Lattice-Superposition NOMA for Near-Far Users

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***Abstract*—A novel non-orthogonal multiple access scheme based on lattice superposition is proposed. Sixteen points from the D_4 lattice are picked as the signal constellation in the four-dimensional real space. Superposition coding for two users is applied at the downlink of the communication system. While the far user detects its intended signal directly, the near user performs interference cancellation before detects its own intended signal. The performance of our proposed scheme is compared with a benchmark orthogonal scheme using 16-QAM. When the link gain difference between the two users is large, our proposed lattice superposition scheme outperforms the benchmark significantly. In particular, when the link gain of the far user is 20 dB smaller than that of the near user, with a word error probability of 10^{-3}, an energy gain of 6 dB can be obtained.**

***Index Terms*—Non-orthogonal multiple access (NOMA), lattice, superposition coding, interference cancellation.**

I. Introduction

Non-Orthogonal Multiple Access (NOMA) is a promising candidate for 5G wireless communications because of its high spectral efficiency [1]. According to [2], NOMA can be broadly classified into power-domain NOMA and code-domain NOMA, depending on whether users are separated by using different power levels or using different codes. In this work, we consider power-domain NOMA. The principle of power-domain NOMA is based on superposition coding and successive interference cancellation, which has been known for many decades. From the viewpoint of information theory, it achieves the capacity of the Gaussian broadcast channel if Gaussian codebook with long block length is used [3]. In practice, it is inconvenient to use Gaussian codebook because of the lack of fast decoding algorithm. Besides, the use of long block length incurs long delay, which is undesirable considering the recent emphasis of short-delay applications. For this reason, we propose a simple practical scheme based on the use of lattices [4]. Different from the works [5], [6], which use lattice partition, our scheme is based on lattice superposition, which will be described in the next section. Afterwards, we will present our simulation results on its error performance. Our main contribution is the design and evaluation of a new NOMA scheme which can effectively reap the near-far gain induced by the difference of link quality between two users.

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II. Lattice Superposition

Consider the downlink of a two-user NOMA system. The base station transmits a signal vector $\boldsymbol{x} \in \mathbb{R}^L$, whose Euclidean norm is given by $\sqrt{E_T}$, where E_T is the total energy. The received vector of user i, where $i = 1, 2$, is given by

$$\boldsymbol{y}_i = h_i \boldsymbol{x} + \boldsymbol{n}_i,$$

where h_i is the amplitude gain and $\boldsymbol{n}_i$ is a random noise vector. We assume that the components of $\boldsymbol{n}_i$ are independent and identically distributed, each of which is a Gaussian random variable with mean zero and variance $N_0/2$, where N_0 is the noise spectral density. Without loss of generality, user 1 is assumed the near user while user 2 is the far user. The relative link gain in energy between the two users is $\beta \triangleq h_2^2/h_1^2 < 1$.

Superposition coding is used for transmission, i.e.,

$$\boldsymbol{x} = \sqrt{\alpha E_T}\boldsymbol{x}_1 + \sqrt{(1-\alpha)E_T}\boldsymbol{x}_2,$$

where $\boldsymbol{x}_i$ is the signal vector for user i, which is of unit norm, and $\alpha \le 1$ is the energy splitting factor between the two users.

We consider the case where $L = 4$. The signal constellation for each user consists of 16 points, which are chosen from the D_4 lattice. Recall that the D_4 lattice consists of all integer vectors in $\mathbb{R}^4$ with an even element sum [4]. We pick 16 points that are closest to the origin and normalize them by the factor of $1/\sqrt{2}$. The normalized 16 vectors are listed as the rows of the following matrix:

$$\boldsymbol{C} = \frac{1}{\sqrt{2}} \begin{bmatrix} \boldsymbol{C}_1 \\ \boldsymbol{C}_2 \end{bmatrix}, \text{ where } \boldsymbol{C}_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix},$$

and $\boldsymbol{C}_2$ is a 12×4 integer matrix, whose rows are the 12 weight-two vectors with one component equal to 1 and the other equal to -1. We call $\boldsymbol{C}$ the signal constellation matrix. Note that each of its rows is of unit norm, and the distance between any two rows is at least one. To transmit four data bits to user i, the transmitter let x_i be the row of $\boldsymbol{C}$ whose index has binary representation equal to the four data bits. In other words, eight data bits are transmitted in four-symbol duration, so the energy per bit, E_b, is given by $E_T/8$.

