Spherical Code Superposition NOMA and Its User Pairing Strategy

Jun Zou[1, 2], Kenneth W. Shum[3] and Chi Wan Sung[2]
[1]School of Electronic and Optical Engineering, Nanjing University of Science and Technology, China
[2]Department of Electrical Engineering, City University of Hong Kong, Hong Kong SAR
[3]School of Science and Engineering, The Chinese University of Hong Kong (Shenzhen), China
Email: jun.zou@cityu.edu.hk, wkshum@cuhk.edu.cn, albert.sung@cityu.edu.hk

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***Abstract*—A novel non-orthogonal multiple access (NOMA) scheme with spherical code superposition and its user pairing strategy are proposed. A transmitter transmits the superposition of the signals of two users, each of which is selected from the high dimensional spherical code on the same time-frequency resource by power-domain multiplexing NOMA. Upper bounds of the word error probabilities of the two users are derived. Based on them, a power allocation scheme for the two users is proposed, which guarantees that their word error probabilities are below a certain threshold. Under our power allocation scheme, the optimal user pairing strategy that minimizes the total power consumption in a general multi-user system is analytically found. Numerical results show that our proposed system outperforms some benchmark methods.**

***Index Terms*—Non-orthogonal multiple access (NOMA), spherical code, user pairing.**

I. Introduction

With the rapid development of the Internet of Things and augmented/virtual reality, 5G communications systems are required to have a 1,000-fold increase in capacity and a 10-fold increase in the number of access devices [1]. According to the Shannon capacity formula, increasing bandwidth can directly improve system capacity and accommodate more devices. However, the frequency spectrum is scarce. Due to the advance of signal processing and hardware technologies, it is time to consider the non-orthogonal multiple access (NOMA), which has higher spectral efficiency than the conventional orthogonal multiple access (OMA) scheme.

In contrast to OMA, NOMA allows two or more users sharing the same resource with inter-user interference. With successive interference cancellation (SIC), the near user can first decode the signal intended for the far user, and then subtracts it from his received signal before decoding his own signal [2]. Most current works on practical NOMA schemes are based on two-dimensional modulation such as quadrature amplitude modulation (QAM). In fact, we can use multiple resource elements (RE) jointly to increase the modulation dimension. For example, two resource elements together can provide four dimensions. To achieve the same spectral efficiency, we can increase the number of bits transmitted on each constellation point. Due to the increased dimension, a larger Euclidean distance between constellation points is possible, which yields better symbol detection performance [3]. This idea led to the development of the four-dimensional lattice superposition NOMA in [4]. In this work, we consider the use of spherical codes for general dimensions, which densely pack the constellation points, similar to the sphere packing problem [5].

In a NOMA system with many users, instead of allowing all of them to superimpose their signals for transmissions, it is more practical to group two users together to share a single resource, which not only reduces decoding complexity but also avoids error propagation. This method is commonly called *user pairing*, which has gained substantial attention [6]–[9]. To save the total transmit power, a cache-based NOMA with user pairing scheme is proposed in [6]. With side information in cache, index coding can be used to reduce the requirement on transmit power. To guarantee the proportional fairness of the users, a prediction-based particle swarm optimization algorithm is discussed in [7]. To improve the sum data rate of cooperative NOMA networks, a close-to-user pairing based full-duplex NOMA is proposed in [8]. Most of the existing works on user pairing in NOMA system are based on the analysis of signal-to-interference-plus-noise ratio and the use of Shannon capacity formula without considering the actual modulation scheme.

In this paper, we use spherical codes as the modulation constellation matrix and investigate its user pairing strategy using power-domain NOMA. Our main contribution is the analysis of a practical NOMA scheme, which includes the analysis of word error probability, the proposal of a power allocation rule between two users, and the derivation of the optimal user pairing strategy.

II. System Model

A. Spherical codes

Let Ω_L be the surface of a unit sphere in the L dimensional space, i.e.,

$$\Omega_L = \left\{ (x_1, x_2, \dots, x_L) \in \mathbb{R}^L \,\middle|\, \sum_{l=1}^{L} x_l^2 = 1 \right\}. \quad (1)$$

