y_j = \boldsymbol{g}_j^H \boldsymbol{x}_2 + \boldsymbol{g}_{j,1}^H \boldsymbol{x}_1 + n_j, \text{ for } j \in \{3,4\}, \tag{4}$$

in which y_l and n_l indicates the received message and noise at user l, $l \in \{1,2,3,4\}$. $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are given by (1) and (2), respectively. In addition, H denotes the Hermitian transpose. We assume the noise is additive white Gaussian noise (AWGN) with zero mean and variance σ_l^2, $l \in \{1,2,3,4\}$.

In each cell, SIC is performed among the multiplexed users. Therefore, we should consider the decoding order of the users for each cell. For cell 1, define Π_1 as the set of all possible permutations of $\{1,2\}$, i.e., $\Pi_1 = \{(1,2),(2,1)\}$. Denote by $\boldsymbol{\pi}_1 \in \Pi_1$ the decoding order of users in cell 1. For $i \in \{1,2\}$, let $\pi_1(i)$ be its i-th component, i.e., $\boldsymbol{\pi}_1 = (\pi_1(1), \pi_1(2))$, which means user $\pi_1(1)$ decode its signal by treating the signal of user $\pi_1(2)$ as noise, meanwhile user $\pi_1(2)$ first decodes the signal of user $\pi_1(1)$, subsequently subtracting this part, and finally decodes its desired signal. This is the principle of SIC [10]. We define Π_2 and $\boldsymbol{\pi}_2$ for cell 2 in the same way.

In this work, the inter-cell interference of each user is regarded as AWGN. For cell 1, let $I_{\pi_1(i)}$ be the inter-cell interference plus noise of user $\pi_1(i)$, and it is represented as follows:

$$I_{\pi_1(i)} \triangleq p_3|\boldsymbol{g}_{\pi_1(i),2}^H \boldsymbol{w}_3|^2 + p_4|\boldsymbol{g}_{\pi_1(i),2}^H \boldsymbol{w}_4|^2 + \sigma_{\pi_1(i)}^2, \tag{5}$$

where $i \in \{1,2\}$.

Similarly, we define

$$I_{\pi_2(i)} \triangleq p_1|\boldsymbol{g}_{\pi_2(i),1}^H \boldsymbol{w}_1|^2 + p_2|\boldsymbol{g}_{\pi_2(i),1}^H \boldsymbol{w}_2|^2 + \sigma_{\pi_2(i)}^2 \tag{6}$$

as the inter-cell interference plus noise of user $\pi_2(i)$ who is attached to BS 2, where $i \in \{1,2\}$.

For each cell, denote by θ_j the signal to interference plus noise ratio (SINR) of user j who is not performing SIC to decode s_j. In addition, for user i who performs SIC, let ξ_i and γ_i be the SINRs of user i to decode s_{-i} and to decode s_i after subtracting s_{-i} using SIC, respectively, where user $-i$ indicates the other user who is associated to the same cell with user i. We have auxiliary functions as follows:

$$\theta_{\pi_j(1)}(\boldsymbol{\pi}_j, \boldsymbol{p}, \boldsymbol{w}) = \frac{p_{\pi_j(1)}|\boldsymbol{g}_{\pi_j(1)}^H \boldsymbol{w}_{\pi_j(1)}|^2}{p_{\pi_j(2)}|\boldsymbol{g}_{\pi_j(1)}^H \boldsymbol{w}_{\pi_j(2)}|^2 + I_{\pi_j(1)}}, \tag{7}$$

$$\xi_{\pi_j(2)}(\boldsymbol{\pi}_j, \boldsymbol{p}, \boldsymbol{w}) = \frac{p_{\pi_j(1)}|\boldsymbol{g}_{\pi_j(2)}^H \boldsymbol{w}_{\pi_j(1)}|^2}{p_{\pi_j(2)}|\boldsymbol{g}_{\pi_j(2)}^H \boldsymbol{w}_{\pi_j(2)}|^2 + I_{\pi_j(2)}}, \tag{8}$$

