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Research Article

Joint Channel-Network Coding for the Gaussian Two-Way Two-Relay Network

Ping Hu,[1] Chi Wan Sung,[1] and Kenneth W. Shum[2]

[1] *Department of Electronic Engineering, City University of Hong Kong, Hong Kong*
[2] *Department of Information Engineering, The Chinese University of Hong Kong, Hong Kong*

Correspondence should be addressed to Kenneth W. Shum, wkshum@inc.cuhk.edu.hk

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New aspects arise when generalizing two-way relay network with one relay to two-way relay network with multiple relays. To study the essential features of the two-way multiple-relay network, we focus on the case of two relays in our work. The problem of how two terminals, equipped with multiple antennas, exchange messages with the help of two relays is studied. Five transmission strategies, namely, amplify-forward (AF), hybrid decode amplify forward (HLC), hybrid decode amplify forward (HMC), decode forward (DF), and partial decode forward (PDF), are proposed. Their designs are based on a variety of techniques including network coding, multiplexed coding, multi-input multi-output transmission, and multiple access with common information. Their performance is compared with the cut-set outer bound. It is shown that there is no dominating strategy and the best strategy depends on the channel conditions. However, by studying their multiplexing gains at high signal-tonoise (SNR) ratio, it is shown that the AF scheme dominates the others in high SNR regime.

1. Introduction

Relay channel, which considers the communication between a source node and a destination with the help of a relay node, was introduced by van der Meulen in [1]. Based on this channel model, Cover and El Gamal developed coding strategies known as decode-forward (DF) and compress-forward (CF) in [2]. These techniques now become standard building blocks for cooperative and relaying networks, which have been extensively studied in the literature (e.g., [3, 4]).

For many applications, communication is inherently two-way. A typical example is the telephone service. In fact, the study of two-way channel is not new and can be traced back to Shannon's work in 1961 [5]. However, the model of two-way relay channel, though natural, did not attract much attention. Recently, probably due to the advent of network coding [6] in the last decade, there is a growing interest in this model. The application of DF and CF to two-way relay channel was considered in [7]. The half-duplex case was studied in [8, 9]. The results in [10] showed that feedback is beneficial only in a two-way transmission. Network coding for the two-way relay channel was studied in [11, 12]. Physical layer network coding based on lattices is considered recently [13], and shown to be within 0.5 bit from the capacity in some special cases [14].

All the aforementioned works are for one relaying node. It is easy to envisage that in real systems, more than one relay can be used. Schein in [15] started the investigation of the network with one source-destination pair and two parallel relays in between. This model was further studied in [16] under the assumption of half-duplex relay operations. For one-way multiple-relay networks in general, cooperative strategies were proposed and studied in [17]. We remark that a notable feature that does not exist in the single-relay case is that the multiple relays can act as a virtual antenna array so that beamforming gain can be reaped at the receiver. In this paper, we follow this line of research and consider two-way communications. Two-relays are assumed, for this simple model already captures the essential features of the more general multiple-relay case. We are interested in knowing how different techniques can be used to construct transmission strategies for the two-way two-relay network and how they perform under different channel conditions. In particular, we apply the idea of network coding to both the physical layer and the network layer. Besides, channel coding techniques for multiple access channel (MAC) and multi-input multi-output (MIMO) channel are also employed. Several transmission strategies are thus constructed and their achievable rate regions are derived.

We remark that the channel model that we consider in this paper is also called the *restricted* two-way two-relay channel [7]. This means that the signal from a source node depends only on the message to be transmitted, but not on the received signal at the source. Besides, our results are obtained under the half-duplex assumption, which is realistic for practical systems. Each node is assumed to transmit one half of the time and receive during the other half of the time. The performance of our proposed strategies can be further improved if the ratio of transmission time and receiving time is optimized. We do not consider this more general case, since it complicates the analysis but provides no new insights.

This paper is organized as follows. Our network model is described in Section 2. Some basic coding techniques are reviewed in Section 3. Based on these coding techniques, several transmission strategies are devised in Section 4. Their performance at high signal-to-noise ratio regime is analyzed in Section 5. The rate regions of these strategies are compared under some typical channel realizations in Section 6. The conclusion is drawn in Section 7.

2. Channel Model and Notations

The two-way two-relay (TWTR) network consists of four nodes: two terminals A and B, and two parallel relays 1 and 2 (see Figure 1). Terminals A and B want to exchange messages with the help of the two relays. We assume there is no direct link between the two terminals and between the two-relays. Furthermore, all of the nodes are half-duplex. The total communication time, $2N$, are divided into two stages, each of which consists of N time slots. In the first stage, the terminals send signals and the relays receive. In the second stage, the relays send signals and the terminals receive. The solid arrows in Figure 1 correspond to stage 1 and the dashed arrows correspond to stage 2.

Suppose that terminals A and B are equipped with n antennas, whereas each of relays 1 and 2 has only one antenna. For $i \in \{A, B\}$ and $j \in \{1, 2\}$, we use $\mathbf{X}_i(t) \in \mathbb{R}^n$ to denote the transmit signal from node i, and $Z_j(t) \in \mathbb{R}$ to denote independently and identically distributed (i.i.d.) Gaussian noise with distribution $\mathcal{N}(0, \sigma^2)$. The channel is assumed static and the channel gain from node i to j is denoted by an n-dimensional column vector $\mathbf{h}_{ij}$. We assume channel reciprocity holds so that $\mathbf{h}_{ij} = \mathbf{h}_{ji}$. In the first stage, the outputs of the network at time $t = 1, 2, \dots, N$, are given by

$$Y_1(t) = \mathbf{h}_{A1}^T \mathbf{X}_A(t) + \mathbf{h}_{B1}^T \mathbf{X}_B(t) + Z_1(t), \tag{1}$$

$$Y_2(t) = \mathbf{h}_{A2}^T \mathbf{X}_A(t) + \mathbf{h}_{B2}^T \mathbf{X}_B(t) + Z_2(t). \tag{2}$$

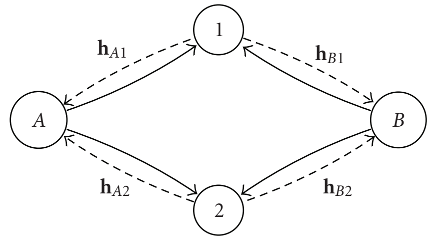

Figure 1: Model of two-way two-relay network. The labels of the arrows indicate the corresponding link gains.

In the second stage, for $t = N + 1, N + 2, \dots, 2N$, the outputs at the terminal nodes are

$$\mathbf{Y}_A(t) = \mathbf{h}_{A1} X_1(t) + \mathbf{h}_{A2} X_2(t) + \mathbf{Z}_A(t), \tag{3}$$

$$\mathbf{Y}_B(t) = \mathbf{h}_{B2} X_2(t) + \mathbf{h}_{B1} X_1(t) + \mathbf{Z}_B(t), \tag{4}$$

where $X_j(t) \in \mathbb{R}$, $j \in \{1, 2\}$ is the transmit symbol of relay j, $\mathbf{Z}_i(t) \in \mathbb{R}^n$ for $i \in \{A, B\}$ is a Gaussian random vector with each component i.i.d according to $\mathcal{N}(0, \sigma^2)$. We assume that the link gains $\mathbf{h}_{A1}$, $\mathbf{h}_{A2}$, $\mathbf{h}_{B1}$, and $\mathbf{h}_{B2}$ are time-invariant and known to all nodes. We have the following power constraints in each stage:

$$\frac{1}{N} \sum_{t=1}^{N} \mathbf{X}_i(t)^T \mathbf{X}_i(t) \le P_i \tag{5}$$

for $i \in \{A, B\}$, and

$$\frac{1}{N} \sum_{t=N+1}^{2N} X_j^2(t) \le P_j \tag{6}$$

for $j \in \{1, 2\}$, where P_A, P_B, P_1, and P_2 denote the power constraints on terminals A and B and relays 1 and 2, respectively.

Let R_A and R_B be the data rates of terminal A and B, respectively. In a period consisting of $2N$ channel symbols (N symbols for each phase), terminal A wants to send one of the 2^{2NR_A} symbols to terminal B, and terminal B wants to send one of the 2^{2NR_B} symbols to terminal A. A $(2^{2NR_A}, 2^{2NR_B}, 2N)$ code for the TWTR network consists of two message sets $M_A = \{1, 2, \dots, 2^{2NR_A}\}$ and $M_B = \{1, 2, \dots, 2^{2NR_B}\}$, two encoding functions

$$f_i : M_{\mathrm{i}} \longrightarrow (\mathbb{R}^n)^N, \quad i \in \{A, B\}, \tag{7}$$

two relay functions

$$\phi_j : \mathbb{R}^N \longrightarrow \mathbb{R}^N, \quad j \in \{1, 2\}, \tag{8}$$

and two decoding functions

$$\begin{aligned} g_A &: (\mathbb{R}^n)^N \times M_A \longrightarrow M_B, \\ g_B &: (\mathbb{R}^n)^N \times M_B \longrightarrow M_A. \end{aligned} \tag{9}$$

For $i = A, B$, terminal i transmits the codeword $f_i(m_i)$ in stage one, where m_i is the message to be transmitted. For

$j = 1, 2$, relay j applies the function ϕ_j to its received signal and transmits the resulting signal in the second stage. Let the received signals at terminals A and B be $\mathbf{Y}_A^N$ and $\mathbf{Y}_B^N$, respectively. In this paper, we will use a superscript "$\mathbf{Y}^N$" to indicate a sequence of length N. So $\mathbf{Y}_A^N$ and $\mathbf{Y}_B^N$ are sequences of length N, with each component equal to a vector in $\mathbb{R}^n$. After the second stage, terminal i decodes the message from the other source node by g_i. We note that the decoding function g_i uses the message from source terminal i as input as well. We say that a decoding error occurs if $g_A(\mathbf{Y}_A^N, m_A) \neq m_B$ or $g_B(\mathbf{Y}_B^N, m_B) \neq m_A$. The average probability of error is

$$P_e^{2N} \triangleq \frac{1}{|M_A||M_B|} \times \sum_{\substack{(m_A,m_B) \\ \in M_A \times M_B}} \Pr\Big\{g_A\Big(\mathbf{Y}_A^N, m_A\Big) \neq m_B, \text{or} \quad g_B\Big(\mathbf{Y}_B^N, m_B\Big) \neq m_A \mid (m_A, m_B) \text{ is sent}\Big\}. \tag{10}$$

A rate pair (R_A, R_B) is said to be achievable if there exists a sequence of $(2^{2NR_A}, 2^{2NR_B}, 2N)$ codes, satisfying the power constraints in (5) and (6), with $P_e^{2N} \to 0$ as $N \to \infty$.