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A spherical code $\boldsymbol{S}(L, M, \phi)$ is an M-subset of Ω_L with the property that for all pairs $\boldsymbol{s}_1, \boldsymbol{s}_2 \in \boldsymbol{S}$ ($\boldsymbol{s}_1 \neq \boldsymbol{s}_2$),

$$\boldsymbol{s}_1^T \boldsymbol{s}_2 \leq \cos\phi, \tag{2}$$

where $0 < \phi \leq \pi$ is an upper bound on the angles between any pair of vectors. For example, if we put the center of the tetrahedron to the origin, then the four vertices together form an $\boldsymbol{S}(3, 4, 109.47^\circ)$ spherical code. According to the Law of Cosines, the minimum Euclidean distance between two points in the set $\boldsymbol{S}(L, M, \phi)$ is $d_{\min} = \sqrt{2 - 2\cos\phi}$. It means that the larger the value of ϕ, the larger the minimum distance between two points in the set $\boldsymbol{S}(L, M, \phi)$. Therefore, given L and M, we can use a spherical code with the largest value of ϕ as the constellation matrix [10]. Sloane, Hardin and Smith have given the arrangements in 3, 4 and 5 dimensions with $M = 4, 5, \ldots, 130$ [11]. The higher the dimension, the larger the minimum distance. For example, the minimum angle of 16 points in 4 dimensions is $\phi = 67.193^\circ$ while the minimum angle of 16 points in 5 dimensions is $\phi = 78.463^\circ$ [11]. From this viewpoint, high dimensional modulation can improve the performance of symbol detection.

B. System description

Consider a single base station with $2K$ active users, who are indexed by elements in $\mathcal{K} \triangleq \{1, 2, 3, \ldots, 2K\}$. Power-domain NOMA with superposition coding is used to improve the system capacity [12]. Two users are scheduled into a group using the same time-frequency resource provided by the base station.

In each group g, where $g = 1, 2, \ldots, K$, there are two users. The one who is closer to the base station (in the sense that the channel gain is larger) is called the near user and labeled as user 1 of group g. The other one is called the far user and labeled as user 2 of group g. For $i = 1, 2$, let $\boldsymbol{x}_i^g \in \mathbb{R}^L$ be the transmitted signal vector for user i of group g. The received signal of user i can be written as

$$\boldsymbol{y}_i^g = h_i^g \boldsymbol{x}^g + \boldsymbol{n}_i^g, \tag{3}$$

where $h_i^g \in \mathbb{R}$ is the channel gain of user i in the g-th group, which is assumed to be a constant in one signal vector transmit period, $\boldsymbol{n}_i^g \in \mathbb{R}^L$ is the random white noise vector of user i, each component of which is a Gaussian random variable with mean zero and variance $N_0/2$, $\boldsymbol{x}^g$ is the superposition of the transmitted signal vectors from the two users. For simplicity, we drop the group index g when we analyse the NOMA strategy in one group, that is,

$$\boldsymbol{y}_i = h_i \boldsymbol{x} + \boldsymbol{n}_i. \tag{4}$$

According to our user labeling, we have $|h_1| > |h_2|$. We define the relative power gain between these two users as the ratio of the squares of these two channel gains, which is given by

$$\beta \triangleq h_2^2 / h_1^2 < 1. \tag{5}$$

In our design, superposition coding is used to form the combined transmit signal vector, that is,

$$\boldsymbol{x} = \sqrt{\alpha E_T}\boldsymbol{x}_1 + \sqrt{(1-\alpha)E_T}\boldsymbol{x}_2, \tag{6}$$

where $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are the signal vectors selected from the constellation matrix $\boldsymbol{C}$ for user 1 and user 2, respectively. The constellation matrix $\boldsymbol{C}$ is an $L \times M$ real matrix which is generated according to the spherical code $\boldsymbol{S}(L, M, \phi)$. Each real number is conveyed by I/Q channel in the practical system. These vectors are assumed to be uniformly distributed in the set of constellation points with unit average energy, i.e., $\mathcal{E}[\boldsymbol{x}_1^T \boldsymbol{x}_1] = \mathcal{E}[\boldsymbol{x}_2^T \boldsymbol{x}_2] = 1$. As a result, E_T is the total transmit energy, and $\alpha < 1$ is the power allocation factor between the two users within a group. In each time slot, because of the superposition in NOMA, $2\log_2 M$ data bits, $\log_2 M$ for each user, are transmitted in one signal duration. Therefore, the average energy per bit is $\frac{E_T}{2\log_2 M}$.

At the receiver side, we assume that user i has the knowledge of his own channel gain h_i and minimum distance detection is used. For user 2, it detects the value of $\boldsymbol{x}_2$ by treating the interfering signal from user 1 as noise and finding the column of $h_2\sqrt{(1-\alpha)E_T}\boldsymbol{C}$ closest to the received signal $\boldsymbol{y}_2$. If the detected column is different from the transmitted column for user 2, we define it as a word error of user 2.

