speeds from 50 to 200 km/h with 25-dB SNR. This figure shows how the bit loading gap increases for higher speeds (the prediction error increases for fixed prediction range), resulting in a throughput loss for high MS speeds. In all cases, the proposed scheme reduces this loss significantly, compared with the uncompensated case.

VI. Conclusion

We have proposed a characterization of the prediction error for prediction-based resource allocation for OFDMA downlink over mobile wireless channel when imperfect channel state information is available. Based on the large amount of frequency data samples available in a typical OFDMA system, we derived an empirical approach based on histograms for the characterization of the prediction error for the different prediction horizons considered in the prediction window.

We evaluated the proposed scheme under realistic channel conditions, system parameters, and a practical channel predictor that is feasible for implementation at MSs. Simulation results indicate that the proposed scheme outperforms similar prediction-based resource RAs that disregard the prediction error.

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Achieving the Outage Capacity of the Diamond Relay Network to Within One Bit and Even Less

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***Abstract*—A new forwarding strategy is proposed for the wireless diamond relay network under slow fading. The key feature is that it can adaptively switch between decode–forward and compress–forward according to the instantaneous received signal strength. It has four control parameters, which can be optimized by an alternating optimization procedure. Its outage performance is compared with two lower bounds. Analytically, it is proven to achieve the outage capacity to within 1 bit and within 50% for any signal-to-noise ratio (SNR). Empirically, it is shown to be nearly optimal for some fading scenarios.**

***Index Terms*—Alamouti code, diamond relay network, outage capacity.**

I. Introduction

The parallel relay network, in which a pair of source–destination nodes is connected by a number of parallel relay nodes, was proposed in [1]. Due to its potential application to mobile cellular systems, it has attracted much attention [2]–[6]. The particular case where there are two relays is called the diamond relay network, as the topology looks like a diamond, as shown in Fig. 1. This model was first studied in [7] and later investigated in [8]–[11]. Its channel capacity, however, remains unknown, even for the nonfading case with only additive white Gaussian noise (AWGN).

In a slow-fading environment with channel state information (CSI) available at receivers but not at transmitters, full diversity of the parallel relay network can be achieved by using distributed space-time code, together with the decode-and-forward (DF) method [2]. For the diamond relay network, one can adopt the Alamouti code, and this particular scheme is called ACDF [11]. The full-diversity result ensures that the outage probability curve of ACDF can be made the steepest possible at high signal-to-noise ratio (SNR). On the other hand, it does not rule out the possibility that the curve can be

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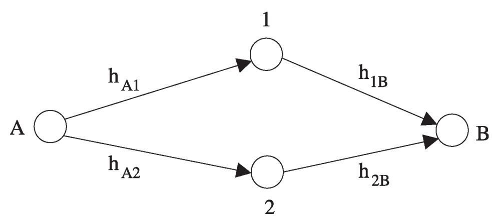

Fig. 1. Diamond relay network model.

horizontally shifted to the left to yield a power gain. In this paper, we try to squeeze the outage capacity as much as possible by proposing a new strategy for the diamond relay network. The main idea is that we allow a relay, when it fails to decode the message, to compress and forward (CF) its received signal to the destination node. In other words, the forwarding strategy adaptively switches between the DF mode and the CF mode, depending on whether the SNR is high enough for successful decoding. We remark that the strategy of having one relay adopt DF and the other adopt CF is also considered in [8] but only for the AWGN setting.

Our proposed strategy has four control parameters. To optimize them, we propose an alternating optimization method, which is shown to converge very fast. To evaluate its outage performance, we compare it with our derived lower bounds. Under independent and identically distributed (i.i.d.) Rayleigh fading, it is analytically shown to achieve outage capacity to within 1 bit and within 50% for the whole SNR regime. Numerical results show that it is very close to optimality for some fading scenarios.

II. System Model

A source node A is sending messages to a destination node B via two parallel relay nodes 1 and 2, as shown in Fig. 1. Each of them is assumed to have a single antenna. Node i is subject to power constraint P_i, $i \in \{A, 1, 2\}$. The transmission of source A is assumed to be orthogonal with the transmission of the relays. This can be achieved, for example, by using half-duplex operations at the relays. For simplicity, the bandwidths of the two orthogonal channels are assumed equal. For systems in which the division of bandwidth is a tunable parameter, an optimization can be performed on top of our relaying strategies.

We consider the slow-fading scenario where the link gains are random but remain constant for one codeword of length N. Let h_{Ai} and h_{iB} be the link gains from source A to relay i and from relay i to destination B, $i = 1, 2$, respectively. We assume that CSI is available at receivers (CSIR) but not at transmitters, (i.e., without CSIT). In other words, relay i knows h_{Ai}, and node B knows h_{Ai} and h_{iB} for $i = 1, 2$. Define $\boldsymbol{h}_B \triangleq (h_{1B}, h_{2B})$ and $\boldsymbol{h} \triangleq (h_{A1}, h_{A2}, h_{1B}, h_{2B})$. In addition, we assume that each relay knows the probability distribution of $\boldsymbol{h}_B$.