$$\gamma_{\pi_j(2)}(\boldsymbol{\pi}_j, \boldsymbol{p}, \boldsymbol{w}) = \frac{p_{\pi_j(2)}|\boldsymbol{g}_{\pi_j(2)}^H \boldsymbol{w}_{\pi_j(2)}|^2}{I_{\pi_j(2)}}, \tag{9}$$

in which $\boldsymbol{\pi}_j \in \Pi_j$, $j \in \{1,2\}$, $\boldsymbol{p} \triangleq (p_1,p_2,p_3,p_4)$, and $\boldsymbol{w} \triangleq (\boldsymbol{w}_1,\boldsymbol{w}_2,\boldsymbol{w}_3,\boldsymbol{w}_4)$.

For $i,j \in \{1,2\}$, denote by $R_{\pi_j(i)}$ the data rate of user $\pi_j(i)$ in cell j. Assume capacity-achieving coding scheme is applied; therefore, the data rate of each user can be expressed by Shannon capacity formula. For user $\pi_j(2)$, we have

$$R_{\pi_j(2)} = C(\gamma_{\pi_j(2)}(\boldsymbol{\pi}_j, \boldsymbol{p}, \boldsymbol{w})), \tag{10}$$

in which $C(\chi) \triangleq B\log_2(1+\chi)$, where B and χ denote the system bandwidth and the SINR, respectively. Note that, since the signal of user $\pi_j(1)$ should be decodable at user $\pi_j(1)$ and $\pi_j(2)$, respectively, $R_{\pi_j(1)}$ is upper-bounded by

$$R_{\pi_j(1)} \leq \min\{C(\theta_{\pi_j(1)}(\boldsymbol{\pi}_j, \boldsymbol{p}, \boldsymbol{w})), C(\xi_{\pi_j(2)}(\boldsymbol{\pi}_j, \boldsymbol{p}, \boldsymbol{w}))\}. \tag{11}$$

B. Problem Formulation

This article targets minimizing the total power consumption subject to each user's data rate constraint. Mathematically, it is formulated as

$$\min ||\boldsymbol{p}||_1 \tag{12}$$

subject to

$$C1: \ C(\theta_{\pi_j^*(1)}(\boldsymbol{\pi}_j^*, \boldsymbol{p}, \boldsymbol{w})) \geq \bar{R}_{\pi_j^*(1)}, \ j \in \{1,2\}, \quad (13)$$

$$C2: \ C(\xi_{\pi_j^*(2)}(\boldsymbol{\pi}_j^*, \boldsymbol{p}, \boldsymbol{w})) \geq \bar{R}_{\pi_j^*(1)}, \ j \in \{1,2\}, \quad (14)$$

$$C3: \ C(\gamma_{\pi_j^*(2)}(\boldsymbol{\pi}_j^*, \boldsymbol{p}, \boldsymbol{w})) \geq \bar{R}_{\pi_j^*(2)}, \ j \in \{1,2\}, \quad (15)$$

where $\bar{R}_{\pi_j^*(i)}$ represents the data rate requirement of user $\pi_j^*(i)$ who is attached to BS j, in which $i = \{1,2\}$ and $j = \{1,2\}$. $\boldsymbol{\pi}_j^*$ indicates the optimal decoding order of users in cell j in terms of power consumption, where $j = \{1,2\}$. $||\boldsymbol{p}||_1$ is the l_1-norm of vector $\boldsymbol{p}$, and it is defined as the sum of all components in $\boldsymbol{p}$. (13) is applied to guarantee that user $\pi_j^*(1)$ could decode its own signal while taking the message of user $\pi_j^*(2)$ as noise. Meanwhile, (14) is necessary at user $\pi_j^*(2)$ so that the signal of user $\pi_j^*(1)$ is decodable and canceled by SIC in decoding $s_{\pi_j^*(2)}$. In addition, for successfully decoding the signal of user $\pi_j^*(2)$ after SIC, (15) is applied.