For user 1, SIC is used to improve the probability of successful detection. It first detects the value of $\boldsymbol{x}_2$ using the same minimum distance detection at the receiver of user 2. Assuming the detected vector is $\boldsymbol{x}_2'$, the signal after removing the signal for user 2 is given by

$$\boldsymbol{y}_1' = h_1\sqrt{\alpha E_T}\boldsymbol{x}_1 + h_1\sqrt{(1-\alpha)E_T}(\boldsymbol{x}_2 - \boldsymbol{x}_2') + \boldsymbol{n}_1. \tag{7}$$

Afterwards, minimum distance detection is used again to find the column of $h_1\sqrt{\alpha E_T}\boldsymbol{C}$ closest to $\boldsymbol{y}_1'$. If the detected column is different from the transmitted column of user 1, we define it as a word error of user 1.

In this paper, different user groups are transmitted in time division multiplexing (TDM) manner, which means that the base station transmits a combined signal to each group in a distinct time slot. Our goal is to minimize the total energy consumption of K time slots subject to a requirement of word error probability.

III. Power Allocation between Two Users

In this section, we derive a common upper bound for the word error probabilities of both users in a group. In addition, we obtain the relationship between the relative power gain and the power allocation factor.

A. Error Analysis for the Far User

From the previous analysis, the pairwise word error probability of user 2, the far user, can be calculated by

$$\begin{aligned} &P_2(\boldsymbol{x}_2 \to \boldsymbol{x}_2') \\ &= P\left(\left|\boldsymbol{y}_2 - h_2\sqrt{(1-\alpha)E_T}\boldsymbol{x}_2\right|^2 \geq \left|\boldsymbol{y}_2 - h_2\sqrt{(1-\alpha)E_T}\boldsymbol{x}_2'\right|^2\right) \\ &= P\left((\boldsymbol{x}_2 - \boldsymbol{x}_2')^T\left(-2h_2\boldsymbol{n}_2 - h_2^2\sqrt{E_T}\boldsymbol{v}(\alpha)\right) \geq 0\right) \end{aligned} \tag{8}$$

where $\boldsymbol{x}_2' \neq \boldsymbol{x}_2$, $|\cdot|$ is the norm of a vector and

$$\boldsymbol{v}(\alpha) \triangleq \sqrt{1-\alpha}(\boldsymbol{x}_2 - \boldsymbol{x}_2') + 2\sqrt{\alpha}\boldsymbol{x}_1. \tag{9}$$

According to the Chernoff inequality [13], there is an upper bound for (8), that is

$$P_2(\boldsymbol{x}_2 \to \boldsymbol{x}_2') \le \mathcal{E}\left(\exp\left(-2h_2\lambda(\boldsymbol{x}_2 - \boldsymbol{x}_2')^T \boldsymbol{n}_2\right)\right) \cdot \exp\left(-\lambda h_2^2 \sqrt{E_T}(\boldsymbol{x}_2 - \boldsymbol{x}_2')^T \boldsymbol{v}(\alpha)\right) \tag{10}$$

where $\lambda > 0$ and $\mathcal{E}(\cdot)$ is the expectation operator.

The variance of each component in $\boldsymbol{n}_2$ is $\sigma^2 = N_0/2$. Using the moment generating function of a Gaussian random variable, we can rewrite (10) as

$$\begin{aligned} &P_2(\boldsymbol{x}_2 \to \boldsymbol{x}_2') \\ &\le \exp(N_0 h_2^2 \lambda^2 |\boldsymbol{x}_2 - \boldsymbol{x}_2'|^2 - \lambda h_2^2 \sqrt{E_T}(\boldsymbol{x}_2 - \boldsymbol{x}_2')^T \boldsymbol{v}(\alpha)). \end{aligned} \tag{11}$$

The right term of the inequality (11) is a quadratic function of λ. It attains the minimum value when

$$\lambda = \frac{\sqrt{E_T}(\boldsymbol{x}_2 - \boldsymbol{x}_2')^T \boldsymbol{v}(\alpha)}{2N_0|\boldsymbol{x}_2 - \boldsymbol{x}_2'|^2}. \tag{12}$$

Since $\lambda > 0$, the power allocation factor α should satisfy

$$\sqrt{1-\alpha}|\boldsymbol{x}_2 - \boldsymbol{x}_2'|^2 + 2\sqrt{\alpha}(\boldsymbol{x}_2 - \boldsymbol{x}_2')^T \boldsymbol{x}_1 > 0 \tag{13}$$

for arbitrary $\boldsymbol{x}_1$, $\boldsymbol{x}_2$ and $\boldsymbol{x}_2'$. The minimum distance between two constellation points in $\boldsymbol{C}$, $\min|\boldsymbol{x}_2 - \boldsymbol{x}_2'|$, is $d_{\min}$ and the norm of the point in $\boldsymbol{C}$, $|\boldsymbol{x}_1|$, is 1. When

$$d_{\min}\sqrt{1-\alpha} > 2\sqrt{\alpha}, \tag{14}$$

or equivalently, $\alpha < \frac{d_{\min}^2}{4+d_{\min}^2}$, (13) is always satisfied. From now on, we assume $0 < \alpha < \frac{d_{\min}^2}{4+d_{\min}^2}$.