Let $X_i[m]$ be the transmitter symbol from node i at time m, $i \in \{A, 1, 2\}$. We also let $Y_i[m]$ and $w_i[m]$ be the received symbol and the thermal noise at node i at time m, $i \in \{1, 2, B\}$, respectively. Each channel's input–output relationship is represented by the following formulas:

$$Y_i[m] = h_{Ai}\sqrt{P_A}X_A[m] + w_i[m] \text{ for } i = 1, 2 \tag{1}$$

$$Y_B[m] = \sum_{i=1}^{2} h_{iB}X_i[m] + w_B[m] \tag{2}$$

where $m = 1, 2, \ldots, N$. The transmitted symbols are subject to $\mathrm{Var}\{X_A[m]\} \le 1$ and $\mathrm{Var}\{X_i[m]\} \le P_i$ for $i = 1, 2$. We assume that $P_1 = P_2 \triangleq \kappa P_A$ and $w_i[m], w_B[m] \sim \mathcal{CN}(0, n)$. Define $\Gamma \triangleq P_A/n$. The instantaneous received SNR at relay i is denoted by $\Gamma_i(h_{Ai})$ and is equal to $|h_{Ai}|^2\Gamma$.

An outage event occurs if the instantaneous end-to-end rate $R(\boldsymbol{h})$ falls below a certain threshold R_{thd}. Outage probability is the probability of occurrence of an outage event, i.e., $p_{\mathrm{out}} \triangleq \Pr\{R(\boldsymbol{h}) < R_{\mathrm{thd}}\}$. The ϵ-outage rate R_ϵ for a certain scheme is defined as the value of R_{thd} such that $p_{\mathrm{out}} = \epsilon$ holds. The maximum achievable ϵ-outage rate is called the ϵ-outage capacity, which is denoted by C_ϵ.

III. Adaptive Decode–Forward and Compress–Forward

In the ACDF strategy [11], if a relay cannot decode the message, it will simply keep silent. The information contained in its received signal is thus wasted. To improve its performance, we propose that a relay, if it fails to decode the message, should compress its received signal into an index and forward it to the destination [15], [16]. In other words, a relay should use either DF or CF, depending on whether decoding is successful or not. We call this strategy adaptive decode–forward and compress–forward (ADFCF).

Depending on the realizations of the random link gains, there are three different scenarios: 1) Both relays adopt DF. 2) Both relays adopt CF. 3) One relay adopts DF, whereas the other one adopts CF. In the first scenario, both relays can decode the message, and the transmission in the second hop is just like a 2×1 MISO system with Alamouti code. Therefore, the received SNR at the destination is given by $(|h_{1B}|^2 + |h_{2B}|^2)\kappa\Gamma$ [18]. According to the Shannon capacity formula for complex symbols, we have

$$R(\mathbf{h}) \le C\left(\left(|h_{1B}|^2 + |h_{2B}|^2\right)\kappa\Gamma\right) \tag{3}$$

where $C(x) \triangleq \log_2(1 + x)$. An outage occurs if $C((|h_{1B}|^2 + |h_{2B}|^2)\kappa\Gamma) < R_{\mathrm{thd}} \triangleq \log_2(1 + \Gamma_{\mathrm{thd}})$, which occurs with probability

$$q_1 = \Pr\left\{\left(|h_{1B}|^2 + |h_{2B}|^2\right)\kappa\Gamma < \Gamma_{\mathrm{thd}}\right\}. \tag{4}$$

In the second scenario, each relay adopts CF, and the achievable rate is given by [8, Theorem 5.8]:

Proposition 1: For fixed channel gain realization $\boldsymbol{h}$, nonnegative rate R is achievable if

$$R(\mathbf{h}) \le C\left(\frac{|h_{A1}|^2 P_A}{n + D_1} + \frac{|h_{A2}|^2 P_A}{n + D_2}\right) \tag{5}$$

for some nonnegative D_1 and D_2 that satisfy

$$C\left(\frac{1}{D_1}\left(n + \frac{|h_{A1}|^2(n + D_2)P_A}{|h_{A2}|^2 P_A + n + D_2}\right)\right) \le C\left(|h_{1B}|^2\frac{\beta_1 P_1}{n}\right) \tag{6}$$

$$C\left(\frac{1}{D_2}\left(n + \frac{|h_{A2}|^2(n + D_1)P_A}{|h_{A1}|^2 P_A + n + D_1}\right)\right) \le C\left(|h_{2B}|^2\frac{\beta_2 P_2}{n}\right) \tag{7}$$

$$C\left(\frac{|h_{A1}|^2 P_A + n}{D_1} + \frac{|h_{A2}|^2 P_A + n}{D_2} + \frac{\left(\sum_{i=1}^{2}|h_{Ai}|^2 P_A + n\right)n}{D_1 D_2}\right) \le C\left(\sum_{i=1}^{2}|h_{iB}|^2\frac{\beta_i P_i}{n}\right) \tag{8}$$

where $\beta_i P_i$ represents the transmit power of relay i (with $0 \le \beta_i \le 1$), and D_i represents the variance of the compression noise at relay i, $i = 1, 2$.