Although the terminals are equipped with n antennas, the transmitted signals from the terminals are essentially 2 dimensional. To see this, we observe that the first term in the right hand side of (1), namely, $\mathbf{h}_{A1}^T \mathbf{X}_A(t)$, is a projection of $\mathbf{X}_A(t)$ in the direction of $\mathbf{h}_{A1}$. Any signal component of $\mathbf{X}_A(t)$ orthogonal to $\mathbf{h}_{A1}$ will not be picked up by relay 1. Likewise, from (2), we see that any signal component of $\mathbf{X}_A(t)$ orthogonal to $\mathbf{h}_{A2}$ will not be sensed by relay 2. There is no loss of generality, if we assume that the signals transmitted from the terminals take the following form:

$$\mathbf{X}_i(t) = \mathbf{H}_i \boldsymbol{\lambda}_i(t) \tag{11}$$

for $i \in \{A, B\}$, where $\mathbf{H}_i \triangleq [\mathbf{h}_{i1} \ \ \mathbf{h}_{i2}]$ is an $n \times 2$ matrix, and the two components in $\boldsymbol{\lambda}_i(t) \triangleq [\lambda_{i1}(t) \ \ \lambda_{i2}(t)]^T$ represent the projections of $\mathbf{X}_i(t)$ on $\mathbf{h}_{i1}$ and $\mathbf{h}_{i2}$. We consider the 2-dimensional vector $\boldsymbol{\lambda}_i(t)$ as the input to the channel at node i. The power constraint in (5) can be written as

$$\frac{1}{N} \sum_{t=1}^{N} \boldsymbol{\lambda}_i(t)^T \mathbf{H}_i^T \mathbf{H}_i \boldsymbol{\lambda}_i(t) \leq P_i, \tag{12}$$

for $i \in \{A, B\}$.

Notations. We will treat 2×1 random vectors $\boldsymbol{\lambda}_A$ and $\boldsymbol{\lambda}_B$ as input signals at terminal A and B, respectively, and let $\mathbf{K}_A$ and $\mathbf{K}_B$ denote their corresponding 2×2 covariance matrices. For $i \in \{A, B\}$ and $j \in \{1, 2\}$, let

$$\Gamma_j^i \triangleq \frac{\mathbf{h}_{ij}^T \mathbf{H}_i \mathbf{K}_i \mathbf{H}_i^T \mathbf{h}_{ij}}{\sigma^2} \tag{13}$$

be the signal to noise ratio of the signal received at relay j from terminal i. Shannon's capacity formula is denoted by $C(x) \triangleq 0.25 \log_2(1 + x)$. Also, for $n \times n$ matrices, we let $C_n(\mathbf{X}) \triangleq 0.25 \log_2 \det(\mathbf{I}_n + \mathbf{X})$, where $\mathbf{I}_n$ denote the $n \times n$ identity matrix. The reason for the factor of 0.25 before the log function, instead of a factor of 0.5 in the original capacity formula, is due to the fact that the total transmission time is divided into two stages of equal length. All logarithms in this paper are in base 2. The set of non-negative real numbers is denoted by $\mathbb{R}_+$. Gaussian distribution with mean zero and covariance matrix $\mathbf{K}$ is denoted by $\mathcal{N}(0, \mathbf{K})$.

3. Review of Coding Techniques and Capacity Regions from Information Theory

The proposed transmission strategies are based on a host of existing coding techniques and capacity results. A review of them is given in this section.

3.1. Physical-Layer Network Coding. In wireless channel, the channel is inherently additive; the received signal is a linear combination of the transmitted signals. This fact is exploited for the two-way relay channel in [18–21]. Consider the following single-antenna two-way network with two sources and one relay in between. There is no direct link between the two sources, and the exchange of data is done via the relay node in the middle. Let $x_i(t)$ be the transmitted signal from source i, for $i = 1, 2$. The transmission is divided into two phases. In the first phase, the relay receives

$$y(t) = x_1(t) + x_2(t) + z(t), \tag{14}$$

where $z(t)$ is an additive noise. For simplicity, it is assumed that both link gains from the sources to the relay are equal to one. In the second phase, the relay amplifies the received signal $y(t)$, and transmits a scaled version $\zeta y(t)$ of $y(t)$, where ζ is a scalar chosen so that the power requirement is met. Since source 1 knows $x_1(t)$, the component $\zeta x_1(t)$ within the received signal at source 1 can be treated as known interference, and hence be subtracted. Similarly, source 2 can subtract $\zeta x_2(t)$ from the received signal. Decoding is then based on the signal after interference subtraction.

3.2. Multiplexed Coding. Multiplexed coding [22] is a useful coding technique for multi-user scenarios in which some user knows the message of another user *a priori*. Consider the two-way relay channel as in the previous paragraph. Node 1 wants to send message m_1 to node 2 via the relay node, and node 2 wants to send message m_2 to node 1 via the relay node. For $i = 1, 2$, let n_i be the number of bits used to represent message m_i. The transmission of the nodes is divided into two phases. In the first phase, the two source nodes transmit. Suppose that the relay node is able to decode m_1 and m_2. For the encoder at the relay, we generate a $2^{n_1} \times 2^{n_2}$ array of codewords. Each codeword is independently drawn according to the Gaussian distribution such that the total power of each codeword is less than or equal to P. In the second phase, the relay node sends the codeword in the (m_1, m_2)-entry in this array. Suppose that the received signal at source node i is corrupted by additive white Gaussian noise with variance σ_i^2, for $i = 1, 2$. At source 1, since m_1 is known, the decoder knows that one of the 2^{n_2} codewords in the row corresponding to m_1 had been transmitted. Out of these 2^{n_2} codewords, it then declares the one based on the maximal likelihood criterion. By the channel coding theorem for the point-to-point Gaussian channel, source 1 can decode reliably at a rate of $0.5\log(1 + P/\sigma_1^2)$. Likewise, by considering the columns in the array of codewords, source 2 can decode at a rate of $0.5\log(1 + P/\sigma_2^2)$.

Multiplexed coding can be implemented using concepts from network coding. We assume, without loss of generality, that $n_2 \geq n_1$. We identify the 2^{n_2} possible messages from source node 2 with the vectors in the n_2-dimensional vector space over the finite field of size 2, $\mathbb{F}_2^{n_2}$, and identify the 2^{n_1} messages from source node 1 with a subspace of $\mathbb{F}_2^{n_2}$ of dimension n_1, say $\mathcal{V}_1$. We generate 2^{n_2} Gaussian codewords independently, one for each vector in $\mathbb{F}_2^{n_2}$. To send messages m_1 and m_2 in the second phase, the relay node transmits the codeword corresponding to $m_1 + m_2$, where the addition is performed using arithmetics in $\mathbb{F}_2^{n_2}$. The output of the decoder at node 1 is a vector in $\mathbb{F}_2^{n_2}$. We subtract from it the vector in $\mathcal{V}_1$ corresponding to m_1. If there is no decoding error, this gives the codeword corresponding to m_2, and the value of m_2 is recovered.

Now let us consider node 2. Since m_2 is known a priori, node 2 is certain that the signal transmitted from the relay is associated with one of the vectors in the affine space $m_2 + \mathcal{V}_1$. The message m_1 can be estimated by comparing the likelihood function of the 2^{n_1} codewords associated with $m_2 + \mathcal{V}_1$. It can be seen that the maximal data rate is the same as in the array approach mentioned in the previous paragraph, but the size of the codebook at the relay reduces from $2^{n_2+n_1}$ to 2^{n_2}.

3.3. Capacity Region for MIMO Channel. Consider a MIMO channel with n_T transmit antennas and n_R receive antennas, with the link gain matrix denoted by a real $n_R \times n_T$ matrix $\mathbf{H}$. The channel output equals

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}, \tag{15}$$

where $\mathbf{X}$ is the n_T-dimensional channel input and $\mathbf{Z}$ is an n_R-dimensional zero-mean colored Gaussian noise vector with covariance matrix $\mathbf{K}_Z$. Without loss of information, we whiten the noise by pre-multiplying both sides of (15) by $\mathbf{K}_Z^{-1/2}$. The transformed channel output is thus

$$\mathbf{Y}' = \mathbf{K}_Z^{-1/2}\mathbf{H}\mathbf{X} + \mathbf{K}_Z^{-1/2}\mathbf{Z}. \tag{16}$$

The covariance matrix of the noise vector $\mathbf{K}_Z^{-1/2}\mathbf{Z}$ is now the $n_R \times n_R$ identity matrix. By the capacity formula for MIMO channel with white Gaussian noise [23], the capacity for the MIMO channel in (15) is given by

$$\frac{1}{2}\log\det\left(\mathbf{I}_{n_R} + \mathbf{K}_Z^{-1/2}\mathbf{H}\mathbf{K}_X\mathbf{H}^T\mathbf{K}_Z^{-1/2}\right), \tag{17}$$

where $\mathbf{K}_X$ denotes the $n_R \times n_R$ covariance matrix of $\mathbf{X}$. Using the identity

$$\det(\mathbf{I}_n + \mathbf{A}\mathbf{B}) \equiv \det(\mathbf{I}_m + \mathbf{B}\mathbf{A}), \tag{18}$$

which holds for any $n \times m$ matrix $\mathbf{A}$ and $m \times n$ matrix $\mathbf{B}$, we rewrite (17) as

$$\frac{1}{2}\log\det\left(\mathbf{I}_{n_T} + \mathbf{H}^T\mathbf{K}_Z^{-1}\mathbf{H}\mathbf{K}_X\right). \tag{19}$$

3.4. Capacity Region for Multiple-Access Channel (MAC). The channel output of the two-user single-antenna Gaussian multiple-access channel is given by

$$y = x_1 + x_2 + z, \tag{20}$$

where x_i is the signal from user i, for $i = 1, 2$, and z is an additive white Gaussian noise with variance σ^2. Each of the two users wants to send some bits to the common receiver. Suppose that the power of user i is limited to P_i, for $i = 1, 2$. The rate pair (R_1, R_2), where R_i is the data rate of user i, is achievable in the above 2-user MAC if and only if it belongs to

$$\mathcal{C}_{\text{mac}}\left(\frac{P_1}{\sigma^2}, \frac{P_2}{\sigma^2}\right) \triangleq \Big\{(R_1, R_2) \in \mathbb{R}_+^2 : \tag{21}$$

$$R_1 \leq 0.5\log_2\left(1 + \frac{P_1}{\sigma^2}\right) \tag{22}$$

$$R_2 \leq 0.5\log_2\left(1 + \frac{P_2}{\sigma^2}\right) \tag{23}$$

$$R_1 + R_2 \leq 0.5\log_2\left(1 + \frac{(P_1 + P_2)}{\sigma^2}\right)\Big\}. \tag{24}$$

We refer the reader to [24] for more details on the optimal coding scheme for MAC.