Substituting (12) into (11), we obtain

$$P_2(\boldsymbol{x}_2 \to \boldsymbol{x}_2') \le \exp\left(-\frac{h_2^2 E_T \left((\boldsymbol{x}_2 - \boldsymbol{x}_2')^T \boldsymbol{v}(\alpha)\right)^2}{4N_0|\boldsymbol{x}_2 - \boldsymbol{x}_2'|^2}\right) \tag{15}$$

By Cauchy-Schwarz inequality,

$$\begin{aligned} \frac{(\boldsymbol{x}_2 - \boldsymbol{x}_2')^T \boldsymbol{v}(\alpha)}{|\boldsymbol{x}_2 - \boldsymbol{x}_2'|} &\ge \frac{\sqrt{1-\alpha}|\boldsymbol{x}_2 - \boldsymbol{x}_2'|^2 - 2\sqrt{\alpha}|\boldsymbol{x}_1||\boldsymbol{x}_2 - \boldsymbol{x}_2'|}{|\boldsymbol{x}_2 - \boldsymbol{x}_2'|} \\ &\ge d_{\min}\sqrt{1-\alpha} - 2\sqrt{\alpha}, \end{aligned} \tag{16}$$

since $|\boldsymbol{x}_2 - \boldsymbol{x}_2'| \ge d_{\min}$ and $|\boldsymbol{x}_1| = 1$. Due to (14), the lower bound in (16) is always non-negative.

Substituting (16) into (15), we obtain

$$P_2(\boldsymbol{x}_2 \to \boldsymbol{x}_2') \le \exp\left(-\frac{h_2^2(d_{\min}\sqrt{1-\alpha} - 2\sqrt{\alpha})^2 E_T}{4N_0}\right), \tag{17}$$

which is an upper bound of user 2's word error probability, since it does not depend on the transmit signal $\boldsymbol{x}_2$.

B. Error Analysis for the Near User

For the near user, user 1, it should detect user 2's information bits first. The word error probability analysis on user 2's information bits at user 1 is nearly the same as the previous analysis except the channel gain. Let F be the event that user 1 makes an error in detecting user 2's information bits. According to (17),

$$\begin{aligned} P(F) &\le \exp\left(-\frac{h_1^2(d_{\min}\sqrt{1-\alpha} - 2\sqrt{\alpha})^2 E_T}{4N_0}\right) \\ &= \left(\exp\left(-\frac{h_2^2(d_{\min}\sqrt{1-\alpha} - 2\sqrt{\alpha})^2 E_T}{4N_0}\right)\right)^{1/\beta}, \end{aligned} \tag{18}$$

which also represents the upper bound of SIC failure probability. From (18), we can see that the upper bound of SIC failure probability is much smaller than the upper bound of user 2's word error probability when the value of β is small.

Then, the pairwise word error probability of user 1 is given by

$$\begin{aligned} P_1(\boldsymbol{x}_1 \to \boldsymbol{x}_1') = P(F^c)P_1(\boldsymbol{x}_1 \to \boldsymbol{x}_1'|F^c) \\ + P(F)P_1(\boldsymbol{x}_1 \to \boldsymbol{x}_1'|F) \end{aligned} \tag{19}$$

where F^c is the complement of F.

When the SIC is successful, the remaining signal after removing user 2's signal is given by

$$\boldsymbol{y}_1' = h_1\sqrt{\alpha E_T}\boldsymbol{x}_1 + \boldsymbol{n}_1. \tag{20}$$

Similar to (8)-(17), it can be easily shown that the upper bound of the word error probability of user 1 with successful SIC is

$$P_1(\boldsymbol{x}_1 \to \boldsymbol{x}_1'|F^c) \le \exp(-\frac{d_{\min}^2 h_1^2 \alpha E_T}{4N_0}). \tag{21}$$

An upper bound of (19) is

$$P_1(\boldsymbol{x}_1 \to \boldsymbol{x}_1') \le P_1(\boldsymbol{x}_1 \to \boldsymbol{x}_1'|F^c) + P(F). \tag{22}$$