Given β_i and D_i, for $i = 1, 2$, an outage occurs if (6)–(8) are not satisfied or $C((|h_{A1}|^2 P_A)/(n + D_1) + (|h_{A2}|^2 P_A)/(n +$

$D_2)) < R_{\text{thd}}$. Denote the probability of its occurrence, conditioning on the events $|h_{Ai}|^2\Gamma < \Gamma_{\text{thd}}$, for $i = 1, 2$, by $q_2(\beta_1, D_1, \beta_2, D_2)$.

In the third scenario, one relay adopts DF, whereas the other adopts CF. We consider the case where relay 1 can decode the message and hence adopts DF, whereas relay 2 fails to decode the message and hence adopts CF. Relay 1 regenerates the codeword and is assumed to transmit it with power P_1. Relay 2 compresses its received signal and transmits it with power $\beta_2 P_2$, where $0 \leq \beta_2 \leq 1$. The variance of the compression noise is denoted by D_2.[1]

Suppose the destination, i.e., node B, decodes the compression index from relay 2 first. Then, it subtracts the signal of relay 2 from its own received signal. Afterward, it decodes the source message using the compression index as side information. The achievable rate is then given by [8]:

Proposition 2: For fixed channel link realization $\boldsymbol{h}$, nonnegative rate R is achievable if

$$R(\mathbf{h}) \leq C\left(\frac{|h_{1B}|^2 P_1}{n} + \frac{|h_{A2}|^2 P_A}{n + D_2}\right) \tag{9}$$

for some nonnegative D_2 that satisfies

$$D_2 \geq \frac{|h_{A2}|^2 + |h_{1B}|^2 + 1/\Gamma}{|h_{2B}|^2 \beta_2} n. \tag{10}$$

Note that the rate given by (9) is achievable only if the realization of the link gains satisfies (10). If the constraint is violated, the destination node can try to directly decode the message by treating the transmission of relay 2 as noise. In that case, the following rate is achievable:

$$R \leq C\left(\frac{|h_{1B}|^2 P_1}{|h_{2B}|^2 \beta_2 P_2 + n}\right). \tag{11}$$

It can be shown that the upper bound in (11) is less than that in (9). Therefore, an outage occurs if the upper bound in (9) is less than R_{thd}, or if the constraint in (10) is violated and the upper bound in (11) is less than R_{thd}. Conditioning on the events $|h_{A1}|^2\Gamma \geq \Gamma_{\text{thd}}$ and $|h_{A2}|^2\Gamma < \Gamma_{\text{thd}}$, we denote the probability that an outage occurs in this scenario by $q_3(\beta_2, D_2)$. Similarly, we define $q_4(\beta_1, D_1)$ in the same way for the case where relay 1 adopts CF and relay 2 adopts DF.

Let p_i be the probability that relay i fails to decode the signal from node A. The outage probability of ADFCF can then be obtained by averaging over the four events that can occur at the first hop

$$\begin{aligned} p_{\text{out}}^{\text{ADFCF}} &= (1 - p_1)(1 - p_2)q_1 + p_1 p_2 q_2(\beta_1, D_1, \beta_2, D_2) \\ &\quad + (1 - p_1)p_2 q_3(\beta_2, D_2) + p_1(1 - p_2)q_4(\beta_1, D_1). \end{aligned} \tag{12}$$

IV. Optimization of Control Parameters

The outage probability of the ADFCF strategy, as stated in (12), can be expressed as a function of β_1, β_2, D_1, and D_2. To minimize it, these four parameters should be properly chosen. However, a brute-force approach requires solving a four-dimensional problem, which is very time consuming. To reduce the computational complexity, we propose the following iterative alternating optimization method:

1) Initialize $\beta_2^{(0)}$ and $D_2^{(0)}$ randomly. Let $n := 0$.
2) Update β_1 and D_1 as follows:

$$\left(\beta_1^{(n+1)}, D_1^{(n+1)}\right) = \arg\max_{\beta_1, D_1} p_{\text{out}}^{\text{ADFCF}}\left(\beta_1, D_1, \beta_2^{(n)}, D_2^{(n)}\right). \tag{13}$$

3) Update β_2 and D_2 as follows:

$$\left(\beta_2^{(n+1)}, D_2^{(n+1)}\right) = \arg\max_{\beta_2, D_2} p_{\text{out}}^{\text{ADFCF}}\left(\beta_1^{(n)}, D_1^{(n)}, \beta_2, D_2\right). \tag{14}$$

4) Let $n := n + 1$, and go to step 2 until the percentage decrease in outage probability is smaller than a predetermined constant.