III. Minimal Transmit Power of a Base station and the Optimal Decoding Order

In this section, we first determine the minimal power needed of a BS to satisfy the required data rate of its associated users under the given decoding order and beamforming vectors, and then derive the optimal decoding order which minimizes the total power consumption.

For convenience, let

$$\Gamma_i \triangleq 2^{\frac{\bar{R}_i}{B}} - 1, \ i \in \{1,2,3,4\} \quad (16)$$

be the corresponding SINR criteria for user i, due to $\bar{R}_i$. Therefore, using (13)-(15), the data rate requirements of each user can be re-written as the following SINR requirements:

$$\theta_{\pi_j(1)}(\boldsymbol{\pi}_j, \boldsymbol{p}, \boldsymbol{w}) \geq \Gamma_{\pi_j(1)}, \quad (17)$$

$$\xi_{\pi_j(2)}(\boldsymbol{\pi}_j, \boldsymbol{p}, \boldsymbol{w})\} \geq \Gamma_{\pi_j(1)}, \quad (18)$$

$$\gamma_{\pi_j(2)}(\boldsymbol{\pi}_j, \boldsymbol{p}, \boldsymbol{w}) \geq \Gamma_{\pi_j(2)}. \quad (19)$$

Consider an arbitrary cell j, $j \in \{1,2\}$. Assume we fix the decoding order $\boldsymbol{\pi}_j$, and we need to satisfy the SINR requirements of its two attached users, i.e., (17)-(19). Define $f_j(\boldsymbol{\pi}_j, \boldsymbol{p})$ as the minimum needed power of BS j to meet (17)-(19), given that beamforming vectors $\boldsymbol{w}$ and the transmit power in the other cell are fixed. Therefore, we have the following result:

Lemma 1. *Given any $\boldsymbol{\pi}_j \in \Pi_j$, we have*

$$f_j(\boldsymbol{\pi}_j, \boldsymbol{p}) = p^*_{\pi_j(1)} + p^*_{\pi_j(2)}, \quad (20)$$

where

$$p^*_{\pi_j(2)} = \frac{I_{\pi_j(2)}}{|\boldsymbol{g}^H_{\pi_j(2)} \boldsymbol{w}_{\pi_j(2)}|^2} \Gamma_{\pi_j(2)}, \quad (21)$$

$$p^*_{\pi_j(1)} = \Gamma_{\pi_j(1)} \cdot \max \left\{ \frac{p^*_{\pi_j(2)} |\boldsymbol{g}^H_{\pi_j(1)} \boldsymbol{w}_{\pi_j(2)}|^2 + I_{\pi_j(1)}}{|\boldsymbol{g}^H_{\pi_j(1)} \boldsymbol{w}_{\pi_j(1)}|^2}, \frac{p^*_{\pi_j(2)} |\boldsymbol{g}^H_{\pi_j(2)} \boldsymbol{w}_{\pi_j(2)}|^2 + I_{\pi_j(2)}}{|\boldsymbol{g}^H_{\pi_j(2)} \boldsymbol{w}_{\pi_j(1)}|^2} \right\}. \quad (22)$$

Proof: Based on (9) and (19), we can obtain the power constraint of user $\pi_j(2)$ and it is given by

$$p_{\pi_j(2)} \geq \frac{I_{\pi_j(2)}}{|\boldsymbol{g}^H_{\pi_j(2)} \boldsymbol{w}_{\pi_j(2)}|^2} \Gamma_{\pi_j(2)}, \quad (23)$$

which indicates that the minimum transmit power of user $\pi_j(2)$ is achieved when the equality in (23) holds, i.e., (21).