When β is small enough, $P(F)$ will tend to zero, that is,

$$\lim_{\beta\to 0} P(F) = 0, \tag{23}$$

and

$$\lim_{\beta\to 0} P_1(\boldsymbol{x}_1 \to \boldsymbol{x}_1') \le \exp(-\frac{d_{\min}^2 h_1^2 \alpha E_T}{4N_0}). \tag{24}$$

When $P(F)$ is non-negligible,

$$\begin{aligned} P_1(\boldsymbol{x}_1 \to \boldsymbol{x}_1') \le \exp(-\frac{d_{\min}^2 h_1^2 \alpha E_T}{4N_0}) \\ + \exp\left(-\frac{h_1^2(d_{\min}\sqrt{1-\alpha} - 2\sqrt{\alpha})^2 E_T}{4N_0}\right). \end{aligned} \tag{25}$$

C. Common Upper Bound and Power Allocation Factor

According to (17), we can see that the upper bound of user 2's word error probability increases as α increases. On the contrary, the upper bound of user 1's word error probability decreases as α increases according to (25). The system performance is decided by the worse case of two users in one group, so it is a good solution to improve the system performance by letting the upper bound of two user's word error probability be the same. For the ease of computing, we ignore the second term of the upper bound in (25) and let

$$\exp\left(-\frac{d_{\min}^2 h_1^2 \alpha E_T}{4N_0}\right) = \exp\left(-\frac{h_2^2(d_{\min}\sqrt{1-\alpha} - 2\sqrt{\alpha})^2 E_T}{4N_0}\right) \tag{26}$$

instead. Then the relationship between the power allocation factor α^* and the relative power gain β can be obtained by solving (26), with

$$\alpha^* = \frac{1}{1+\left(\frac{2}{d_{\min}}+\beta^{-\frac{1}{2}}\right)^2}. \quad (27)$$

Under the assumption of (26), the upper bound of user 1's word error probability is

$$P_1(\boldsymbol{x}_1 \to \boldsymbol{x}_1') \leq \exp\left(-\frac{d_{\min}^2 h_1^2 \alpha^* E_T}{4N_0}\right) + \left(\exp\left(-\frac{d_{\min}^2 h_1^2 \alpha^* E_T}{4N_0}\right)\right)^{\frac{1}{\beta}} \quad (28)$$

A looser bound for (28) is

$$P_1(\boldsymbol{x}_1 \to \boldsymbol{x}_1') < 2\exp\left(-\frac{d_{\min}^2 h_1^2 \alpha^* E_T}{4N_0}\right) \quad (29)$$

for arbitrary relative power gain β. Note that it is also an upper bound of the word error probability of user 2. In the following analysis, α^* is used as the power allocation factor for any power link gain.

IV. User Pairing with Minimum Power

For a given word error probability P_w, the transmit power required by user 1 and user 2 according to (29) is

$$E_T \geq \frac{-4\left(1+\left(\frac{2}{d_{\min}}+\beta^{-\frac{1}{2}}\right)^2\right)N_0 \ln\frac{P_w}{2}}{d_{\min}^2 h_1^2}. \quad (30)$$

In the following analysis, we will use (30) as the minimum transmit power requirement.

As we describe in the system model, there are $2K$ users in a cell which need to be scheduled as K groups. The total energy consumption of K groups of users can be written as

$$\begin{aligned} E_{total} &= \sum_{g=1}^{K} \frac{-4\left(1+\left(\frac{2}{d_{\min}}+\frac{|h_1^g|}{|h_2^g|}\right)^2\right)N_0\ln\frac{P_w}{2}}{d_{\min}^2 |h_1^g|^2} \\ &= \frac{-4N_0\ln\frac{P_w}{2}}{d_{\min}^2}\sum_{g=1}^{K}\left(\frac{1}{|h_1^g|^2}+\frac{1}{|h_2^g|^2}\right)+ \\ &\quad \frac{-4N_0\ln\frac{P_w}{2}}{d_{\min}^2}\sum_{g=1}^{K}\left(\frac{4}{|h_1^g|^2 d_{\min}^2}+\frac{4}{|h_1^g||h_2^g|d_{\min}}\right). \end{aligned} \quad (31)$$

Our goal is to minimize the total energy consumption in (31) by dividing users into groups. This combinatorial problem is called user pairing. The first term in (31) is constant under any user pairing method. Without loss of generality, we assume that the channel gains of these $2K$ users are in descending order, i.e., $|h_1| > |h_2| > \cdots > |h_{2K}|$. Therefore, our optimization problem is equivalent to solving

$$\min_{\pi} \sum_{g=1}^{K}\left(\frac{4}{|h_{\pi(2g-1)}|^2 d_{\min}^2}+\frac{4}{|h_{\pi(2g-1)}||h_{\pi(2g)}|d_{\min}}\right) \quad (32)$$

with the minimum taken over all bijective function $\pi(\cdot)$ from $\{1,2,...,2K\}$ to itself, where $h_1^g \triangleq h_{\pi(2g-1)}$ and $h_2^g \triangleq h_{\pi(2g)}$.