This algorithm always converges since the outage probability function is bounded below by zero and monotonically decreasing while the algorithm iterates. In Steps 2 and 3, we have to solve a 2-D optimization problem, which can be done by exhaustive grid search. With this method, the dimension of the problem is reduced from four to two, which greatly reduces the computation time.

As this method does not necessarily produce the optimal solution, we may compare the solution so generated to the case where $\beta_1 = \beta_2 = 0$, which corresponds to the degenerate case of ACDF. (In that case, the values of D_1 and D_2 do not matter.) By adopting the better of these two solutions, we can guarantee that ADFCF based on the preceding parameter optimization always performs no worse than ACDF.

V. Lower Bounds on Outage Probability

In this section, we derive two lower bounds for the achievable outage probability. The first one is obtained by viewing the two relays as one virtual relay node with two antennas. It can be interpreted as if there is a genie, who let each relay know the received signal of the other relay without error and delay. Hence, we call it the *genie-aided* bound. While assuming that the two relays (or equivalently, antennas in a virtual node) can cooperate in transmission and reception, we keep the original power constraint for each relay. In other words, power sharing between the two relays is *not* allowed. In addition, as in the original model, we assume that the two relays do not have CSIT. The optimal outage performance of this genie-aided system is better than that of the original model since we have relaxed the constraint that the two relay nodes cannot cooperate by introducing a genie. Thus, the minimum outage probability can serve as a lower bound of the original system. In particular, we assume that the random gain vector in the second hop has the property of circular symmetry [18, p. 500] so that a nice expression can be obtained. Otherwise, one needs to optimize a covariance matrix, which is, in general, difficult to solve.

Theorem 3: Given that $\boldsymbol{h}_B$ is circular symmetric, the achievable outage probability of the diamond relay network has the following lower bound:

$$p_{\text{out}} \geq 1 - (1 - \epsilon_1)(1 - \epsilon_2) \tag{15}$$

where $\epsilon_1 = \Pr\{|h_{A1}|^2 + |h_{A2}|^2 < \Gamma_{\text{thd}}/\Gamma\}$, and $\epsilon_2 = \Pr\{|h_{1B}|^2 + |h_{2B}|^2 < \Gamma_{\text{thd}}/\kappa\Gamma\}$.

Proof: To find the minimum outage probability of the genie-aided system, we note that X_A, (Y_1, Y_2), and Y_B form a Markov chain. According to the data processing inequality [17, Sec. 2.8], node B cannot successfully decode the message if the relay node cannot. In other words, outage at node B occurs if outage occurs at the first hop. If the virtual relay node can decode the message, then the transmission is successful if there is no outage in the second hop. By the theory of probability, we obtain (15), where ϵ_1 represents the minimum outage probability in the first hop, and ϵ_2 represents the minimum outage probability in the second hop under the condition that no outage occurs at the relay node.

It remains to derive the expressions for ϵ_1 and ϵ_2. Since the first hop is equivalent to a 1×2 single-input–multiple-output (SIMO) channel, according to [18, Sec. 5.4.2], we immediately have the expression for ϵ_1. Consider the second hop. Since $\boldsymbol{h}_B$ is circular symmetric,

[1] Detailed description of the operation performed by relay 2 can be found in [8].

correlation between the transmissions of the two relay nodes cannot improve outage performance [18, Sec. 5.4.3]. In other words, we can assume without loss of generality that the optimal correlation matrix of the transmitted signals from the two relays is diagonal. Since power allocation across the two relay nodes is not allowed, we get the expression for ϵ_2. ■

Note that i.i.d. Rayleigh fading satisfies the circular symmetry property. After algebraic simplification, we have the following result:

Corollary 4: For i.i.d. Rayleigh fading, the achievable outage probability of the diamond relay network has the following lower bound:

$$p_{\rm out} \geq 1 - \left(1 + \left(1 + \frac{1}{\kappa}\right) x + \frac{1}{\kappa} x^2\right) e^{-\left(1+\frac{1}{\kappa}\right)x} \triangleq \underline{p}_{\rm out}^{\rm genie}(x) \qquad (16)$$

where $x \triangleq \Gamma_{\rm thd}/\Gamma$.