We assume that the receiver of user i knows the value of h_i. User 2, the far user, treats the intended signal for user 1 as noise, and performs nearest-neighbor detection by finding

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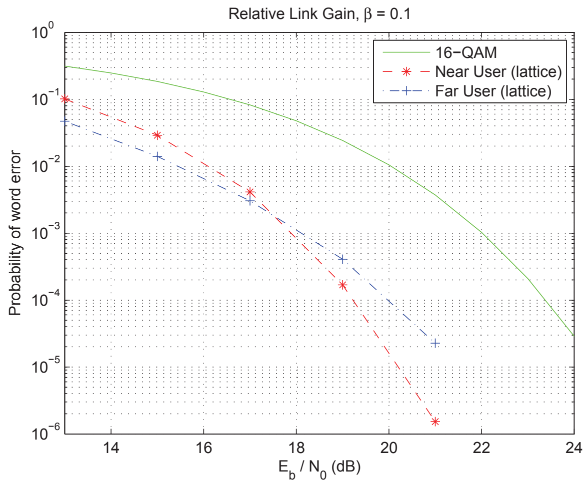

Fig. 1. Error Probabilities of lattice superposition and 16-QAM ($\beta = 0.1$).

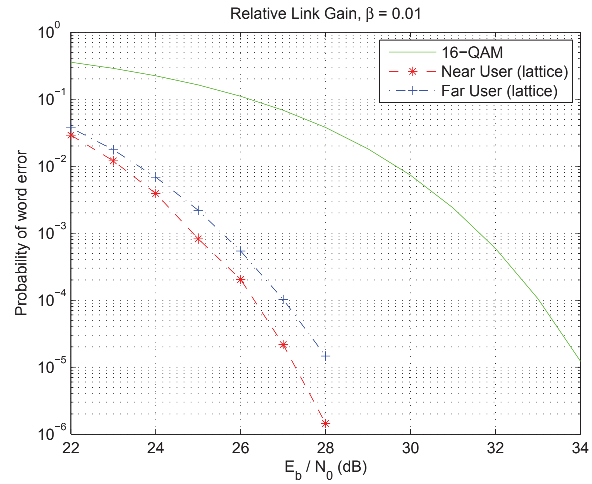

Fig. 2. Error Probabilities of lattice superposition and 16-QAM ($\beta = 0.01$).

the row vector in $h_2\sqrt{(1-\alpha)E_T}\boldsymbol{C}$ that is closest to $\boldsymbol{y}_2$. If the detected signal vector differs from the transmitted one, we say that a *word error* occurs at the far user, which means that one or more of the four data bits are in error. The probability of its occurrence is denoted by P_{w2}.

User 1, the near user, performs interference cancellation. It first detects the signal vector intended for user 2 by finding the row vector in $h_1\sqrt{(1-\alpha)E_T}\boldsymbol{C}$ that is closest to $\boldsymbol{y}_1$. Let it be $\hat{\boldsymbol{r}}$. It then subtracts $\hat{\boldsymbol{r}}$ from $\boldsymbol{y}_1$ to obtain

$$\boldsymbol{y}_1' = h_1\sqrt{\alpha E_T}\boldsymbol{x}_1 + \left(h_1\sqrt{(1-\alpha)E_T}\boldsymbol{x}_2 - \hat{\boldsymbol{r}}\right) + \boldsymbol{n}_1,$$

which is the received vector after interference cancellation. It then performs another nearest-neighbor detection by finding the row vector in $h_1\sqrt{\alpha E_T}\boldsymbol{C}$ that is closest to $\boldsymbol{y}_1'$. If the detected vector differs from the transmitted one, a *word error* occurs at the near user, which occurs with probability P_{w1}.

III. Simulation Results

In this section, we evaluate the performance of our proposed NOMA scheme with a benchmark orthogonal multiple access (OMA) scheme. Without loss of generality, the link gain for the near user is normalized to 1. We investigate how the probability of word error varies with E_b/N_0. For the OMA scheme, we assign two symbols to each of the two users. Each symbol is modulated by 4-PAM, so the two symbols together becomes 16-QAM, which represent four bits. Again a word error is defined as the event that one or more of the four bits are in error. The energy splitting factor, α, is chosen to be $\beta/(\beta+1)$, so that the two users under the OMA scheme have identical word error probability [7]:

$$P_w^{(OMA)} = 1 - \left[1 - \frac{3}{2}\,Q\left(\sqrt{\frac{\alpha E_T}{10N_0}}\right)\right]^2.$$

First, we consider the case where $\beta = 0.1$, which means that the link gain of the far user is 10 dB smaller than that of the near user. For the NOMA scheme, we let $\alpha = 0.05$. The error probabilities under the two schemes are plotted in Fig. 1, which shows lattice-superposition NOMA outperforms OMA. For example, to achieve a word error probability of 10^{-3}, lattice-superposition NOMA has 4 dB energy gain.

Next, we consider the case where $\beta = 0.01$, which means the link gain of the far user is 20 dB smaller. For the NOMA scheme, we let $\alpha = 0.01$. The error probabilities are plotted in Fig. 2. Since the difference between the link gains of the two users becomes larger, the advantage of NOMA increases. To achieve a word error probability of 10^{-3}, lattice-superposition NOMA has 6 dB energy gain for the far user and even more for the near user. This result agrees with the well-known fact that NOMA has larger benefit when the link gains between the two users are significantly different.

IV. Conclusion

A novel lattice-superposition NOMA scheme is constructed for the two-user Gaussian broadcast channel. It outperforms OMA especially when the link gain difference between the two users is large. We are extending the work to a larger system.

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