We first focus on how to achieve

$$\min_{\pi}\sum_{g=1}^{K}\frac{1}{|h_{\pi(2g-1)}||h_{\pi(2g)}|}, \quad (33)$$

without considering the first term in (32).

Lemma 1. *Define $\mathcal{U}_1 \triangleq \{1,2,\ldots,K\}$ as the index set of the K nearest users and $\mathcal{U}_2 \triangleq \mathcal{K}\backslash U_1$ as that of the K furthest users. The optimal solution to (33) must satisfy $\pi(2g-1) \in \mathcal{U}_1$ and $\pi(2g) \in \mathcal{U}_2$ for $g = 1,2,\ldots,K$.*

Proof: We prove by contradiction. Suppose at an optimal solution, group i consists of two near users, i.e., $\pi(2i-1), \pi(2i) \in \mathcal{U}_1$. Then, there must be a group j which consists of two far users, i.e., $\pi(2j-1), \pi(2j) \in \mathcal{U}_2$. Since

$$\left(\frac{1}{|h_{\pi(2i-1)}|}-\frac{1}{|h_{\pi(2j-1)}|}\right)\left(\frac{1}{|h_{\pi(2i)}|}-\frac{1}{|h_{\pi(2j)}|}\right) > 0, \quad (34)$$

we have

$$\frac{1}{|h_1^i||h_2^i|}+\frac{1}{|h_1^j||h_2^j|} > \frac{1}{|h_1^i||h_2^j|}+\frac{1}{|h_2^i||h_1^j|}. \quad (35)$$

That means, if we change the pairing combination according to (35), the total energy consumption will strictly decrease, which contradicts with the assumption that the original pairing is optimal. ■

Theorem 2. *The optimality in (32) can be achieved by the permutation $\pi(i) = (i+1)/2$ if i is odd and $\pi(i) = 2K-(i-2)/2$ if i is even.*

Proof: According to Lemma 1, it suffices to consider only the pairing between near users and far users. By assumption, we have $\frac{1}{|h_1|} < \frac{1}{|h_2|} < \cdots < \frac{1}{|h_K|}$ and $\frac{1}{|h_{K+1}|} < \frac{1}{|h_{K+2}|} < \cdots < \frac{1}{|h_{2K}|}$. The rearrangement inequality in [14] gives a lower bound on the objective function in (33):

$$\begin{aligned} &\sum_{g=1}^{K}\frac{1}{|h_{\pi(2g-1)}||h_{\pi(2g)}|} \\ &\geq \frac{1}{|h_1||h_{2K}|}+\frac{1}{|h_2||h_{2K-1}|}+\cdots+\frac{1}{|h_K||h_{K+1}|}, \end{aligned} \quad (36)$$

which can be achieved by pairing up the i-th nearest user with the i-th furthest user to form group i, for $i = 1,2,\ldots,K$. This pairing strategy is represented by the given permutation $\pi(i)$. It can also minimize the objective function in (32), since the channel gains of the users in $\mathcal{U}_1$ are larger than those of the users in $\mathcal{U}_2$. ■

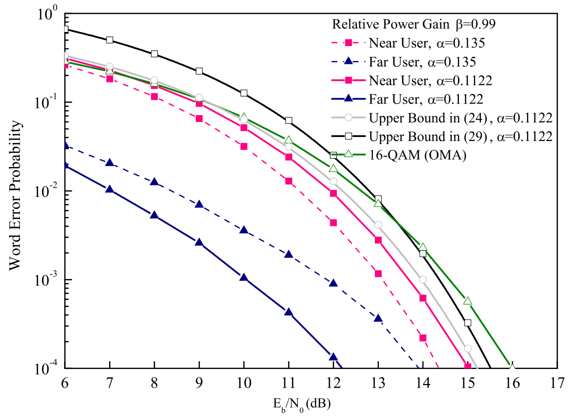

Fig. 1. Word error probability of 4-dimensional spherical code superposition with different power allocation factor ($\beta = 0.99$)