4. Channel-Network Coding Strategies

We develop five transmission schemes for TWTR network. In the first scheme (AF), the received signals at both relay nodes are amplified and forwarded back to terminals A and B. In the second and third scheme (HLC, HMC), one of the relays employs the amplify forward strategy, while the other decodes the messages from terminals A and B. In the fourth scheme (DF), both relays decode the messages from terminals A and B. In the last strategy (PDF), another mixture of decode-forward and amplify-forward strategy is described.

4.1. Amplify Forward (AF). In this strategy, relay node j ($j \in \{1, 2\}$) buffers the signal received in the first stage, and amplifies it by a factor of ζ_j. The amplified signal

$$X_j(t) = \zeta_j\left(\mathbf{h}_{Aj}^T\mathbf{X}_A(t) + \mathbf{h}_{Bj}^T\mathbf{X}_B(t) + Z_j(t)\right) \tag{25}$$

is then transmitted in the second stage. At the end of the second stage, each terminal, who has the information of itself, subtracts the corresponding term and obtains the desired message from the residual signal.

By putting (25) into (3), we can write the received signal at terminal A as

$$\begin{aligned}\mathbf{Y}_A(t) = {} & \left(\zeta_1 \mathbf{h}_{A1}\mathbf{h}_{A1}^T + \zeta_2 \mathbf{h}_{A2}\mathbf{h}_{A2}^T\right)\mathbf{H}_A \boldsymbol{\lambda}_A(t) \\ & + \left(\zeta_1 \mathbf{h}_{A1}\mathbf{h}_{B1}^T + \zeta_2 \mathbf{h}_{A2}\mathbf{h}_{B2}^T\right)\mathbf{H}_B \boldsymbol{\lambda}_B(t) \\ & + \zeta_1 \mathbf{h}_{A1} Z_1(t) + \zeta_2 \mathbf{h}_{A2} Z_2(t) + \mathbf{Z}_A(t).\end{aligned} \tag{26}$$

Here, we have replaced $\mathbf{X}_A(t)$ and $\mathbf{X}_B(t)$ by their 2-dimensional representations $\mathbf{H}_A\boldsymbol{\lambda}_A(t)$ and $\mathbf{H}_B\boldsymbol{\lambda}_B(t)$. Since terminal A knows its own input $\boldsymbol{\lambda}_A(t)$ as well as the link gains and amplifying factors, the signal component containing $\boldsymbol{\lambda}_A(t)$ as a factor can be subtracted from $\mathbf{Y}_A(t)$. The residual signal is

$$\begin{aligned}& \left(\zeta_1 \mathbf{h}_{A1}\mathbf{h}_{B1}^T + \zeta_2 \mathbf{h}_{A2}\mathbf{h}_{B2}^T\right)\mathbf{H}_B \boldsymbol{\lambda}_B(t) \\ & \quad + \zeta_1 \mathbf{h}_{A1} Z_1(t) + \zeta_2 \mathbf{h}_{A2} Z_2(t) + \mathbf{Z}_A(t).\end{aligned} \tag{27}$$

The message from terminal B can then be decoded using a decoding algorithm for point-to-point MIMO channel. The received signal at terminal B is treated similarly.

Theorem 1. *A rate pair (R_A, R_B) is achievable by the AF strategy if*

$$\begin{aligned}R_A &\le C_2\left(\mathbf{H}_A^T \mathbf{H}_{\text{af}}^T \left(\mathbf{N}_{\text{af}}^B\right)^{-1} \mathbf{H}_{\text{af}} \mathbf{H}_A \mathbf{K}_A\right), \\ R_B &\le C_2\left(\mathbf{H}_B^T \mathbf{H}_{\text{af}} \left(\mathbf{N}_{\text{af}}^A\right)^{-1} \mathbf{H}_{\text{af}}^T \mathbf{H}_B \mathbf{K}_B\right),\end{aligned} \tag{28}$$

where

$$\begin{aligned}\mathbf{N}_{\text{af}}^i &\triangleq \left(\zeta_1^2 \mathbf{h}_{i1}\mathbf{h}_{i1}^T + \zeta_2^2 \mathbf{h}_{i2}\mathbf{h}_{i2}^T + \mathbf{I}_n\right)\sigma^2, \quad i \in \{A, B\}, \\ \mathbf{H}_{\text{af}} &\triangleq \zeta_1 \mathbf{h}_{B1}\mathbf{h}_{A1}^T + \zeta_2 \mathbf{h}_{B2}\mathbf{h}_{A2}^T,\end{aligned} \tag{29}$$

$\zeta_1, \zeta_2 \in \mathbb{R}$ and $\mathbf{K}_A$ and $\mathbf{K}_B$ are 2×2 covariance matrices, such that the following power constraints:

$$\operatorname{Tr}\left(\mathbf{H}_i \mathbf{K}_i \mathbf{H}_i^T\right) \le P_i, \quad \text{for } i = A, B, \tag{30}$$

$$\left(\Gamma_j^A + \Gamma_j^B + 1\right)\zeta_j^2 \sigma^2 \le P_j, \quad \text{for } j = 1, 2, \tag{31}$$

are satisfied.

Proof. The residual signal (27) at terminal A can be written as $\mathbf{H}_{\text{af}}^T \mathbf{H}_B \boldsymbol{\lambda}_B(t)$ plus a noise vector with covariance matrix $\mathbf{N}_{\text{af}}^A$. The residual signal at terminal B equals $\mathbf{H}_{\text{af}} \mathbf{H}_A \boldsymbol{\lambda}_A(t)$ plus a noise vector with covariance matrix $\mathbf{N}_{\text{af}}^B$. Therefore, after self-signal subtraction, the resultant channels can be considered MIMO channels with two transmit antennas and n receive antennas. From (19), we obtain the rate constraints in (28). The inequalities in (30) are the power constraints for terminals A and B, and those in (31) are the power constraints for relays 1 and 2. □

4.2. Hybrid Decode-Amplify Forward with Linear Combination (HLC). In this strategy, relay 1 decodes the messages from terminals A and B, and meanwhile, relay 2 employs the amplify-forward strategy. In order to obtain beamforming gain, after decoding the two messages, relay 1 reconstructs the codewords corresponding to the decoded messages and sends a linear combination of them in the second stage.

In the first stage, relay 1 and terminals A and B form a multiple-access channel with relay 1 as the destination node. We use the optimal encoding scheme for MAC at terminals A and B, and the optimal decoding scheme at relay 1. In the second stage, relay 1 decodes and reconstructs $\mathbf{X}_A(t)$ and $\mathbf{X}_B(t)$, and then transmits a linear combination

$$X_1(t) = \mathbf{z}_A^T \mathbf{X}_A(t) + \mathbf{z}_B^T \mathbf{X}_B(t) \tag{32}$$

for some $\mathbf{z}_A$ and $\mathbf{z}_B \in \mathbb{R}^n$. Relay 2 amplifies $Y_2(t)$ by a scalar factor ζ and transmits $X_2(t) = \zeta Y_2(t)$.

At terminal A, after subtracting the signal component that involves $\mathbf{X}_A(\mathrm{t})$, we get

$$\left(\mathbf{h}_{A1}\mathbf{z}_B^T + \zeta \mathbf{h}_{A2}\mathbf{h}_{B2}^T\right)\mathbf{H}_B \boldsymbol{\lambda}_B(t) + \zeta \mathbf{h}_{A2} Z_2(t) + \mathbf{Z}_A(t). \tag{33}$$

At terminal B, the residual signal after subtraction is

$$\left(\mathbf{h}_{B1}\mathbf{z}_A^T + \zeta \mathbf{h}_{B2}\mathbf{h}_{A2}^T\right)\mathbf{H}_A \boldsymbol{\lambda}_A(t) + \zeta \mathbf{h}_{B2} Z_2(t) + \mathbf{Z}_B(t). \tag{34}$$

The decoding is done by using decoding method for MIMO channel.