Additionally, we have the other two conditions (17) and (18); combining them with (7), (8) and the achieved $p^*_{\pi_j(2)}$ in (21), we have

$$p_{\pi_j(1)} \geq \frac{p^*_{\pi_j(2)} |\boldsymbol{g}^H_{\pi_j(1)} \boldsymbol{w}_{\pi_j(2)}|^2 + I_{\pi_j(1)}}{|\boldsymbol{g}^H_{\pi_j(1)} \boldsymbol{w}_{\pi_j(1)}|^2} \Gamma_{\pi_j(1)}, \quad (24)$$

and also

$$p_{\pi_j(1)} \geq \frac{p^*_{\pi_j(2)} |\boldsymbol{g}^H_{\pi_j(2)} \boldsymbol{w}_{\pi_j(2)}|^2 + I_{\pi_j(2)}}{|\boldsymbol{g}^H_{\pi_j(2)} \boldsymbol{w}_{\pi_j(1)}|^2} \Gamma_{\pi_j(1)}, \quad (25)$$

which imply (22). This completes the proof. □

For cell j, the minimum transmit power of its attached users with fixed decoding order can be obtained according to (21) and (22) in Lemma 1, respectively. The sum of these two values equals the power consumption of BS j. Therefore, the minimum transmit power of BS j can be found by searching decoding orders $\boldsymbol{\pi}_j \in \Pi_j$. The optimal decoding order, $\boldsymbol{\pi}_j^*$, is related to this minimum total transmit power instance. That is,

$$\boldsymbol{\pi}_j^* \triangleq \underset{\boldsymbol{\pi}_j \in \Pi_j}{\operatorname{argmin}} f_j(\boldsymbol{\pi}_j, \boldsymbol{p}), \quad (26)$$

where $j \in \{1,2\}$ and $f_j(\boldsymbol{\pi}_j, \boldsymbol{p})$ is given by (20).

IV. Distributed Power Allocation Algorithm

In this section, the proposed iterative distributed power allocation algorithm is first introduced. Then, we show that there exists some cases where the iterative algorithm converges to a unique fixed point that could be an optimal solution to problem (12). Based on which, some modifications are made to deal with the non-convergent but feasible cases, which could increase the convergence probability of such instances. We assume fixed beamforming scheme, matched filter (MF) beamfomer is used in the NOMA system.

Let $p_i^{(t)}$ for $i \in \{1,2,3,4\}$ be the transmit power of user i at iteration t. Denote by $\boldsymbol{p}^{(t)} \triangleq (p_1^{(t)}, p_2^{(t)}, p_3^{(t)}, p_4^{(t)})$ and $\boldsymbol{p}_j^{(t)} \triangleq (p^{(t)}_{\pi_j^*(1)}, p^{(t)}_{\pi_j^*(2)})$. Let $I^{(t)}_{\pi_j(i)}$ be the inter-cell interference plus noise of user $\pi_j(i)$ at t-th iteration, which are given by (5) and (6). At iteration t, the BS performs power allocation so as that the SINR requirements of its associated users are satisfied. With the aforementioned definitions, the iterative power allocation algorithm is expressible as follows:

$$p^{(t)}_{\pi_j^*(2)} = \frac{I^{(t-1)}_{\pi_j^*(2)}}{|\boldsymbol{g}^H_{\pi_j^*(2)} \boldsymbol{w}_{\pi_j^*(2)}|^2} \Gamma_{\pi_j^*(2)}, \quad (27)$$

$$p^{(t)}_{\pi_j^*(1)} = \Gamma_{\pi_j^*(1)} \cdot \max \left\{ \frac{p^{(t)}_{\pi_j^*(2)} |\boldsymbol{g}^H_{\pi_j^*(1)} \boldsymbol{w}_{\pi_j^*(2)}|^2 + I^{(t-1)}_{\pi_j^*(1)}}{|\boldsymbol{g}^H_{\pi_j^*(1)} \boldsymbol{w}_{\pi_j^*(1)}|^2}, \frac{p^{(t)}_{\pi_j^*(2)} |\boldsymbol{g}^H_{\pi_j^*(2)} \boldsymbol{w}_{\pi_j^*(2)}|^2 + I^{(t-1)}_{\pi_j^*(2)}}{|\boldsymbol{g}^H_{\pi_j^*(2)} \boldsymbol{w}_{\pi_j^*(1)}|^2} \right\}, \quad (28)$$

where $j \in \{1,2\}$, $\boldsymbol{\pi}_j^* = \text{argmin}_{\boldsymbol{\pi}_j \in \Pi_j} f_j(\boldsymbol{\pi}_j, \boldsymbol{p}^{(t-1)})$.