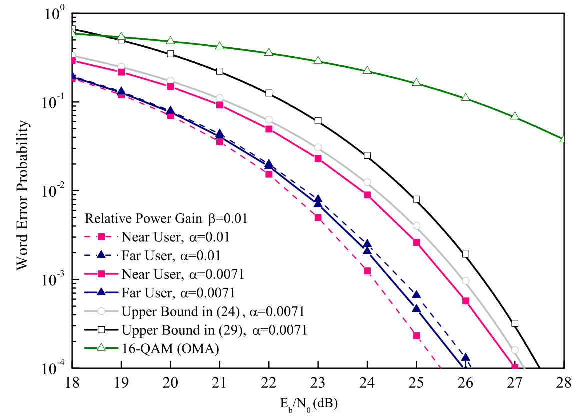

Fig. 2. Word error probability of 4-dimensional spherical code superposition with different power allocation factor ($\beta = 0.01$)

V. Simulation Results

In this section, we use a spherical code $\boldsymbol{S}(4, 16, 67.19^\circ)$ as an example to show the performance of our proposed spherical code superposition NOMA. The constellation matrix is

$$\boldsymbol{C} = \begin{bmatrix} -0.5126 & 0.3971 & -0.2620 & 0.7148 \\ -0.2222 & 0.4164 & -0.8541 & -0.2184 \\ -0.2235 & -0.4142 & -0.1452 & -0.8703 \\ 0.2263 & -0.6144 & -0.7548 & -0.0385 \\ -0.0141 & -0.6087 & 0.0161 & 0.7931 \\ 0.8451 & 0.1125 & -0.2752 & -0.4443 \\ 0.5033 & -0.7967 & 0.2726 & -0.1942 \\ -0.2356 & 0.2149 & 0.7654 & 0.5590 \\ -0.7911 & -0.4281 & -0.4296 & 0.0798 \\ 0.2358 & 0.9466 & 0.1311 & 0.1767 \\ -0.5142 & -0.6103 & 0.5978 & -0.0759 \\ 0.2345 & 0.1159 & 0.8401 & -0.4752 \\ 0.5049 & 0.2108 & -0.5873 & 0.5965 \\ 0.0141 & 0.6087 & -0.0161 & -0.7931 \\ -0.8327 & 0.4198 & 0.2612 & -0.2493 \\ 0.7818 & 0.0285 & 0.4401 & 0.4407 \end{bmatrix}^T. \quad (37)$$

The corresponding minimum distance is $d_{\min} = 1.1066$.

A. Power allocation between near-far users

In this subsection, we consider a two-user system, and focus on how the word error probability varies with different bit SNR. Without loss of generality, the channel gain for the near user is normalized to 1. Since each word can convey eight bits, the average bit SNR is $E_b/N_0 = E_T/8N_0$. Monte-Carlo simulation is adopted.

Fig. 1 and Fig. 2 show the simulation results and the mathematical results of the word error probability with different relative power gains. We also add the traditional 16-QAM with orthogonal multiple access (OMA) as reference. The principle of the power allocation for OMA system is to let the two users achieving the same E_b/N_0. Therefore, the word error probability can be calculated as [3]

$$P_{\text{16-QAM}} \approx 3Q\left(\sqrt{\frac{4\beta E_b}{5N_0(\beta+1)}}\right). \quad (38)$$

We can see that even when the relative power gain, β, is 0.99, our proposed spherical code NOMA is still better than the OMA performance. As the relative gain increases, the proposed spherical code NOMA obtains more and more improvement than the OMA. The power allocation factor α of the solid lines in Fig. 1 and Fig. 2 is determined by (27) while the α of the dash lines is set by manual selection whose performance is better than the derived α and close to the best. Therefore, our derived power allocation factor is a feasible solution with a derived upper bound of word error probability, maybe not the best, but is achievable and deterministic for arbitrary relative power gain.

The gap between analytical upper bound of word error probability and the worse case of the two users' word error probability by simulation with the same α is less than 1 dB. And the gap between analytical upper bound of word error probability with the derived α and the simulation results with manual selection α is about 1 dB with $\beta = 0.99$ and less than 1.5 dB with $\beta = 0.01$.

Besides, we can see that the upper bound in (24) can also be the upper bound of word error probability of the two users in one group from the simulation results in Fig. 1 for $\beta = 0.99$. If we use (24) to calculate the upper bound of transmit power, the user pairing strategy described in section IV is still the best. Because, the relationship between (24) and (29) is linear.