Theorem 2. *A rate pair (R_A, R_B) is achievable by the HLC strategy if*

$$(R_A, R_B) \in \frac{1}{2}\mathcal{C}_{\text{mac}}\left(\Gamma_1^A, \Gamma_1^B\right), \tag{35}$$

$$R_A \le C_2\left(\left(\mathbf{H}_{\text{hlc}}^A\right)^T \left(\mathbf{N}_{\text{hlc}}^B\right)^{-1} \mathbf{H}_{\text{hlc}}^A \mathbf{K}_A\right), \tag{36}$$

$$R_B \le C_2\left(\left(\mathbf{H}_{\text{hlc}}^B\right)^T \left(\mathbf{N}_{\text{hlc}}^A\right)^{-1} \mathbf{H}_{\text{hlc}}^B \mathbf{K}_B\right), \tag{37}$$

where

$$\begin{aligned}\mathbf{H}_{\text{hlc}}^A &\triangleq \left(\mathbf{h}_{B1}\mathbf{z}_A^T + \zeta \mathbf{h}_{B2}\mathbf{h}_{A2}^T\right)\mathbf{H}_A, \\ \mathbf{H}_{\text{hlc}}^B &\triangleq \left(\mathbf{h}_{A1}\mathbf{z}_B^T + \zeta \mathbf{h}_{A2}\mathbf{h}_{B2}^T\right)\mathbf{H}_B, \\ \mathbf{N}_{\text{hlc}}^i &\triangleq \left(\zeta^2 \mathbf{h}_{i2}\mathbf{h}_{i2}^T + \mathbf{I}_n\right)\sigma^2, \quad \text{for } i = A, B,\end{aligned} \tag{38}$$

$\mathbf{z}_A, \mathbf{z}_B \in \mathbb{R}^n$, $\zeta \in \mathbb{R}$, and $\mathbf{K}_A$ and $\mathbf{K}_B$ are 2×2 covariance matrices such that the following power constraints:

$$\operatorname{Tr}\left(\mathbf{H}_i \mathbf{K}_i \mathbf{H}_i^T\right) \le P_i, \quad \text{for } i = A, B, \tag{39}$$

$$\mathbf{z}_A^T \mathbf{H}_A \mathbf{K}_A \mathbf{H}_A^T \mathbf{z}_A + \mathbf{z}_B^T \mathbf{H}_B \mathbf{K}_B \mathbf{H}_B^T \mathbf{z}_B \le P_1, \tag{40}$$

$$\left(\Gamma_2^A + \Gamma_2^B + 1\right)\zeta^2 \sigma^2 \le P_2 \tag{41}$$

are satisfied.

In (35), the product of a real number x and a set $\mathcal{A}$ is defined as $x\mathcal{A} \triangleq \{xa : a \in \mathcal{A}\}$.

Proof. From the rate constraints for MAC channel in (22)–(24), we have the rate constraints for relay 1 in (35). We multiply by a factor of one half because the first phase only occupies half of the total transmission time.

The conditions in (36) and (37) are derived from the capacity formula for MIMO channel with colored noise in (19). The inequalities in (39) are the power constraints for sources A and B. The inequalities in (40) and (41) are the power constraints for relays 1 and 2, respectively. □

The parameters $\mathbf{z}_A$, $\mathbf{z}_B$, $\mathbf{K}_A$, and $\mathbf{K}_B$ can be obtained by running an optimization algorithm. For example, we can aim at maximizing a weighted sum $w_A R_A + w_B R_B$. The values of $\mathbf{z}_A$, $\mathbf{z}_B$, $\mathbf{K}_A$ and $\mathbf{K}_B$ which maximize the weighted sum $w_A R_A + w_B R_B$ are chosen.

4.3. Hybrid Decode-Amplify Forward with Multiplexed Coding (HMC). As in the previous strategy, relay 1 decodes and forwards the messages from A and B, and relay 2 amplifies and transmits the received signal. However, in this strategy, relay 1 re-encodes the messages into a new codeword to be sent out in the second stage. Terminals A and B decode the desired messages based on multiplexed coding.

Theorem 3. *A rate pair* (R_A, R_B) *is achievable by the HMC strategy if* R_A *and* R_B *satisfy*

$$(R_A, R_B) \in \frac{1}{2}\, \mathcal{C}_{\text{mac}}\left(\Gamma_1^A, \Gamma_1^B\right), \tag{42}$$

$$R_A \le C_n\left(\mathbf{G}_{\text{hmc}}^A \left(\mathbf{N}_{\text{hmc}}^B\right)^{-1}\right), \tag{43}$$

$$R_B \le C_n\left(\mathbf{G}_{\text{hmc}}^B \left(\mathbf{N}_{\text{hmc}}^A\right)^{-1}\right), \tag{44}$$

where

$$\mathbf{G}_{\text{hmc}}^A \triangleq \mathbf{h}_{B1}\mathbf{h}_{B1}^T P_1 + \zeta^2 \mathbf{h}_{B2}\mathbf{h}_{A2}^T \mathbf{H}_A \mathbf{K}_A \mathbf{H}_A^T \mathbf{h}_{A2}\mathbf{h}_{B2}^T, \tag{45}$$

$$\mathbf{G}_{\text{hmc}}^B \triangleq \mathbf{h}_{A1}\mathbf{h}_{A1}^T P_1 + \zeta^2 \mathbf{h}_{A2}\mathbf{h}_{B2}^T \mathbf{H}_B \mathbf{K}_B \mathbf{H}_B^T \mathbf{h}_{B2}\mathbf{h}_{A2}^T, \tag{46}$$

$$\mathbf{N}_{\text{hmc}}^i \triangleq \left(\zeta^2 \mathbf{h}_{i2}\mathbf{h}_{i2}^T + \mathbf{I}_n\right)\sigma^2, \quad \textit{for } i \in \{A, B\}, \tag{47}$$

$\mathbf{K}_A$, $\mathbf{K}_B$ *are* 2×2 *covariance matrices satisfying* (39)*, and* $\zeta \in \mathbb{R}$ *satisfies* (41)*.*

Proof. The proof is by random coding argument and we will sketch the proof below. More details can be found in [25].

Our objective is to show that any rate pair (R_A, R_B) that satisfies the condition in the theorem is achievable. For $i = A, B$, terminal i randomly generates a Gaussian codebook with 2^{2NR_i} codewords with length N, satisfying the power constraint in (5). Label the codewords by $\mathbf{X}_i^N(m_i)$, for $m_i \in M_i$. For relay 1, we generate a $2^{2NR_A} \times 2^{2NR_B}$ array of Gaussian codewords of length N and power P_1. The codeword in row m_A and column m_B is denoted by $X_1^N(m_A, m_B)$, and satisfies the power constraint in (6).

After the first stage, relay 1 is required to decode both messages from terminals A and B. This can be accomplished with arbitrarily small probability of error if the rate constraints for MAC in (22) to (24) are satisfied. This corresponds to the rate constraint in (42). Let the estimated messages from A and B be $\hat{m}_A$ and $\hat{m}_B$.

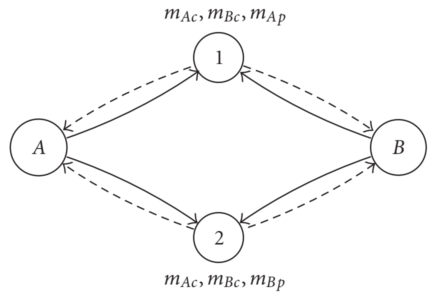

Figure 2: Decoded messages at the two-relays in the DF strategy.

In the second stage, relay 1 transmits $X_1^N(\hat{m}_A, \hat{m}_B)$. Relay 2 amplifies its received signal and transmits $\zeta Y_2(t)$. From (41), the amplified signal has average power no more than P_2.

After subtracting the term $\zeta \mathbf{h}_{A2}\mathbf{h}_{A2}^T \mathbf{X}_A(t)$, which is known to terminal A, the residual signal at terminal A is

$$\left[\mathbf{h}_{A1} X_1(\hat{m}_A, \hat{m}_B)(t) + \zeta \mathbf{h}_{A2}\mathbf{h}_{B2}^T \mathbf{X}_B(t)\right] + \zeta \mathbf{h}_{A2} Z_2(t) + \mathbf{Z}_A(t). \tag{48}$$

Note that terminal A knows its message m_A, and $\hat{m}_A = m_A$ with probability arbitrarily close to one if (42) is satisfied. The idea of multiplexed coding can then be used. In (48), the covariance matrix of the signal in square bracket is given by $\mathbf{G}_{\text{hmc}}^B$ in (46), and the covariance of the noise term is given by $\mathbf{N}_{\text{hmc}}^A$. Applying the capacity expression, we obtain the rate constraint in (44). In a similar manner, we obtain (43). □

4.4. Decode Forward (DF). In the DF strategy, terminal node i, $(i \in \{A, B\})$ splits the message m_i into two parts: the common part m_{ic} and the private part m_{ip}. The two common messages are transmitted via both relay nodes. The private message m_{Ap} is decoded by relay 1 only, and can be interpreted as going through the path from terminal A to relay 1 to terminal B. Symmetrically, the private part of message m_{Bp} is decoded by relay 2 only, and can be interpreted as going through the path from terminal B to relay 2 to terminal A. After the first stage, relay 1 decodes the common messages of both terminals and the private message of terminal A. Relay 2 decodes the common messages of both terminals and the private message of terminal B. The encoding and decoding schemes in the first stage is similar to those developed by Han and Kobayashi for the interference channel (IC) in [26]. Since both relays have access to the common messages, the channel in the second stage can be considered a multiple access channel with common information. Furthermore, since terminals A and B have information of themselves, we can further improve the rate region by the idea of multiplexed coding.

We have the following characterization of the rate region for the DF strategy:

Theorem 4. *For $i \in \{A, B\}$, let R_{ip} and R_{ic} be the rates of the private and common messages, respectively, from terminal i. Let Γ_j denote P_j/σ^2 for $j = 1, 2$, and let $\mathbf{K}_{Ac}$, $\mathbf{K}_{Ap}$, $\mathbf{K}_{Bc}$, and $\mathbf{K}_{Bp}$ denote 2×2 covariance matrices, and*

$$\Gamma_j^{ik} \triangleq \frac{\mathbf{h}_{ij}^T \mathbf{H}_i \mathbf{K}_{ik} \mathbf{H}_i^T \mathbf{h}_{ij}}{\sigma^2} \tag{49}$$

for $i \in \{A, B\}$, $j \in \{1, 2\}$ and $k \in \{p, c\}$. For $j = 1, 2$. A rate pair (R_A, R_B) is achievable if we can decompose $R_A = R_{Ap} + R_{Ac}$ and $R_B = R_{Bp} + R_{Bc}$ such that

$$(R_A, R_{Bc}) \in \frac{1}{2}\mathcal{C}_{\text{mac}}\left(\frac{\Gamma_1^{Ap} + \Gamma_1^{Ac}}{\Gamma_1^{Bp} + 1}, \frac{\Gamma_1^{Bc}}{\Gamma_1^{Bp} + 1}\right), \tag{50}$$

$$(R_{Ac}, R_B) \in \frac{1}{2}\mathcal{C}_{\text{mac}}\left(\frac{\Gamma_2^{Ac}}{\Gamma_2^{Ac} + 1}, \frac{\Gamma_2^{Bp} + \Gamma_2^{Bc}}{\Gamma_2^{Ac} + 1}\right), \tag{51}$$