Obviously, to apply this power allocation scheme, BS j needs the following two pieces of information, i.e., 1) the intra-cell link gain vectors, $\boldsymbol{g}_{\pi_j(i)}$, $i \in \{1,2\}$, and 2) the inter-cell interference plus noise value at the users who are served by BS j, i.e., $I_{\pi_j(i)}$, $i \in \{1,2\}$. All these informations can be estimated within each cell, control information exchanges among BSs is not needed. Therefore, the designed power allocation method is distributed.

Noted that this distributed power allocation scheme is not always guaranteed to converge. However, there exists some cases (conditions) where the algorithm converges to a unique fixed point that might be the optimal solution. To elaborate this, we start from Yate's power control framework.

According to [15], the users' SINR constraints (17), (18) and (19) in problem (12) can be re-written into the following form:

$$\boldsymbol{p} \succeq \mathbf{I}(\boldsymbol{p}), \tag{29}$$

where $\mathbf{I}(\boldsymbol{p})$ represents the interference function and its i-th component indicates the total interference that user i needs to overcome such that its required SINR is satisfied. It follows that the corresponding power optimization problem is solvable via an iterative power allocation algorithm which is given by

$$\boldsymbol{p}^{(t+1)} = \mathbf{I}(\boldsymbol{p}^{(t)}). \tag{30}$$

In addition, we say $\mathbf{I}(\boldsymbol{p})$ is standard if for all $\boldsymbol{p} \succeq \mathbf{0}$, the following three criteria are satisfied [15].

(1) *Positivity:* $\mathbf{I}(\boldsymbol{p}) \succ \mathbf{0}$.
(2) *Monotonicity:* If $\boldsymbol{p} \succeq \boldsymbol{p}'$, then $\mathbf{I}(\boldsymbol{p}) \succeq \mathbf{I}(\boldsymbol{p}')$.
(3) *Scalability:* For all $\alpha > 1$, then $\alpha\mathbf{I}(\boldsymbol{p}) \succ \mathbf{I}(\alpha\boldsymbol{p})$.

If $\mathbf{I}(\boldsymbol{p})$ is standard, the power allocation strategy in (30) is called *standard power control algorithm*. It was proved in [15] that the algorithm converges to a unique fixed point given that the problem has feasible solution. The following lemma depicts the existence of the optimal solution to problem (12).

Lemma 2. *Suppose the feasible region of problem* (12) *is non-empty. Then, there exists an optimal solution.*

Proof: Since (12) is feasible, there exists some decoding orders and power vectors such that (13)-(15) are satisfied. For the fixed feasible decoding order, problem (12) becomes a linear programming problem. According to [16, Corrolary 2.3], either the optimal value is $-\infty$ or there exists an optimal solution. Since the power vector in problem (12) is bounded below by zero vector and there are at most four feasible decoding orders, our problem has an optimal solution. □

Next, we show that the optimal solution is a fixed point.

Theorem 3. *The optimal solution* $(\boldsymbol{\pi}^*, \boldsymbol{p}^*)$ *to problem* (12) *is a fixed point, where* $\boldsymbol{\pi}^* = (\boldsymbol{\pi}_1^*, \boldsymbol{\pi}_2^*)$ *and* $\boldsymbol{p}^* = (p_1^*, p_2^*, p_3^*, p_4^*)$.

Proof: We prove this by contradiction and assume $\boldsymbol{p}^*$ is not a fixed point. Let $\boldsymbol{p}^*$ be the initial power vector of the iterative power allocation method (27)-(28) with fixed decoding order $\boldsymbol{\pi}^*$. Since $\boldsymbol{p}^*$ is a feasible vector, we have

$$\boldsymbol{p}^{(0)} = \boldsymbol{p}^* \succeq \mathbf{I}(\boldsymbol{p}^*) = \boldsymbol{p}^{(1)}. \tag{31}$$

Since $\boldsymbol{p}^*$ is not a fixed point, there exists at least one user satisfies $p_i^* > p_i^{(1)}$, which contradicts that $\boldsymbol{p}^*$ is an optimal solution. □

Therefore, we can draw the following result by the aforementioned framework.