The second bound is obtained by straightforward application of the cut-set theorem [17, Sec. 14.10]: For fixed channel realization $\boldsymbol{h}$, the achievable rate is bounded above by

$$R(\mathbf{h}) \leq \min\{C(s_{A1} + s_{A2}), C\left(s_{1B} + s_{2B} + 2\sqrt{s_{1B}s_{2B}}\right) \\ C(s_{A2}) + C(s_{1B}), C(s_{A1}) + C(s_{2B})\} \qquad (17)$$

$$\triangleq C_{\min}(\mathbf{h}) \qquad (18)$$

where $s_{Aj} \triangleq |h_{Aj}|^2\Gamma$, and $s_{iB} \triangleq |h_{iB}|^2\kappa\Gamma$. Note that s_{ij} represents the effective SNR from node i to node j. Consequently, we have $p_{\rm out} \geq \Pr\{C_{\min}(\boldsymbol{h}) < R_{\rm thd}\}$.

Note that the genie-aided bound and the cut-set bound do not dominate each other in the sense that, given any realization of the link gains, violation of one bound does not imply the violation of the other. The reason is that, in the derivation of the genie-aided bound, CSIT at the two relays is not assumed. Therefore, it is tighter than the cut-set bound, which implicitly assumes CSIT, in some cases. On the other hand, the cut-set bound is tighter in some other cases since it considers all the four cuts in the network, whereas the genie-aided bound only considers the cuts across the first hop and the second hop. Since these two bounds do not dominate each other, they can be combined to form a tighter bound in the following manner: Given a realization of the link gains, an outage occurs if either the genie-aided bound or the cut-set bound is violated. This combined bound is used in the simulation section.

VI. Bounds on the Outage Rate Under Rayleigh Fading

Capacity results for communication systems are useful as they provide fundamental limits on the maximum transmission rates that those systems can support. However, it is difficult to determine the ϵ-outage capacity of a communication network in general and of the diamond network in particular. It is then natural to ask whether a transmission strategy is close to the capacity limit. For high-rate transmission, it is informative to quantify an additive gap between the achievable rate and the capacity. For low-rate transmission, a multiplicative gap should be determined. If a strategy is shown to have both small additive and multiplicative gaps from the capacity, then it is provably good in both the low-rate and high-rate regimes.

In this section, we consider i.i.d. Rayleigh fading only. Given a certain outage probability ϵ, the required SNR of a transmission scheme is defined as the infimum of all SNRs such that its outage probability is no less than ϵ. The infimum of the required SNR of all possible transmission schemes is denoted by γ_ϵ^*. We define the SNR offset (in decibels) of a transmission scheme as $10\log_{10}(\Gamma_\epsilon(R_{\rm thd})/\gamma_\epsilon^*)$, where $\Gamma_\epsilon(R_{\rm thd})$ is the required SNR of that scheme to achieve rate $R_{\rm thd}$ with outage probability ϵ. Note that ADFCF encompasses ACDF as a particular case. With parameter optimization, it always performs no worse than ACDF. Therefore, we have

$$\underline{p}_{\rm out}^{\rm genie}(x) \leq p_{\rm out}^{\rm ADFCF}(x) \leq p_{\rm out}^{\rm ACDF}(x) \qquad (19)$$

where $p_{\rm out}^{\rm ACDF}$ is shown in [11] to be

$$p_{\rm out}^{\rm ACDF}(x) = 1 - 2e^{-\left(1+\frac{1}{\kappa}\right)x} + \left(1 - \frac{1}{\kappa}x\right) e^{-\left(2+\frac{1}{\kappa}\right)x}. \qquad (20)$$

Theorem 5: The SNR offset of ACDF is less than 3 dB for any ϵ and $R_{\rm thd}$.

Proof: It was proven in [11] that, for all $x \geq 0$

$$p_{\rm out}^{\rm ACDF}(x) \leq \underline{p}_{\rm out}^{\rm genie}(2x). \qquad (21)$$

Therefore, for any $\Gamma_{\rm thd}$, the outage probability of ACDF can be made smaller than the genie-aided bound by doubling the SNR for ACDF. Hence, the SNR offset of ACDF is at most $10\log_{10} 2 \approx 3$ dB. ■

The bound on the SNR offset can be translated into a bound on the ϵ-outage rate.

Theorem 6: ACDF achieves within $\Delta_\epsilon(\Gamma)$ bit of the ϵ-outage capacity of the diamond relay channel, where

$$\Delta_\epsilon(\Gamma) \triangleq 1 - \left[\log_2\left(2^{R_\epsilon^{\rm genie}(\Gamma)} + 1\right) - R_\epsilon^{\rm genie}(\Gamma)\right] \leq 1. \qquad (22)$$

Proof: Given a realization of link gains $\boldsymbol{h}$ and an SNR Γ, we define $\overline{R}(\boldsymbol{h}, \Gamma)$ and $\underline{R}(\boldsymbol{h}, \Gamma)$ as the maximum achievable rate of the genie-aided system described in the previous section and that of ACDF, respectively. According to (21), we have

$$\Pr\left\{\overline{R}(\mathbf{h}, \Gamma) < R_{\rm thd}\right\} \geq \Pr\left\{\underline{R}(\mathbf{h}, 2\Gamma) < R_{\rm thd}\right\}. \qquad (23)$$