$$R_{Ap} \le C\left(\overline{\alpha}_1 \|\mathbf{h}_{B1}\|^2 \Gamma_1\right), \tag{52}$$

$$\begin{aligned} R_A \le C_n\Big(&\Gamma_1 \mathbf{h}_{B1}\mathbf{h}_{B1}^T + \Gamma_2 \mathbf{h}_{B2}\mathbf{h}_{B2}^T \\ &+ \sqrt{\alpha_1\alpha_2\Gamma_1\Gamma_2}\left(\mathbf{h}_{B1}\mathbf{h}_{B2}^T + \mathbf{h}_{B2}\mathbf{h}_{B1}^T\right)\Big), \end{aligned} \tag{53}$$

$$R_{Bp} \le C\left(\overline{\alpha}_2 \|\mathbf{h}_{A2}\|^2 \Gamma_2\right), \tag{54}$$

$$\begin{aligned} R_B \le C_n\Big(&\Gamma_1 \mathbf{h}_{A1}\mathbf{h}_{A1}^T + \Gamma_2 \mathbf{h}_{A2}\mathbf{h}_{A2}^T \\ &+ \sqrt{\alpha_1\alpha_2\Gamma_1\Gamma_2}\left(\mathbf{h}_{A1}\mathbf{h}_{A2}^T + \mathbf{h}_{A2}\mathbf{h}_{A1}^T\right)\Big), \end{aligned} \tag{55}$$

$$\operatorname{Tr}\left(\mathbf{H}_A\left(\mathbf{K}_{Ac} + \mathbf{K}_{Ap}\right)\mathbf{H}_A^T\right) < P_A, \tag{56}$$

$$\begin{aligned} &\operatorname{Tr}\left(\mathbf{H}_B\left(\mathbf{K}_{Bc} + \mathbf{K}_{Bp}\right)\mathbf{H}_B^T\right) < P_B, \\ &\alpha_1 + \overline{\alpha}_1 < 1, \quad \alpha_2 + \overline{\alpha}_2 < 1 \end{aligned} \tag{57}$$

for some nonnegative α_j and $\overline{\alpha}_j$.

Details of the DF coding scheme and the proof of Theorem 4 are given in the Appendix.

4.5. Partial Decode Forward (PDF). In the PDF strategy, both relays decode the message of terminal A. Each relay then subtracts the reconstructed signal of terminal A from the received signal. Call the resulting signal the residual signal. The message of terminal A is re-encoded into a new codeword, and linearly combined with the residual signal. This linear combination is then transmitted in the second stage. Since both relays know the message of terminal A, the two-relays can jointly re-encode the message of terminal A using some encoding scheme for a MIMO channel with two transmit antennas and n receive antennas.

Theorem 5. *A rate pair (R_A, R_B) is achievable by the PDF strategy if it satisfies*

$$R_A \le \min\left\{C\left(\frac{\Gamma_1^A}{\left(\Gamma_1^B + 1\right)}\right), C\left(\frac{\Gamma_2^A}{\left(\Gamma_2^B + 1\right)}\right)\right\}, \tag{58}$$

$$R_A \le C_2\left((\mathbf{H}_B)^T\left(\mathbf{N}_{\text{pdf}}^B\right)^{-1}\mathbf{H}_B\mathbf{K}_R\right), \tag{59}$$

$$R_B \le C_2\left(\left(\mathbf{H}_{\text{pdf}}^B\right)^T\left(\mathbf{N}_{\text{pdf}}^A\right)^{-1}\mathbf{H}_{\text{pdf}}^B\mathbf{K}_B\right), \tag{60}$$

where

$$\begin{aligned} \mathbf{N}_{\text{pdf}}^i &\triangleq \left(\zeta_1^2\mathbf{h}_{i1}\mathbf{h}_{i1}^T + \zeta_2^2\mathbf{h}_{i2}\mathbf{h}_{i2}^T + \mathbf{I}_n\right)\sigma^2, \\ \mathbf{H}_{\text{pdf}}^B &\triangleq \left(\zeta_1\mathbf{h}_{A1}\mathbf{h}_{B1}^T + \zeta_2\mathbf{h}_{A2}\mathbf{h}_{B2}^T\right)\mathbf{H}_B, \end{aligned} \tag{61}$$

and $\zeta_j \in \mathbb{R}$ and $\mathbf{K}_A$, $\mathbf{K}_B$, $\mathbf{K}_R$ are 2×2 covariance matrices such that the following power constraints hold

$$\operatorname{Tr}\left(\mathbf{H}_i\mathbf{K}_i\mathbf{H}_i^T\right) \le P_i, \quad \text{for } i = A, B, \tag{62}$$

$$\mathbf{K}_R(j,j) + \left(\Gamma_j^B + 1\right)\sigma^2\zeta_j^2 \le P_j \tag{63}$$

for $j = 1, 2$. (Here, $\mathbf{K}_R(j,j)$ denotes the jth diagonal entry in $\mathbf{K}_R$.)

Proof. The two-relays treat the signal originated from terminal B as noise, and decode the message of terminal A. The rate requirement in (58) guarantees that the message of terminal A can be decoded with arbitrarily small probability of error at both relays. Let the decoded message of terminal A be denoted by $\hat{m}_A$.

For $j = 1, 2$, the reconstructed signal $\mathbf{h}_{Aj}^T\mathbf{X}_A(t)$ is then subtracted from $Y_j(t)$. The residual signal at relay j is $\mathbf{h}_{Bj}^T\mathbf{X}_B(t) + Z_j(t)$.

At the relays, we employ two Gaussian codebooks for the re-encoding of the message from terminal A. For each message m_A, we generate two correlated codewords $U_{1,m_A}(t)$ and $U_{2,m_A}(t)$, with mean zero and each pair of symbols at any t distributed according to a 2×2 covariance matrix $\mathbf{K}_R$. At relay j, the decoded message $\hat{m}_A$ is re-encoded into $U_{j,\hat{m}_A}(t)$, which is a codeword with power $\mathbf{K}_R(j,j)$. In the second stage, relay j transmits

$$U_{j,\hat{m}_A}(t) + \zeta_j\left(\mathbf{h}_{Bj}^T\mathbf{X}_B(t) + Z_j(t)\right), \tag{64}$$

for some amplifying factor ζ_j. The inequality in (63) ensures that the power constraint is satisfied at the relays.

At the end of stage 2, terminal A subtracts the signal component that involves U_{1,m_A} and U_{2,m_A} from its received signal and obtains

$$\mathbf{H}_{\text{pdf}}^B\boldsymbol{\lambda}_B(t) + \zeta_1\mathbf{h}_{A1}Z_1(t) + \zeta_2\mathbf{h}_{A2}Z_2(t) + \mathbf{Z}_A(t). \tag{65}$$

From the capacity formula for MIMO channel (19), terminal A can recover the message from terminal B reliably if (60) is satisfied.

For the decoding in terminal B, we subtract all terms involving $\mathbf{X}_B(t)$, and get

$$\mathbf{H}_B\begin{bmatrix} U_{1,\hat{m}_A}(t) \\ U_{2,\hat{m}_A}(t)\end{bmatrix} + \zeta_1\mathbf{h}_{B1}Z_1(t) + \zeta_2\mathbf{h}_{B2}Z_2(t) + \mathbf{Z}_B(t). \quad (66)$$

This is equivalent to a MIMO channel with link gain matrix $\mathbf{H}_B$ and colored noise. Recall that $\mathbf{K}_R$ is the covariance matrix of the encoded signal. By the capacity formula of MIMO channel (19), we obtain the rate constraint in (59). □

Remark 1. We note that the matrices $\mathbf{N}^i_{\text{af}}$, $\mathbf{N}^i_{\text{hlc}}$, $\mathbf{N}^i_{\text{hmc}}$ and $\mathbf{N}^i_{\text{pdf}}$, for $i = A, B$, are invertible. Indeed, by checking that $\mathbf{v}^T\mathbf{N}\mathbf{v}$ is strictly positive for all non-zero $\mathbf{v} \in \mathbb{R}^n$, we see that the matrix is positive definite, and hence invertible.

5. Performance in High SNR Regime

In this section, we compare the performance of the five strategies described in the previous section in the high Signal-to-Noise Ratio (SNR) regime.

For fixed powers and link gains, let $C_{\text{sum}}(\sigma^2)$ denote the sum rate $R_A + R_B$ as a function of the noise variance σ^2. We use the *multiplexing gain* (also called *degree of freedom*) [27], defined by

$$M \triangleq \lim_{\sigma^2\to 0}\frac{C_{\text{sum}}(\sigma^2)}{(1/2)\log(\sigma^{-2})}, \quad (67)$$

as the performance measure at high SNR. At high SNR, that is, when σ^2 is very small, we can approximate the sum rate by $(M/2)\log(\sigma^{-2})$ if the multiplexing gain is equal to M.

Consider the multiplexing gain of the AF scheme. When the sum rate $R_A + R_B$ is maximized subject to the rate constraints (28) in Theorem 1, the equalities in (28) hold. We can assume without loss of generality that

$$R_A = C_2\left(\mathbf{H}_A^T\mathbf{H}_{\text{af}}^T\left(\mathbf{N}^B_{\text{af}}\right)^{-1}\mathbf{H}_{\text{af}}\mathbf{H}_A\mathbf{K}_A\right), \quad (68)$$

$$R_B = C_2\left(\mathbf{H}_B^T\mathbf{H}_{\text{af}}\left(\mathbf{N}^A_{\text{af}}\right)^{-1}\mathbf{H}_{\text{af}}^T\mathbf{H}_B\mathbf{K}_B\right). \quad (69)$$

We first suppose that the covariance matrices $\mathbf{K}_A$ and $\mathbf{K}_B$, and the amplifying constants ζ_1 and ζ_2, are fixed. Note that if the power constraint in (31) holds, then it continues to hold if σ^2 becomes smaller. Therefore, when $\sigma^2 \to 0$, the power constraints in (30) and (31) are satisfied.

