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General Solution of a Monopole Loaded by a Dielectric Hemisphere for Efficient Computation

K. W. Leung

***Abstract*—A simple result for the general solution of a monopole loaded by a dielectric hemisphere is presented. The result can be calculated without the need for any numerical integration and, thus, it is computationally very efficient. In addition, the result is very easy to implement and should be useful to the design engineer.**

***Index Terms*—Dielectric antennas, monopole antennas.**

I. INTRODUCTION

In antenna designs, a dielectric loading can be added to change the antenna characteristics, such as the impedance, bandwidth, and radiation patterns [1]. It can also be used to provide insulation with the external medium [2]. There are many shapes of dielectric, but only the spherical one is convenient for an exact analysis [1], [2]. A rigorous solution of a monopole loaded by a dielectric hemisphere can be found in [3], where the exact Greens function was found analytically using the mode- matching method. In using the method of moments (MoM) to find the probe (or monopole) current [3], it is required to evaluate the quadruple integrals for the impedance matrix elements. Although the quadruple integrals can be reduced to double integrals by using the thin-wire approximation, considerable computation time and programming effort are still required to calculate the integrals numerically. In practice, the design engineer should prefer a simpler result that can be implemented easily and calculated quickly. Leung *et al.* [4] employed the single-mode theory to obtain a simple result for a probe along the axis of a dielectric hemisphere. The result, however, is not general and is only limited to frequencies around the TM_{101} mode of the dielectric hemisphere. In this letter, the general solution of a monopole along the axis of a dielectric hemisphere is presented. The result does not involve any numerical integration, and, thus, the computation is extremely fast. Furthermore, the expression is rather simple and can be implemented very easily. It is worth mentioning that for this particular configuration, all TE modes of the dielectric hemisphere vanish and only TM modes can be excited.

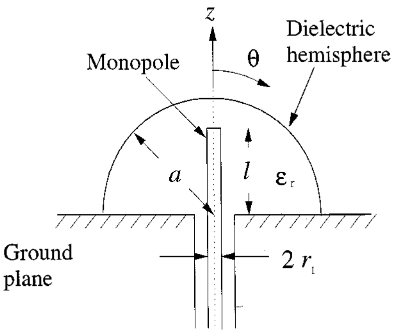

Fig. 1. Geometry of the monopole loaded by a dielectric hemisphere.

II. THEORY

The configuration is shown in Fig. 1. A monopole of length l and radius r_1 is located at the center of a dielectric hemisphere of radius a and dielectric constant ε_r. In the following formulation, z and z' refer to the field and source coordinates, respectively. To begin with, image theory is used to obtain the equivalent problem of a dipole embedded inside a dielectric sphere. Denote $G(z, z')$ as the Green's function for the z-directed E-field due to a point current along the z-axis. Using the result of [3], we have, for a thin monopole ($r_1 \ll l$ and $kr_1 \ll 1$)

$$
\begin{aligned}
G(z, z') = \frac{-j}{\omega\varepsilon}\left(\frac{\partial^2}{\partial z^2} + k^2\right)\frac{e^{-jkR}}{4\pi R} - \frac{1}{4\pi\omega\varepsilon k} \\
\cdot \frac{1}{z^2 z'^2}\sum_{n=1}^{\infty} n(n+1)(2n+1)\alpha_n^{\mathrm{TM}}\hat{J}_n(kz')\hat{J}_n(kz)
\end{aligned}
\tag{1}
$$

where

$$
\alpha_n^{\mathrm{TM}} = \frac{-\lfloor \hat{H}_n^{(2)\prime}(ka)\hat{H}_n^{(2)}(k_0 a) - \sqrt{\varepsilon_r}\hat{H}_n^{(2)}(ka)\hat{H}_n^{(2)\prime}(k_0 a)\rfloor}{\hat{J}_n{}'(ka)\hat{H}_n^{(2)}(k_0 a) - \sqrt{\varepsilon_r}\hat{J}_n(ka)\hat{H}_n^{(2)\prime}(k_0 a)} \tag{2}
$$

is the reflection coefficient at the DR boundary, $R = \sqrt{r_1^2 + (z - z')^2}$, and $k = \sqrt{\varepsilon_r}k_0$. In (1) and (2), $\hat{J}_n(x)$ and $\hat{H}_n^{(2)}(x)$ are the Schelkunoff-type [3] spherical Bessel function of the first kind and Hankel function of the second kind, respectively, both of them of order n. By enforcing the boundary condition that the total E-field vanishes on the (equivalent) dipole surface, an integral equation for the dipole current is obtained. The MoM with the Galerkin's procedure is used to solve the dipole current. The current is first expanded as $I(z) = \sum_{q=1}^{N} I_q f_q(z)$, where $f_q(z)$ is a piecewise sinusoidal (PWS) function given by $f_q(z) = [\sin k(d - |z - z_q|)]/\sin kd$ for $|z - z_q| < d$ and $f_q(z) = 0$ otherwise, with $z_q = -l + qd$ and $d = 2l/(N+1)$. The unknown expansion coefficients I_q's are solved via the matrix equation $[Z_{pq}^P + Z_{pq}^H][I_q] = [f_p(0)]$, where Z_{pq}^P and Z_{pq}^H are the impedance integrals associated with the first term (particular solution) and second term (homogeneous solution) of (1), respectively. The efficient evaluation of the impedance integrals Z_{pq}^P was discussed in

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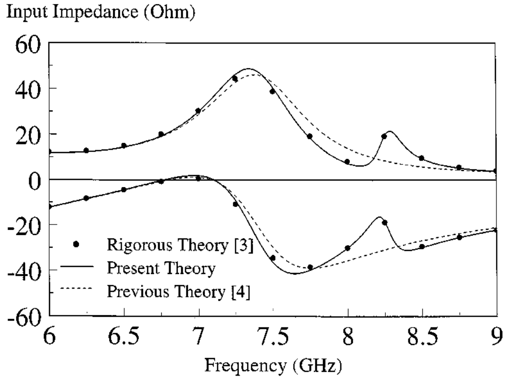

Fig. 2. Comparison of the present, rigorous, and previous simplified theories: $a = 12.5$ mm, $\varepsilon_r = 9.5, l = 5.0$ mm, $r_1 = 0.63$ mm, and $N = 5$.