Theorem 4. *With fixed decoding order, the distributed power control algorithm in* (27)-(28) *will converge to a unique fixed point provided that problem* (12) *is feasible under the given decoding order. Besides, if the given decoding order is optimal, then the algorithm will converge to the optimal solution.*

Proof: The interference functions of users $\pi_j^*(2)$ and $\pi_j^*(1)$ is given by (27) and (28), respectively. For fixed $\boldsymbol{\pi}_j^*$, $j \in \{1,2\}$, it can be seen that (27) is an affine function of the transmit power vector of the other cell. It can be easily confirmed that the aforementioned criteria of standard interference function (SIF) are satisfied; that is to say, the interference function of user $\pi_j^*(2)$ is standard.

Substituting (27) into (28), we can see that each component of (28) is also an affine function of the transmit power of users in the other cell and satisfies the three criteria of SIF. Based on [15, Theorem 5], the maximization of two standard functions is also standard. Therefore, the interference function of user $\pi_j^*(1)$ is also standard. In accordance with [15, Theorem 2], the algorithm will converge to the unique fixed point from any initial power vector. In accordance with Theorem 3, the algorithm will converge to the optimal solution given that the fixed decoding order is optimal. □

Since the varying decoding orders during each iteration of power allocation algorithm (27)-(28) is the key influencing factor of convergence, some modifications are made in order to deal with the non-convergent but feasible cases to complete the scheme. Let $p_{r,j}^{(t)}$ be the probability that BS j follows the algorithm (27)-(28) at iteration t. Otherwise (i.e., with probability $1 - p_{r,j}^{(t)}$), it will keep the decoding order to be the same as in the previous iteration, and only adjusts the transmit power. We assume $p_{r,j}^{(t+1)} = \alpha_j p_{r,j}^{(t)}$, where $\alpha_j \in (0,1)$ can be regarded as the decay factor of user j and $p_{r,j}^{(0)} = 1$ for $j \in \{1,2\}$. In other words, we define $(p_{r,j}^{(1)}, p_{r,j}^{(2)}, \ldots)$ as a decreasing sequence that converges to 0. When t becomes large, each base station will eventually keep its decoding order unchanged, and the improved power control algorithm will guarantee to converge with the assumption that the problem is feasible under the unchangeable decoding order.

We name the improved algorithm as generalized distributed power control (GDPC) algorithm and let $\boldsymbol{\pi}_j^{(t)} = (\pi_j^{(t)}(1), \pi_j^{(t)}(2))$ be the decoding order of cell j at iteration t. The pseudo-code of GDPC for the power update of BS j at iteration t is presented in Algorithm 1.

V. Simulation Results

In this section, we evaluate the system performance of the proposed GDPC algorithm by extensive Monte Carlo simulations. A MISO-NOMA network with two neighboring hexagonal cells is considered, in which each cell contains a

Algorithm 1 The power update of BS j at iteration t

Input: $\boldsymbol{g}_{\pi_j^{(t-1)}(i)}$ and $I^{(t-1)}_{\pi_j^{(t-1)}(i)}$ for $i \in \{1,2\}$ and $p_{r,j}^{(t)}$.

Output: $\boldsymbol{p}_j^{(t)}$.