Each of the expressions in (68) and (69) can be written in the form

$$\frac{1}{4}\log\det\left(\mathbf{I}_2 + \frac{\mathbf{M}}{\sigma^2}\right), \quad (70)$$

where $\mathbf{M}$ is a 2×2 matrix that equals

$$\mathbf{H}_A^T\mathbf{H}_{\text{af}}^T\left(\mathbf{N}^B_{\text{af}}\right)^{-1}\mathbf{H}_{\text{af}}\mathbf{H}_A\mathbf{K}_A, \quad \text{or} \quad (71)$$

$$\mathbf{H}_B^T\mathbf{H}_{\text{af}}\left(\mathbf{N}^A_{\text{af}}\right)^{-1}\mathbf{H}_{\text{af}}^T\mathbf{H}_B\mathbf{K}_B. \quad (72)$$

By singular value decomposition [28, Chapter 7], we can factor $\mathbf{M}$ as $\mathbf{U}\boldsymbol{\Lambda}\mathbf{V}$, where $\mathbf{U}$ and $\mathbf{V}$ are 2×2 unitary matrices, and $\boldsymbol{\Lambda} = [\lambda_{ij}]$ is a diagonal matrix with non-negative diagonal entries $\lambda_{11} \geq \lambda_{22} \geq 0$. The number of positive diagonal entries in $\boldsymbol{\Lambda}$ is precisely the rank of $\mathbf{M}$. We can rewrite (70) as

$$\frac{1}{4}\log\det\left(\mathbf{U}^{-1}\mathbf{V}^{-1} + \frac{\boldsymbol{\Lambda}}{\sigma^2}\right). \quad (73)$$

Suppose that $\mathbf{U}^{-1}\mathbf{V}^{-1}$ is equal to $[a_{ij}]_{i,j=1}^2$. The determinant

$$\begin{vmatrix} a_{11} + \dfrac{\lambda_{11}}{\sigma^2} & a_{12} \\ a_{21} & a_{22} + \dfrac{\lambda_{22}}{\sigma^2}\end{vmatrix} \quad (74)$$

in (73) can be expanded as a polynomial in σ^{-2}, with the degree equal to the rank of $\mathbf{M}$. Therefore, the limit

$$\lim_{\sigma^2\to 0}\frac{(1/4)\log\det(\mathbf{I}_2 + \mathbf{M}/\sigma^2)}{(1/2)\log(\sigma^{-2})} \quad (75)$$

depends only on the rank of the matrix $\mathbf{M}$, and equals 0, 0.5, or 1, if the rank of $\mathbf{M}$ is 0, 1, or 2, respectively. The problem of determining the multiplexing gain now reduces to determining the rank of the matrices in (71) and (72).

Recall that the rank function satisfies the following properties [28, page 13]: (i) if $\mathbf{A}$ and $\mathbf{C}$ are square invertible matrices, then $\text{rank}(\mathbf{ABC}) = \text{rank}(\mathbf{B})$ for all matrix $\mathbf{B}$, whenever the matrix multiplications are well-defined; (ii) for all $m\times n$ matrices $\mathbf{A}$, we have $\text{rank}(\mathbf{A}^T\mathbf{A}) = \text{rank}(\mathbf{A})$. Consider the matrix in (72). After replacing $\mathbf{H}_{\text{af}}$ by its definition, we can express the matrix in (72) as

$$\mathbf{H}_B^T\mathbf{H}_B\mathbf{Z}\mathbf{H}_A^T\left(\mathbf{N}^A_{\text{af}}\right)^{-1}\mathbf{H}_A\mathbf{Z}\mathbf{H}_B^T\mathbf{H}_B\mathbf{K}_B, \quad (76)$$

where $\mathbf{Z}$ denotes the diagonal matrix $\text{diag}(\zeta_1^2, \zeta_2^2)$. We assume that $\mathbf{H}_A$ and $\mathbf{H}_B$ have full rank. This assumption holds with probability one if the link gains are generated from a continuous probability distribution function such as Rayleigh. Also, we assume that $\mathbf{Z}$, $\mathbf{K}_A$, and $\mathbf{K}_B$ are of full rank. This assumption does not incur any loss of generality, because they are design parameters that we can choose. We can perturb them infinitesimally, and the resulting matrices will be of rank two, but the value on the right hand side of (69) deviates negligibly. By property (i), and the fact that $\mathbf{H}_B^T\mathbf{H}_B$, $\mathbf{Z}$, and $\mathbf{K}_B$ are invertible 2×2 matrices, the rank of the matrix in (76) is equal to the rank of $\mathbf{H}_A^T(\mathbf{N}^A_{\text{af}})^{-1}\mathbf{H}_A$. Then we get

$$\begin{aligned} &\text{rank}\left(\mathbf{H}_A^T\left(\mathbf{N}^A_{\text{af}}\right)^{-1}\mathbf{H}_A\right) \\ &\quad = \text{rank}\left(\mathbf{H}_A^T\left(\mathbf{N}^A_{\text{af}}\right)^{-1/2}\left(\mathbf{N}^A_{\text{af}}\right)^{-1/2}\mathbf{H}_A\right) \\ &\quad = \text{rank}\left(\left(\mathbf{N}^A_{\text{af}}\right)^{-1/2}\mathbf{H}_A\right) \quad \text{(by Property (ii))} \\ &\quad = \text{rank}(\mathbf{H}_A) \quad \text{(by Property (i))} \\ &\quad = 2. \end{aligned} \quad (77)$$

Similarly, we can show that the rank of the matrix in (71) is equal to two.

For fixed invertible covariance matrices $\mathbf{K}_A$ and $\mathbf{K}_B$, and positive real numbers ζ_1 and ζ_2,

$$\lim_{\sigma^2 \to 0} \frac{\text{R.H.S. of (69)} + \text{R.H.S. of (69)}}{0.5 \log(\sigma^{-2})} = 2. \tag{78}$$

Since the above argument holds for all invertible $\mathbf{K}_A$ and $\mathbf{K}_B$, and positive ζ_1 and ζ_2, we conclude that the multiplexing gain of the AF strategy is equal to 2.

For HLC and HMC, relay 1 is required to decode the messages of the terminals, and in both schemes the sum rate is subject to the sum rate constraint in the MAC channel in the first phase. The multiplexing gains of both the HLC and HMC strategies are limited by

$$\lim_{\sigma^2 \to 0} \frac{C\left(\Gamma_1^A + \Gamma_1^B\right)}{0.5 \log(\sigma^{-2})} = 0.5. \tag{79}$$

Similarly, the multiplexing gain of DF is also limited by the decoding of messages at the relays. The rate constraints (50) and (51) imply that it is no more than 0.5.

The multiplexing gain of the PDF scheme is somewhere in between the multiplexing gains of AF and DF. The transmission from terminal B to terminal A can be considered AF, while the transmission from terminal A to terminal B in the other direction is limited by the message decoding after stage 1. From (58), we get

$$\lim_{\sigma^2 \to 0} \frac{R_A(\sigma^2)}{0.5 \log(\sigma^{-2})} \le 0.5, \tag{80}$$

and from (60), we have

$$\lim_{\sigma^2 \to 0} \frac{R_B(\sigma^2)}{0.5 \log(\sigma^{-2})} = \frac{1}{2} \operatorname{rank}(\mathbf{H}_A) = 1, \tag{81}$$

provided that the $\mathbf{H}_A$ has full rank. Therefore, its maximal multiplexing gain is 1.5.

We summarize the performance of the five schemes at high SNR in Table 1. We can see that the AF strategy has the highest multiplexing gain. It is well known that the maximal multiplexing gain of the Gaussian MIMO channel with two transmit antennas and two received antennas is equal to two [23]. We see that at high SNR, the AF strategy behaves like a transmission scheme achieving full multiplexing gain in the MIMO channel with two transmit antennas and two received antennas.

6. Numerical Examples

We compare the information rates achievable by the proposed strategies in Section 4 with the cut-set outer bound in [29]. Since the derivation is straightforward, we state the outer bound without proof. For $i, j \in \{1, 2\}$, and $k \in \{A, B\}$, let

$$\Gamma_{ij}^k \triangleq \frac{\mathbf{h}_{ki}^T \mathbf{H}_k \mathbf{K}_k \mathbf{H}_k^T \mathbf{h}_{kj}}{\sigma^2}. \tag{82}$$

Theorem 6 (Outer bound). *A rate pair (R_A, R_B) is achievable in the TWTR network only if it satisfies*

$$\begin{aligned}
R_A \le \min\Big\{ & C\left(\Gamma_1^A + \Gamma_2^A + \Gamma_1^A\Gamma_2^A - \Gamma_{12}^A\Gamma_{21}^A\right), \\
& C\left(\Gamma_2^A\right) + C_n\left(\mathbf{h}_{B1}\mathbf{h}_{B1}^T(1-\rho^2)\Gamma_1\right), \\
& C\left(\Gamma_1^A\right) + C_n\left(\mathbf{h}_{B2}\mathbf{h}_{B2}^T(1-\rho^2)\Gamma_2\right), \\
& C_n\Big(\mathbf{h}_{B1}\mathbf{h}_{B1}^T\Gamma_1 + \mathbf{h}_{B2}\mathbf{h}_{B2}^T\Gamma_2 \\
& \quad + \rho\left(\mathbf{h}_{B1}\mathbf{h}_{B2}^T + \mathbf{h}_{B2}\mathbf{h}_{B1}^T\right)\sqrt{\Gamma_1\Gamma_2}\Big)\Big\}, \\
R_B \le \min\Big\{ & C\left(\Gamma_1^B + \Gamma_2^B + \Gamma_1^B\Gamma_2^B - \Gamma_{12}^A\Gamma_{21}^B\right), \\
& C\left(\Gamma_2^B\right) + C_n\left(\mathbf{h}_{A1}\mathbf{h}_{A1}^T(1-\rho^2)\Gamma_1\right), \\
& C\left(\Gamma_1^B\right) + C_n\left(\mathbf{h}_{A2}\mathbf{h}_{A2}^T(1-\rho^2)\Gamma_2\right), \\
& C_n\Big(\mathbf{h}_{A1}\mathbf{h}_{A1}^T\Gamma_1 + \mathbf{h}_{A2}\mathbf{h}_{A2}^T\Gamma_2 \\
& \quad + \rho\left(\mathbf{h}_{A1}\mathbf{h}_{A2}^T + \mathbf{h}_{A2}\mathbf{h}_{A1}^T\right)\sqrt{\Gamma_1\Gamma_2}\Big)\Big\},
\end{aligned} \tag{83}$$

for some real number ρ between 0 and 1, and 2×2 covariance matrices $\mathbf{K}_A$ and $\mathbf{K}_B$ such that $\operatorname{Tr}(\mathbf{H}_i\mathbf{K}_i\mathbf{H}_i^T) \le P_i$ holds for $i = A, B$.