[5] and is not duplicated here. Instead, we will concentrate on the Z_{pq}^H integral, which can be written as

$$Z_{pq}^H = \frac{1}{4\pi\omega\varepsilon k}\sum_{n=1}^{\infty} n(n+1)(2n+1)\alpha_n^{\mathrm{TM}}\Lambda_n(p)\Lambda_n(q) \tag{3}$$

where, for $i = p,\ q$

$$\Lambda_n(i) = \int_{z_{i-1}}^{z_{i+1}} \frac{\hat{J}_n(kz)}{z^2}\cdot\frac{\sin k(d-|z-z_i|)}{\sin kd}\,dz. \tag{4}$$

To evaluate $\Lambda_n(i)$ analytically, the absolute sign of the PWS function is first removed by breaking (4) into two integrals. Next we apply the identity $\sin k[(d \pm z_i) \mp z] = \sin k(d \pm z_i)\cos kz \mp \cos k(d \pm z_i)\sin kz$ to the integrals. Then the crucial step is to integrate $[\hat{J}_n(kz)/z^2]\cos kz$ and $[\hat{J}_n(kz)/z^2]\sin kz$. To do this, we first note that $\cos\ kz$ and $\sin\ kz$ can be alternatively written as $-\hat{Y}_0(kz)$ and $\hat{J}_0(kz)$, respectively. Then based on an integral formula for a product of two cylindrical Bessel functions $A_\mu(t),\ B_\nu(t)$ [6], the following integral was derived:

$$\begin{aligned}\int^z \frac{\hat{A}_n(t)\hat{B}_m(t)}{t^2}\,dt &= \frac{1}{(n+m+1)(n-m)}\\ &\quad\cdot\Big\{\frac{n-m}{z}\hat{A}_n(z)\hat{B}_m(z)\\ &\qquad - \Big[\hat{A}_{n+1}(z)\hat{B}_m(z) - \hat{A}_n(z)\hat{B}_{m+1}(z)\Big]\Big\}\end{aligned} \tag{5}$$

where $\hat{A}_n(t) = \sqrt{\pi t/2}A_{n+(1/2)}(t)$ and $\hat{B}_m(t) = \sqrt{\pi t/2}\, B_{m+(1/2)}(t)$ denote any two Schelkunoff-type spherical Bessel/Hankel functions and $n \neq m$. Since the order of $\hat{J}_n(kz)$ in $G(z,\ z')$ starts from $n = 1$, it is always unequal to those of $\hat{Y}_0(kz)$ and $\hat{J}_0(kz)$. Therefore, the condition that $n \neq m$ in (5) is satisfied in the present problem. After tedious manipulation, the result of Z_{pq}^H is found to be surprisingly simple

$$Z_{pq}^H = \frac{k}{4\pi\omega\varepsilon}\cdot\frac{1}{\sin^2 kd}\sum_{n=1}^{\infty}\frac{2n+1}{n(n+1)}\alpha_n^{\mathrm{TM}}L_n(p)L_n(q) \tag{6}$$

where, for $i = p,\ q$

$$L_n(i) = \sum_{j=-1}^{1} A(j)\hat{J}_n(ku_{ij}) \tag{7}$$

with $u_{ij} = -l + (i+j)d, A(\pm 1) = 1$, and $A(0) = -2\cos kd$. It can be proved that the result of [4] is only the $n = 1$ term of (6). Note that Z_{pq}^H now does not involve any integration and, thus, the calculation is extremely fast. Moreover, implementation of (6) is very easy. The only care that has to be exercised is that u_{ij} may be zero for some $i,\ j$, at which $\hat{J}_n(ku_{ij}) = 0$. Therefore, u_{ij} should be checked in the program, as the (backward) recurrence formula for $\hat{J}_n(x)$ cannot be used when $x = 0$. Now the overall solution is computationally very efficient, as the other impedance integral, Z_{pq}^P, can also be calculated without the need for any numerical integration [5]. After I_q's are found, the input impedance can be obtained by simply using $Z_{\mathrm{in}} = \beta\sum_{n=1}^{N} I_n f_n(0)$, where $\beta = 1$ for the equivalent dipole configuration and 1/2 for the original (monopole) configuration.

III. RESULTS

In this letter, five current expansion modes ($N = 5$) were used in the calculations. Fig. 2 compares the present theory with the rigorous solution [3] and the previous simplified theory [4]. It is seen that excellent agreement between the present and rigorous solutions is obtained over the whole frequency band. In contrast, the previous simplified theory is valid only in the low-frequency portion, as expected. The programs were run on a SunSPARC 20 Model 612 workstation. It was found that for the present theory the average computation time for a frequency point was only 1 ms, whereas it took a few seconds for the rigorous theory. Other values of $l,\ \varepsilon_r$, and a were used and, in all cases, the present theory was in excellent agreement with the rigorous theory. The results, however, are omitted here for brevity.

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