Generate a random number ξ taken uniformly in $[0,1]$.
if $\xi < p_{r,j}^{(t)}$ **then**
  Determine the decoding order, $\boldsymbol{\pi}_j^{(t)}$, according to criterion (26), i.e.,

$$\boldsymbol{\pi}_j^{(t)} = \operatorname*{argmin}_{\boldsymbol{\pi}_j \in \Pi_j} f_j(\boldsymbol{\pi}_j, \boldsymbol{p}^{(t-1)}).$$

else
  $\boldsymbol{\pi}_j^{(t)} = \boldsymbol{\pi}_j^{(t-1)}$.
end if
Calculate the optimal minimum transmit power of user $\boldsymbol{\pi}_j^{(t)}(2)$ and $\boldsymbol{\pi}_j^{(t)}(1)$ according to (27) and (28), respectively.
return $(p^{(t)}_{\pi_j^{(t)}(1)}, p^{(t)}_{\pi_j^{(t)}(2)})$.

TABLE I
SIMULATION PARAMETERS

Parameters	Value
Cell radius	1000 m
Minimum distance from user to BS	35 m
Distance dependent path loss	$128.1 + 37.6 \log_{10} d$ dB, d is in km
Shadowing of each user	Log-normal, standard deviation 10 dB
Small-scale fading	Rayleigh fading with variance 1
Users distribution scheme	Randomly uniform distribution
Noise power spectral density, N_0	-174 dBm/Hz
System bandwidth, W	10 MHz
Number of transmit antenna, N	2, 3
Data rate requirement, R	1 Mbits/s to 10 Mbit/s
Throughput calculation	Shannon's capacity formula
The decay factor, α_j for $j \in \{1,2\}$	0.99
Termination condition epsilon	10^{-4}
Maximum number of iterations	50

cell center located BS and two uniformly distributed users. The radius of each cell is 1000 m. The system bandwidth W and the noise density N_0 are set to 10 MHz and -174 dBm/Hz, respectively. The noise power can be achieved by multiplying W and N_0. For simplicity, the date rate requirement of each user is assumed to be the same, i.e., $\bar{R}_i = \bar{R}$, $\forall i$. Three factors are included to generate the link gain information, i.e., the distance dependent path loss, the shadowing and the small-scale fading. Specifically, the path loss is given by $128.1 + 37.6 \log_{10} d$, where d represents the distance between the BS and the user and it is measured in km [17]. The correlation of shadowing among intra-cell users is considered, which is given by $(X_i + X)/\sqrt{2}$ for user $i \in \{1,2\}$, where X_i and X represent the independent and identical log-normal random variables, respectively. For users 3 and 4, the shadowing components can be defined similarly. Besides, independent Rayleigh fading with variance 1 is computed to represent the small-scale fading of each user. The simulation parameters are set following [17] and summarized in Table I.

We compare the performance of our GDPC algorithm in NOMA system with fixed MF beamforming (denoted by

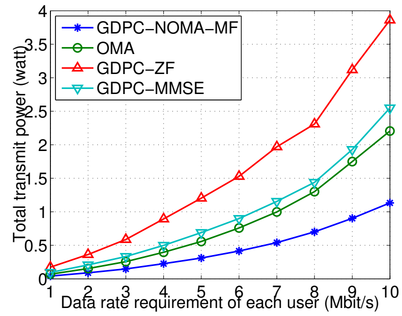

Fig. 1. Total transmit power vs. data rate requirement, $N = 2$

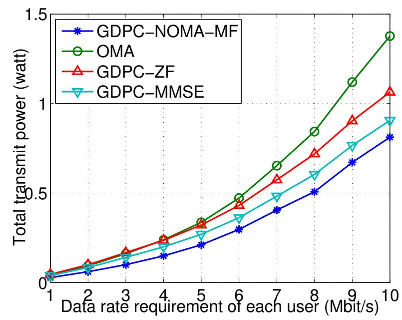

Fig. 2. Total transmit power vs. data rate requirement, $N = 3$

GDPC-NOMA-MF) to the conventional orthogonal scheme, denoted by OMA. In OMA scheme, we assume two time slots according to 3GPP-LTE and MF beamformer is also used. Users 1 and 3 are active in the first time slot, while users 2 and 4 are active in the second time slot. Since Zero-Forcing (ZF) and Minimum Mean Square Error (MMSE) beamforming are two common techniques to mitigate intra-cell interference [18], we also combine the GDPC with these two beamforming schemes as benchmarks, denoted by GDPC-ZF and GDPC-MMSE, respectively.