Recall that $\Gamma_{\rm thd} \triangleq 2^{R_{\rm thd}-1}$. For ACDF, scaling Γ and $\Gamma_{\rm thd}$ by the same amount does not affect its outage probability, which is implied by the following two observations: First, the probability that a relay can decode the message depends only on the ratio between Γ and $\Gamma_{\rm thd}$. Second, the outage probability in the second hop also depends only on that ratio, no matter whether one or two relays can decode the message in the first hop. Therefore

$$\Pr\left\{\underline{R}(\mathbf{h}, 2\Gamma) < R_{\rm thd}\right\} = \Pr\left\{\underline{R}(\mathbf{h}, \Gamma) < \log_2(2^{R_{\rm thd}} + 1) - 1\right\}. \qquad (24)$$

Let $R_\epsilon^{\rm genie}(\Gamma)$ and $R_\epsilon^{\rm ACDF}(\Gamma)$ be the ϵ-outage rate of the genie-aided system and that of the ACDF, respectively. That means that we have

$$\Pr\left\{\overline{R}(\mathbf{h}, \Gamma) < R_\epsilon^{\rm genie}(\Gamma)\right\} = \Pr\left\{\underline{R}(\mathbf{h}, 2\Gamma) < R_\epsilon^{\rm ACDF(\Gamma)}\right\} = \epsilon. \qquad (25)$$

By (23) and (24), we have $R_\epsilon^{\rm ACDF}(\Gamma) \geq \log_2(2^{R_\epsilon^{\rm genie}(\Gamma)} + 1) - 1$, which implies

$$R_\epsilon^{\rm genie}(\Gamma) - R_\epsilon^{\rm ACDF}(\Gamma) \\ \leq 1 - \left[\log_2\left(2^{R_\epsilon^{\rm genie}(\Gamma)} + 1\right) - R_\epsilon^{\rm genie}(\Gamma)\right] \leq 1. \qquad (26)$$

■

To find $R_\epsilon^{\rm genie}(\Gamma)$, we can first let $\underline{p}_{\rm out}^{\rm genie}(x)$ in (16) equal to ϵ, solve for x, and then compute the corresponding $\Gamma_{\rm thd}$. The ϵ-outage rate is then given by $\log_2(1 + \Gamma_{\rm thd})$. An example for the relationship between $\Delta_\epsilon(\Gamma)$ and Γ when $\epsilon = 0.05$ and $\kappa = 1$ is shown in Fig. 2.

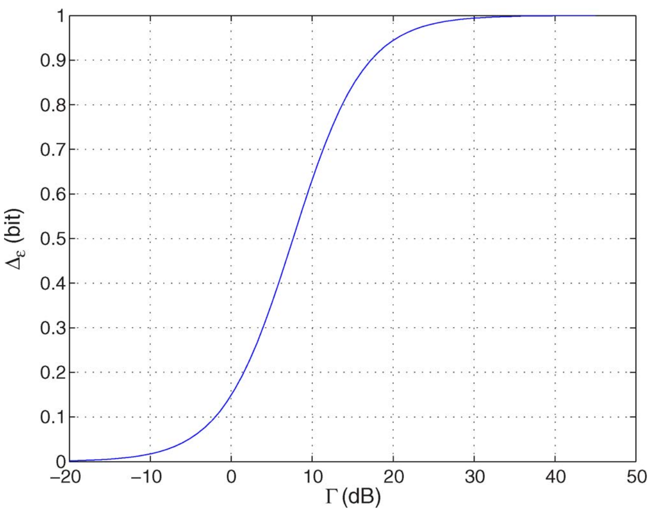

Fig. 2. Relationship between $\Delta_\epsilon(\Gamma)$ and Γ when $\epsilon = 0.05$ and $\kappa = 1$.

Theorem 7: ACDF achieves within a fraction of $\rho_\epsilon(\Gamma)$ of the ϵ-outage capacity of the diamond relay channel, where

$$\rho_\epsilon(\Gamma) \triangleq \left[\log_2\left(2^{R_\epsilon^{\text{genie}}(\Gamma)}+1\right)-1\right] \Big/ R_\epsilon^{\text{genie}}(\Gamma) \geq 0.5 \quad (27)$$

with equality holds when $R_\epsilon^{\text{genie}}(\Gamma) \to 0$ or $\Gamma \to 0$. Furthermore, $\rho_\epsilon(\Gamma) \to 1$ when $\Gamma \to \infty$.