Fig. 3 shows the comparisons of word error probability with different constellation matrix when $\beta = 0.01$, lattice code in [4], 4-D cube code and spherical code. In fact, the 4-D cube code which is the permutation of 1 and -1, can be regarded as a traditional NOMA structure with two independent QPSK tones. Certainly, the 4-D cube code is also a lattice code [11]. The power allocation factor is $\alpha = 0.0071$. We can see that our proposed spherical code superposition NOMA outperforms than the other two scenarios. Because, the minimum distance between two points using spherical code is the largest when there are 16 points with 4 dimensions. The gain is about 0.5 dB when the spherical code is used comparing with the 4-D cube code.

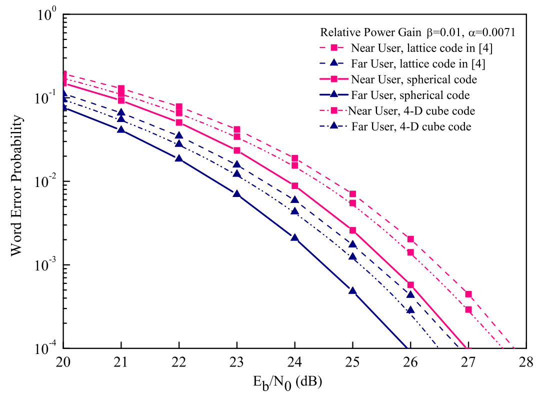

Fig. 3. Comparisons of word error probability with different constellation matrices ($\beta = 0.01$)

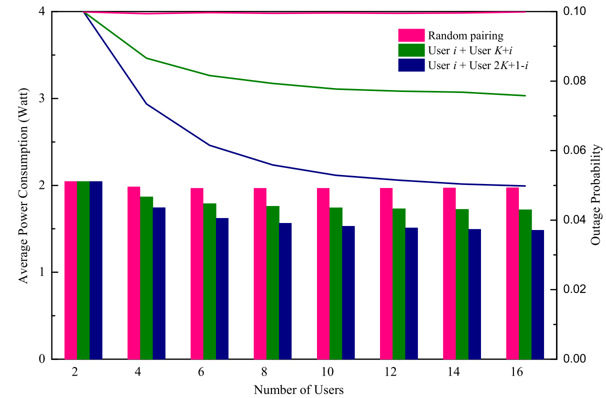

Fig. 4. Comparisons of outage probability and average power consumption with different user pairing strategies ($P_w = 0.001$)

B. Power consumption

In our simulations, the users are randomly and uniformly distributed in a hexagon cell whose radius is 400 m. The path loss model is $128.1 + 37.6\log_{10}d$ dB, where d is the distance between the base station and the user measured in km. The standard deviation of shadow fading is assumed to be 12 dB and one-path Rayleigh fading is considered. The number of users are from 2 to 16. Two users of the same group are transmitted in one time slot with 5 MHz bandwidth. The noise power spectral density is -174 dBm/Hz and the noise figure is 9 dB at the receiver. The maximum transmit power of base station is 40 dBm in this simulation. We define the outage probability as the probability that the required energy exceeds the maximum transmit power of base station.

Fig. 4 shows the comparisons of outage probability (line) and average power consumption per group (bar) with different user pairing strategies when $P_w = 0.001$. One is the derived user pairing strategy in section IV, and the other is to pair up user i with user $K+i$ as a group, for $i = 1, 2, \ldots, K$. We also add the random pairing between all $2K$ users as a reference. We can see that our proposed pairing strategy outperforms the other two strategies. Besides, as the number of users increases, the outage probability decreases. The reason is when the number of users increases, there are more combinations for user pairing, which can present the advantage of user pairing clearly.

The power consumption of the three strategies are obtained by averaging over those instances under which all three strategies experience no outage. When there are 4 users, the power saving ratios of our proposed strategy comparing to the random pairing and the other pairing strategy are 12.1% and 6.6%, respectively. When there are 16 users, the corresponding power saving ratios are 24.8% and 13.7%, respectively. The more users, the larger savings can be obtained.

VI. Conclusions

In this paper, we investigate the NOMA scheme with high dimensional spherical code superposition and its pairing strategy in a cell. In the two-user scenario, we analyze the upper bound of the word error probability using the Chernoff inequality. We derive the power allocation factor, which can guarantee that the word error probabilities of the two users are both below a given threshold. The power allocation factor depends only on the relative power gain of the two users, which allows us to obtain a closed-form expression for the total energy consumption. Simulation results show that the gap between the derived upper bound of word error probability and the simulation results is less than 1 dB. Furthermore, according to the derived lower bound of power consumption, we prove that the optimal user pairing strategy is to pair up the strongest user with the weakest user repeatedly in a nested way.

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