We select several typical channel realizations and show the corresponding achievable rate regions in Figure 3 to Figure 8. To simplify the calculation, we consider the single antenna case where $n = 1$. The power constraint is set to $P = 1$ and the noise variance is set to $\sigma^2 = 1$.

In Figure 3, we plot the rate regions when all link gains are large (the link gain is 10 for all links). As mentioned in the previous section, the AF strategy has the largest multiplexing gain in the high SNR regime. We can see in Figure 3 that the AF strategy achieves the largest sum rate.

In Figures 4 and 5, we consider the case where relay 1 has larger link gains than relay 2. In Figure 4, the link gains h_{A1} and h_{B1} are the same. In this case, HMC dominates all other strategies. In Figure 5, the two link gains, h_{A1} and h_{B1}, are not equal. In this case, HLC dominates HMC. HLC performs better in this asymmetric case because of its ability to adjust power between signals and utilize the beamforming gain.

When both relays are close to one of the terminals, PDF has the best performance, as can be seen in Figure 6. The reason is that both relays are able to decode reliably the message from the closer terminal, and then they cooperatively forward the message to the other terminal using MIMO techniques.

Figures 7 and 8 presents two scenarios in which DF dominates all other transmission strategies. We remark that DF is quite flexible in that it has many tunable parameters. The case where both h_{A1} and h_{B2} are relatively large is shown in Figure 7. Another case where h_{A1} and h_{A2} are larger than h_{B1} and h_{B2} is shown in Figure 8. In both cases, DF is much better than other strategies.

We can further summarize the numerical results in Table 2. It is not supposed to be a precise description on the

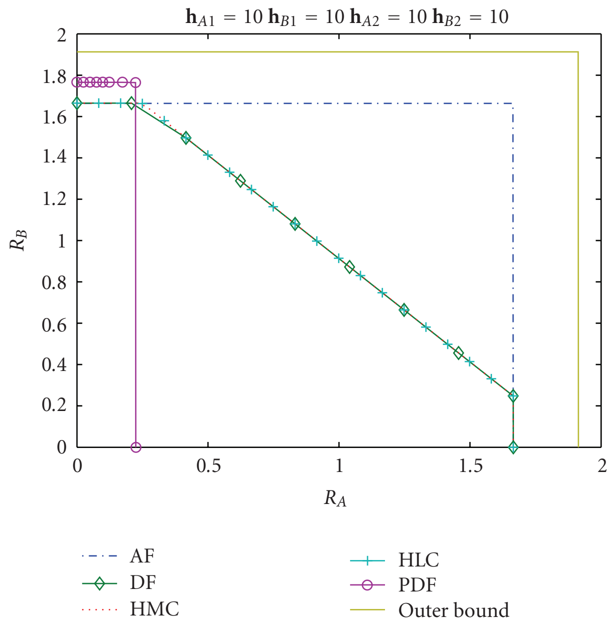

FIGURE 3: The achievable rate regions when all link gains are large.

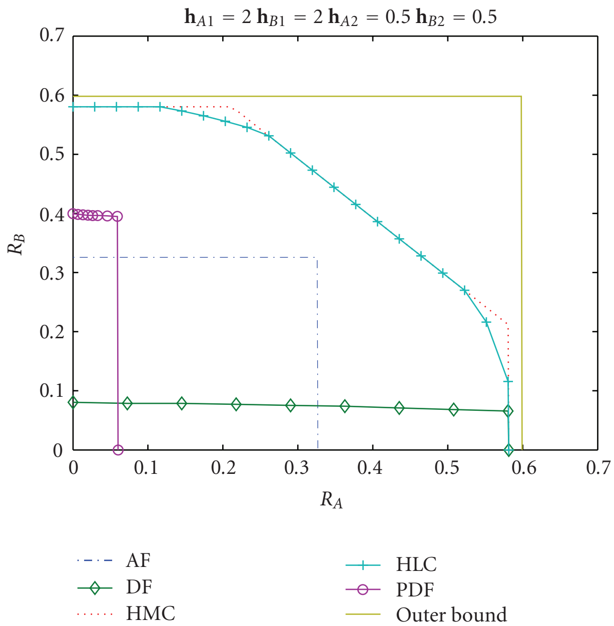

FIGURE 4: The achievable rate regions when one relay has large link gains (symmetric case).

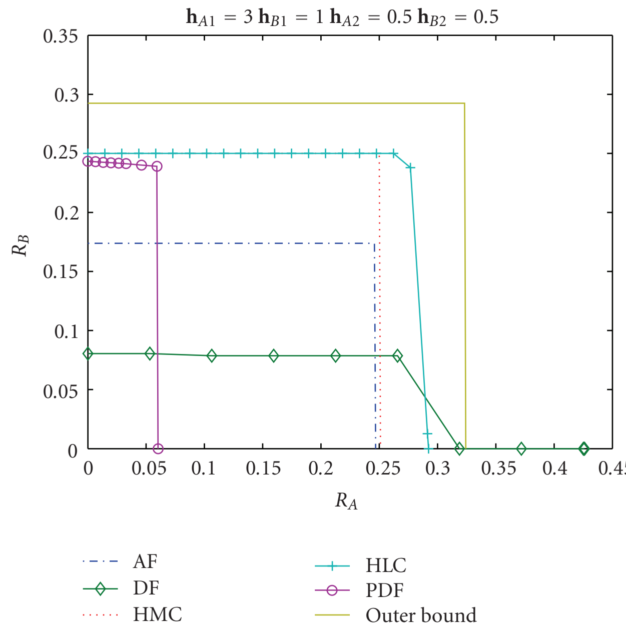

FIGURE 5: The achievable rate regions when one relay has large link gains (symmetric case).

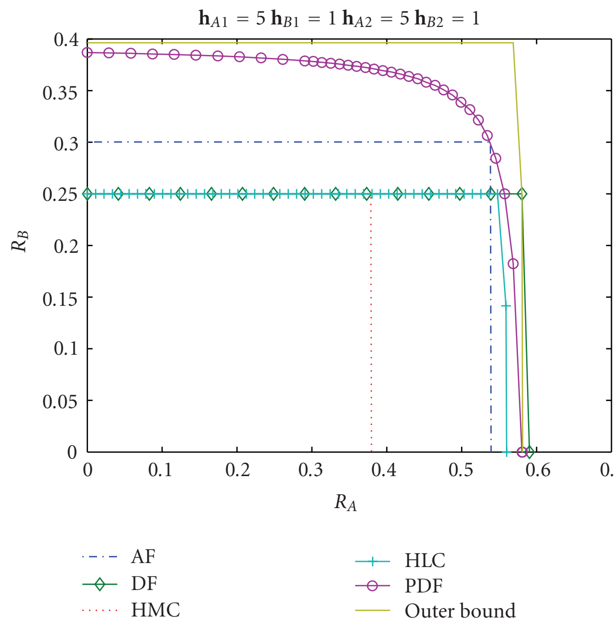

FIGURE 6: The achievable rate regions when both relays are close to terminal A.

relative merits of the schemes. Instead, it provides a rough guideline for easy selection of a suitable scheme. In the table, "G" refers to "the channel condition is good" and "B" refers to "the channel condition is bad." We say that a channel is good if its link gain is two to three times, or more, than the link gain of a bad channel. When all the link gains are large, we should use AF. In the case when one pair of the opposite links of the network is good, whereas the other pair is weak, DF provides larger throughput. If one of the relays is good but the other relay is bad, HMC or HLC should be used.

TABLE 1: Multiplexing gains of the transmission schemes in the high SNR regime.

Scheme	AF	HMC, HLC, DF	PDF
Multiplexing gain	2	0.5	1.5

PDF scheme is the best one in the scenario where one of the sources has large link gains but the other does not.

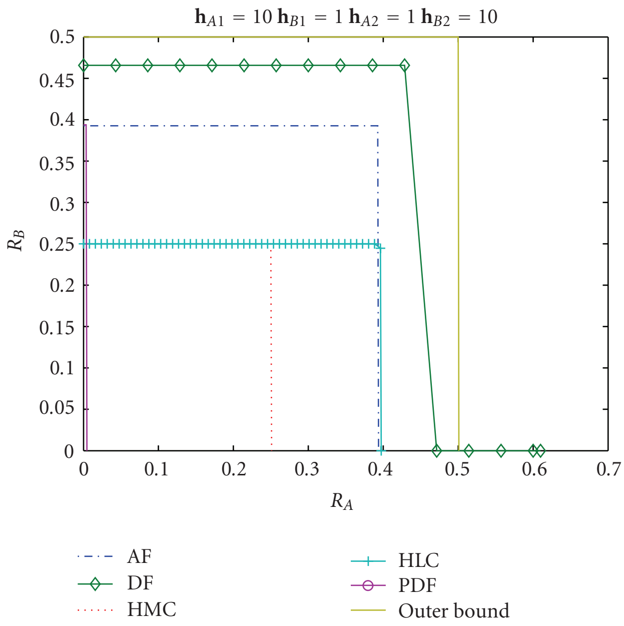

FIGURE 7: The achievable rate regions and the outer bound.

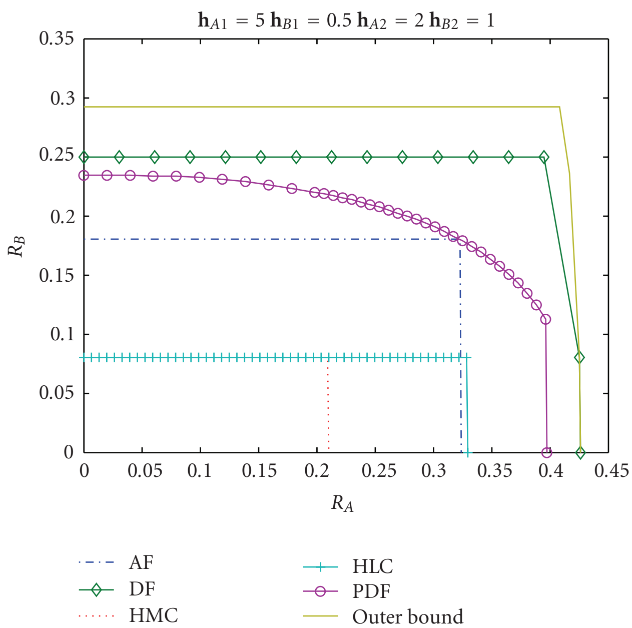

FIGURE 8: The achievable rate regions and the outer bound.