In the following, we evaluate the performance of the aforementioned four schemes from two aspects, i.e., 1) the total power consumption, and 2) the outage probability.

A. Total Transmit Power

In this subsection, we compare the total power consumption of four schemes. For different number of antennas and each target rate, we generate randomly numerous instances in accordance with Table I, until there are $10,000$ instances in which all the four schemes are feasible. Denote by N_{total} the total number of needed instances. Each point in Fig. 1 and Fig. 2 is the average of these $10,000$ feasible values.

Fig. 1 and Fig. 2 show the total transmit power versus the data rate requirements among four different schemes. Since each BS serves two users, the antenna number of Fig. 1 and Fig. 2 is set to be 2 and 3, respectively. We can see that GDPC-NOMA-MF outperforms the other three schemes significantly especially for the cases with high data rate requirements. For example, when $\bar{R}$ is set to be 10 Mbit/s and $N = 2$, GDPC-

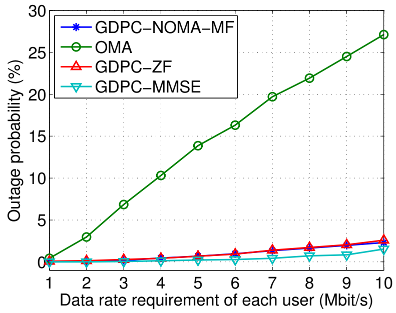

Fig. 3. Outage probability vs. data rate requirement, $N = 2$

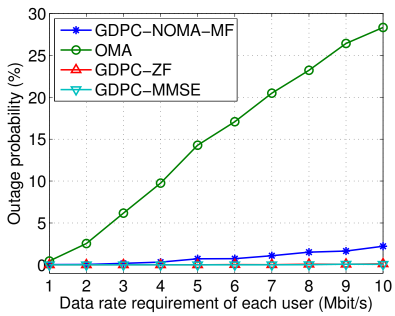

Fig. 4. Outage probability vs. data rate requirement, $N = 3$

NOMA-MF saves power by 48.66%, 70.68% and 55.60% in comparison to OMA, GDPC-ZF and GDPC-MMSE, respectively. Besides, it is worth mentioning that GDPC-MMSE and GDPC-ZF can achieve a better performance than OMA when the number of antenna is larger, i.e., $N = 3$. The reason is, MMSE and ZF become more efficient with the increase of antenna number. Note that for different $\bar{R}$ and N, GDPC-MMSE always outperform GDPC-ZF, see Fig. 1 and Fig. 2. The reason is, ZF does not consider the effect of noise and the resultant noise enhancement factor [18] can introduce serious damage to system performance.

B. Outage Performance

Fig. 3 and Fig. 4 compare the outage performance of the four schemes, in which the antenna number is $N = 2$ and $N = 3$, respectively. For each resource allocation scheme, the outage probability is obtained by dividing the number of infeasible instances by the aforementioned N_{total}. Obviously, with the increasing of required data rate $\bar{R}$, the outage probabilities of all the four schemes increase. We can see that OMA has the worst outage performance of all. Meanwhile, GDPC-NOMA-MF has near outage performance to that of GDPC-ZF and GDPC-MMSE.

VI. Conclusion

We investigate the power minimization problem for a multi-cell MISO-NOMA system with users' data rate requirements taken into account in this work. An iterative distributed power allocation algorithm is designed. For some special cases, we show that the algorithm could converge to the unique fixed point which might be the optimal solution based on Yate's power control framework. Furthermore, some improvements are made for the distributed algorithm to enhance the convergence performance of the instances with feasible solutions. Simulation results illustrate that MISO-NOMA with power control can substantially outperform the power controlled OMA scheme. In addition, the performance of GDPC-ZF and GDPC-MMSE is also evaluated as benchmarks. We are now generalizing the results to the scenario with arbitrary number of cells and users.

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