Proof: The ratio between $R_\epsilon^{\text{ACDF}}(\Gamma)$ and $C_\epsilon(\Gamma)$ is clearly bounded below by $\rho_\epsilon(\Gamma)$. Let $y \triangleq 2^{R_\epsilon^{\text{genie}}(\Gamma)/2}$. If $R_\epsilon^{\text{genie}}(\Gamma) \neq 0$, then (27) is equivalent to $y^2+1 \geq 2y$, or $(y-1)^2 \geq 0$. When $\Gamma \to 0$, we have $R_\epsilon^{\text{genie}}(\Gamma) \to 0$ and $y \to 1$. When $\Gamma \to \infty$, we obtain from (27) that $\rho_\epsilon(\Gamma) \to 1$. ■

VII. Numerical Studies

We compare the outage probabilities of ACDF and ADFCF with the lower bound under different fading environments. The parameters of the model are $\kappa = 1$, $\Gamma_{\text{thd}} = 1$, and $\sigma^2 = 1$. Each point in the figures is obtained by Monte Carlo simulations through averaging over 3×10^6 channel realizations. The control parameters of ADFCF are obtained by the alternating optimization method. Our numerical results indicate that the algorithm converges very fast, typically, with just a few iterations.

We first consider the scenario that the four links experience i.i.d. Rayleigh fading. The result is plotted in Fig. 3, with the lower bound obtained by combining the genie-aided bound with the cut-set bound. We can see that the SNR offsets of ACDF and ADFCF at high SNR are no more than 1.6 and 1.8 dB, respectively. Fig. 4 shows that the lower bound becomes much tightened after combining the two bounds.

We next consider the scenarios that the four links experience independent Rician fading with different parameters. In particular, we consider the following three parameter settings: 1) $|h_{A1}|, |h_{A2}|, |h_{1B}| \sim \text{Rice}(1,1)$, and $|h_{2B}| \sim \text{Rice}(10,1)$; 2) $|h_{A1}|, |h_{A2}| \sim \text{Rice}(1,1)$ and $|h_{1B}|, |h_{2B}| \sim \text{Rice}(10,1)$; and 3) $|h_{A1}|, |h_{2B}| \sim \text{Rice}(100,1)$ and $|h_{A2}|, |h_{1B}| \sim \text{Rice}(1,1)$. The outage curves for these settings are plotted in Figs. 5–7, respectively. As the fading distribution in the second hop is no longer circular symmetric, the lower bound is solely obtained by the cut-set bound. From the figures, it can be seen that ADFCF significantly outperforms ACDF at high SNR, with power gains of 0.93, 1, and 1.5 dB for the three settings, respectively. Note that, for setting 3, the SNR offset of ADFCF is no more than 0.3 dB at high SNR, showing that ADFCF is close to optimal.

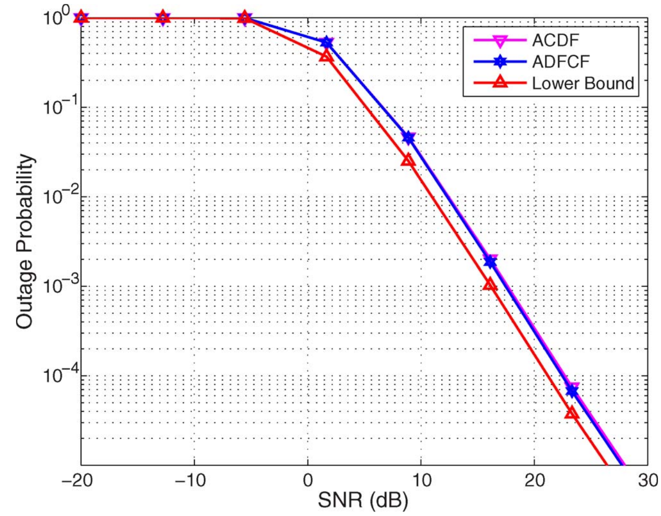

Fig. 3. Outage performance under i.i.d. Rayleigh fading.

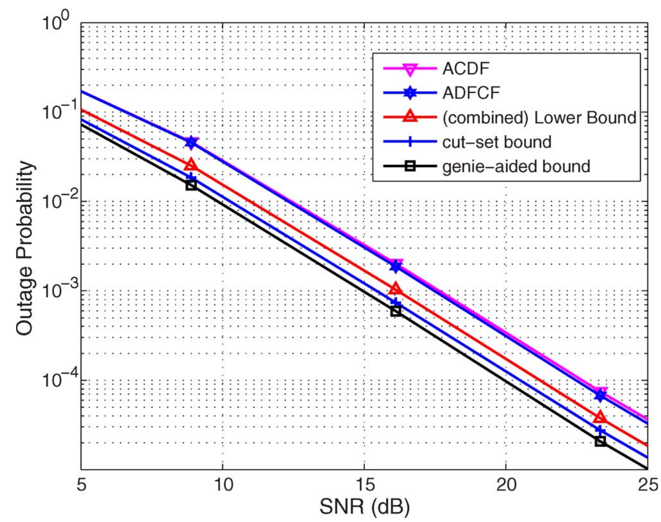

Fig. 4. Outage performance under i.i.d. Rayleigh fading: a comparison between different lower bounds.