TABLE 2: Performance guideline for the two-way two-relay network in the medium SNR regime.

$\|\mathbf{h}_{A1}\|$	$\|\mathbf{h}_{B1}\|$	$\|\mathbf{h}_{A2}\|$	$\|\mathbf{h}_{B2}\|$	Scheme
G	G	G	G	AF
G	B	B	G	DF
G	G	B	B	HMC, HLC
G	B	G	B	PDF

7. Conclusion

We have devised several transmission strategies for the TWTR network, each of which is derived from a mix-and-match of several basic building blocks, namely, amplify-forward strategy, decode-forward strategy, and physical-layer network coding, and so forth. We can see from the numerical examples that there is no single transmission strategy that can dominate all other strategies under all channel realizations. In other words, transmission strategy should be tailor-made for a given environment. In this paper, we have investigated the pros and cons of different building blocks and demonstrated how they can be used to construct transmission strategies for the TWTR network. We believe that the idea can be applied to other relay networks as well.

While in this paper we only consider the case where there are only two-relays, the ideas of our proposed schemes can be applied to the case with more than two-relays. In particular, AF and PDF can be directly implemented without any change. As for DF, HMC, and HLC, the design may be more complicated, since we have to determine which relay to decode which source's message. On the other hand, the idea behind remains the same.

In our work, we have assumed that the channels are static. When link gains are time varying, our result reveals that a static strategy can only be suboptimal. To fully exploit the available capacity of the network, adaptive strategies that can switch between several modes are needed. How to determine a good strategy based on channel state information is an open problem. It is especially difficult if the switching is based on local information only, and we leave it for future work.

Appendix

Proof of Theorem 4

The following information-theoretic argument shows that any rate pair (R_A, R_B) satisfying the conditions in Theorem 4 is achievable.

Codebook Generation . For $i = A, B$, the common message of terminal i is drawn uniformly in $M_{ic} \triangleq \{1, 2, \ldots, 2^{2NR_{ic}}\}$ and the private message from $M_{ip} \triangleq \{1, 2, \ldots, 2^{2NR_{ip}}\}$. For $i = A, B$, we generate $2^{2NR_{ic}}$ independent sequences of length N. In each sequence, the components are 2×1 vectors drawn independently with distribution $\mathcal{N}(0, \mathbf{K}_{ic})$. Label the generated sequences by $\mathbf{U}_i^N(m_{ic})$ for $m_{ic} \in M_{ic}$. Generate $2^{2NR_{ip}}$ independent sequences of length N, with each component drawn independently with distribution $\mathcal{N}(0, \mathbf{K}_{ip})$. Label the generated sequences by $\mathbf{W}_i^N(m_{ip})$ for $m_{ip} \in M_{ip}$. Set

$$\mathbf{X}_i^N\left(m_{ic}, m_{ip}\right) = \mathbf{H}_i\left(\mathbf{U}_i^N(m_{ic}) + \mathbf{W}_i^N\left(m_{ip}\right)\right). \tag{A.1}$$

By (56) and (57), with very high probability the power constraints on node A and node B are satisfied.

There is a common codebook for relay 1 and relay 2. We generate an array of codewords with $2^{2NR_{A_c}}$ rows and $2^{2NR_{B_c}}$

columns. The codewords have length N and each component is drawn independently from $\mathcal{N}(0,1)$. Label the codewords by $V_0^N(m_{Ac}, m_{Bc})$, for $m_{Ac} \in M_{Ac}$ and $m_{Bc} \in M_{Bc}$.

For relay 1, we generate $2^{2N(R_{Ap}+R_{Ac}R_{Bc})}$ codewords, indexed by $m_{Ap} \in M_{Ap}$, $m_{Ac} \in M_{Ac}$, $m_{Bc} \in M_{Bc}$, and denoted by

$$\widetilde{X}_1^N\left(m_{Ap}, m_{Ac}, m_{Bc}\right). \tag{A.2}$$

Each of them is drawn independently with each component generated from $\mathcal{N}(0, \overline{\alpha}_1 P_1)$. Let $X_1^N(m_{Ac}, m_{Bc}, m_{Ap})$ be the linear combination

$$\sqrt{\alpha_1 P_1} V_0^N(m_{Ac}, m_{Bc}) + \widetilde{X}_1^N\left(m_{Ap}, m_{Ac}, m_{Bc}\right). \tag{A.3}$$

Since $\alpha_1 + \overline{\alpha}_1$ is strictly less than 1, $X_1^N(m_{Ac}, m_{Bc}, m_{Ap})$ satisfies the power constraint of node 1 with very high probability.

For relay 2, we generate $2^{2N(R_{Bc}+R_{Bp}+R_{Ac})}$ codewords, labeled by

$$\widetilde{X}_2^N\left(m_{Bp}, m_{Bc}, m_{Ac}\right), \tag{A.4}$$

for $m_{Bp} \in M_{Bp}$, $m_{Bc} \in M_{Bc}$, $m_{Ac} \in M_{Ac}$. The components of each codeword are generated independently from $\mathcal{N}(0, \overline{\alpha}_2 P_2)$. Let $X_2^N(m_{Ac}, m_{Bc}, m_{Bp})$ be

$$\sqrt{\alpha_2 P_2} V_0^N(m_{Ac}, m_{Bc}) + \widetilde{X}_2^N\left(m_{Bp}, m_{Bc}, m_{Ac}\right). \tag{A.5}$$

The codeword $X_2^N(m_{Ac}, m_{Bc}, m_{Bp})$ satisfies the power constraint of node 2 by the hypothesis that $\alpha_2 + \overline{\alpha}_2 < 1$.

Encoding: For source node $i \in \{A, B\}$, to send the message (m_{ic}, m_{ip}), it sends $\mathbf{X}_i^N(m_{ic}, m_{ip})$ to the relays.

In the second stage, relay 1 and relay 2 transmit $X_1^N(\hat{m}_{Ac}, \hat{m}_{Bc}, \hat{m}_{Ap})$ and $X_2^N(\hat{m}_{Ac}, \hat{m}_{Bc}, \hat{m}_{Bp})$. The messages indicated by $\hat{}$ is the estimated version of the original message.

Decoding: For $i = 1, 2$, the channel output at relay i is

$$\begin{aligned} &\mathbf{h}_{Ai}^T \mathbf{H}_A\left(\mathbf{U}_A(m_{Ac})(t) + \mathbf{W}_A\left(m_{Ap}\right)(t)\right) \\ &\quad + \mathbf{h}_{Bi}^T \mathbf{H}_B\left(\mathbf{U}_B(m_{Bc}(t)) + \mathbf{W}_B\left(m_{Bp}\right)(t)\right) + Z_i(t). \end{aligned} \tag{A.6}$$

The receiver at relay 1 treats the signal component $\mathbf{h}_{B1}^T \mathbf{H}_B \mathbf{W}_B(m_{Bp})(t)$ as noise, and tries to decode m_{Ac}, m_{Bc} and m_{Ap}. It reduces to a MAC with two users, but three independent messages; two messages from node A and one message from node B. In order to decode these three messages reliably, we need the requirement in (50). Likewise, we have the requirement in (51) for correct decoding at node 2.

Relay 2 treats the signal component $\mathbf{h}_{A2}^T \mathbf{H}_A \mathbf{W}_A(m_{Ap})(t)$ as noise, and tries to decode m_{Ac}, m_{Bc} and m_{Bp}. This can be done with arbitrarily small error if the condition in (51) holds.

In the second stage, terminal A receives

$$\begin{aligned} \mathbf{Y}_A(t) &= \left[\sqrt{\alpha_1 P_1}\mathbf{h}_{A1} + \sqrt{\alpha_2 P_2}\mathbf{h}_{A2}\right] V_0(\hat{m}_{Ac}, \hat{m}_{Bc})(t) \\ &\quad + \mathbf{h}_{A1}\widetilde{X}_1\left(\hat{m}_{Ap}, \hat{m}_{Ac}, \hat{m}_{Bc}\right)(t) \\ &\quad + \mathbf{h}_{A2}\widetilde{X}_2\left(\hat{m}_{Bp}, \hat{m}_{Bc}, \hat{m}_{Ac}\right)(t) + \mathbf{Z}_A(t). \end{aligned} \tag{A.7}$$

Assuming that $\hat{m}_{Ac} = m_{Ac}$ and $\hat{m}_{Ap} = m_{Ap}$, the channel is equivalent to a two-user MAC with common information, in which both users send m_{Bc}, and one of the users sends the private message m_{Bp}. The decoding is done by typicality as in [30, chapter 8], with the additional functionality of multiplexed coding. The decoder at terminal A searches for $\hat{\hat{m}}_{Bc}$ and $\hat{\hat{m}}_{Bp}$ such that Y_A^N, $V_0^N(m_{Ac}, \hat{\hat{m}}_{Bc})$, $\widetilde{X}_1^N(m_{Ap}, m_{Ac}, \hat{\hat{m}}_{Bc})$ and $\widetilde{X}_2^N(\hat{\hat{m}}_{Bp}, \hat{\hat{m}}_{Bc}, m_{Ac})$ are jointly typical. From the capacity region of MAC with common information [30, page 102], we obtain the following rate requirements

$$\begin{aligned} R_{Bp} &\le I\left(\widetilde{X}_2; Y_A \mid \widetilde{X}_1, V_0\right), \\ R_{Bp} + R_{Bc} &\le I\left(\widetilde{X}_1, \widetilde{X}_2, V_0; Y_A\right), \end{aligned} \tag{A.8}$$

where I is the mutual information function. This gives the conditions in (54) and (55).

Similarly, we have the conditions in (52) and (53) for successful decoding in terminal B. This completes the proof of Theorem 4.

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