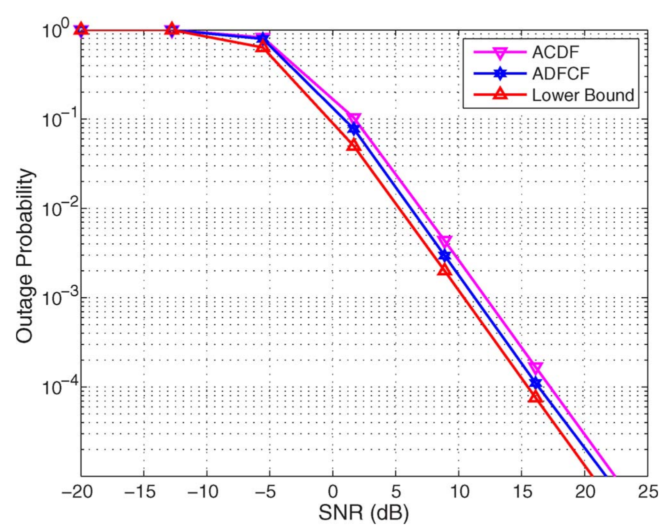

Fig. 5. Outage performance under independent Rician fading with parameter setting 1.

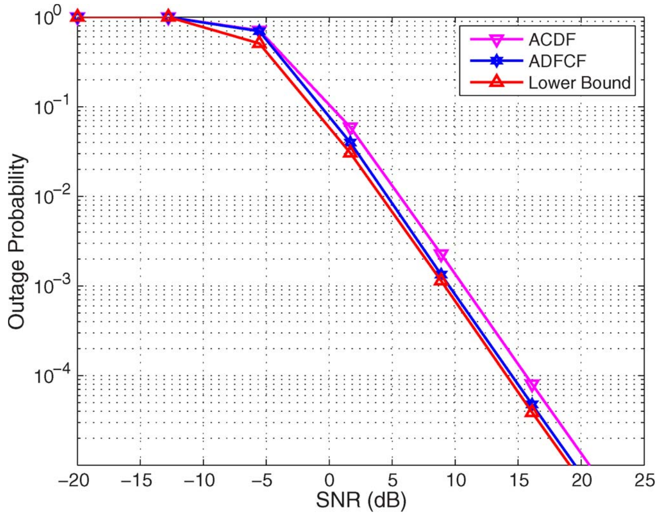

Fig. 6. Outage performance under independent Rician fading with parameter setting 2.

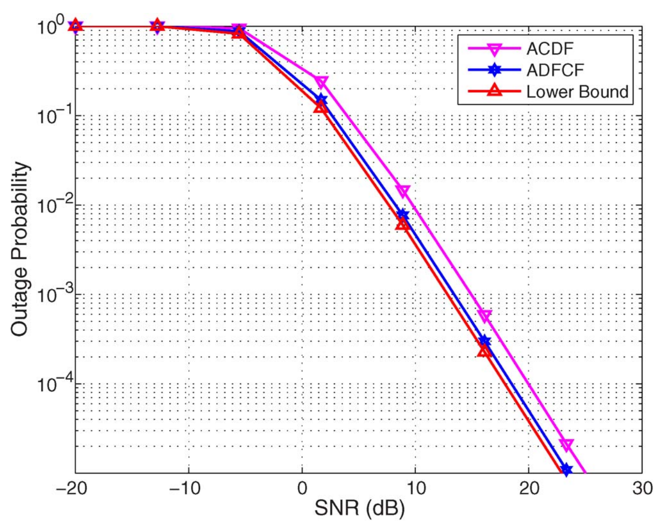

Fig. 7. Outage performance under independent Rician fading with parameter setting 3.

VIII. Conclusion

How to achieve the capacity of a communication network is an important but challenging question. In this paper, we have considered the wireless diamond relay network under slow fading. We have proposed a new forwarding strategy based on a combination of Alamouti code, DF, and CF. Its performance is characterized by deriving analytical bounds. Our result has shown that it can achieve outage capacity to within 1 bit for any SNR. Given the SNR, the bound can be tightened, as shown in Theorem 6. On the other hand, as in the low-SNR regime, the outage capacity goes to zero, and we have compared the ratio between the outage rate of our strategy with the outage capacity. We have proven that our strategy can achieve within one half of the outage capacity for all SNR values. Simulation results have shown that it outperforms a well-known strategy by combining Alamouti code only with DF. We hope that our work increases our understanding of the diamond relay network under fading and provides insights of how to design efficient strategies for it.

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