

# Performance Analysis of Circuit Switched Multi-service Multi-rate Networks with Alternative Routing

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**Abstract**—We consider a circuit-switched multiservice network with non-hierarchical deflection routing and trunk reservation. Based on the fundamental concept of overflow priority classification approximation (OPCA), we develop two approximations for the estimation of the blocking probability: OPCA and service-based OPCA. We also apply the classical Erlang fixed-point approximation (EFPA) for the estimation of the blocking probability in the network and propose the more conservative max(EFPA, service-based OPCA) as a reasonably accurate evaluation. We compare the approximations with simulation results and discuss the accuracy of the blocking probabilities for the various traffic classes under different system parameter values such as service rates, bandwidth requirements, number of channels per trunk, maximum allowable number of deflections and trunk reservation. We also discuss the robustness of the approximations to the shape of the holding time distribution and their performances under asymmetrical cases. We illustrate that when the approximations are used for network dimensioning, their error is acceptable. We further demonstrate that the approximations can be applied in large networks such as the CORONET.

**Index Terms**—blocking probability, circuit switching, alternative routing, trunk reservation, Erlang fixed-point approximation, overflow priority classification approximation

## I. INTRODUCTION

Over the last quarter of a century, the Internet has evolved from a packet switched network that provides only data services, such as email and file transfer, to a network that provides a wide range of services. Nowadays, there is an increasing number of Internet users that transmit extremely large flows of data. Several examples follow. The users of the Internet include cloud service providers (CSP) such as Google, Facebook and Yahoo! that often replicate their content across multiple data centers transmitting massive amount data [1]. In fact, the total CSPs inter-datacenter traffic was over 400 Exabytes during the year 2011 and growing at over 30% yearly growth rate [2]. The Internet users also include the CERNs Large Hadron Collider (LHC) that transmits several Petabytes of data per year [3].

Switching such extremely large flows using packet switching at the IP layer requires unacceptable levels of energy consumption [4]–[6]. Given the potential for orders of magnitude savings in energy per bit using lower (optical) layers [7], [8],

it is envisaged that circuit switching, which has been widely used in telephone networks, will have a renewed and important role in future optical networks [9]–[16]. In packet switching, many energy consuming operations such as buffering packets, processing individual packet headers, performing table look-up, and counting the number of packets at the destination node [17] are avoided by circuit switching [4]. Note that these advantages of circuit-switching for *wide bandwidth networks* were pointed out nearly a quarter of a century ago [18] in terms of *simplicity* rather than energy consumption.

If the bit-rate offered to a circuit-switched multimedia network is sufficiently high and the traffic is well engineered, such a network can guarantee quality-of-service (QoS) to customers in a way that can even lead to efficient transmission resource utilization and low energy consumption. For example, a 100 GByte burst transmitted from the Large Hadron Collider (LHC) can be efficiently transmitted by setting up a circuit of one or more wavelengths which will be fully utilized during its holding time. Other benefits provided by circuit switching include overload control without congestion collapse, robustness brought by the fast recovering of circuit switching equipments, and relatively simple provision of synchronization.

Different rates, different holding times and different bandwidth requirements are relevant to future circuit switching applications in the Internet. Two classes of circuit switched capacity allocation are envisioned [19]. The first is a commonly used class of circuit-switched long-lived connections which are based on setting up a lightpath that provides permanent or semi-permanent connections between two metropolitan edge routers that normally serve many flows that come and go and use that lightpath. Such long-lived connections are used by Internet service providers, or by corporations as leased lines to create private networks.

The second is characterized by dynamic resource allocation for relatively short-lived circuit-switched connections that serve dynamic demands and provide lightpath connections whenever and wherever they are needed. They can be provided end-to-end or edge-to-edge. These short-lived dynamic connections require quick connection set-up, but when the capacity resource is allocated, it can be efficiently utilized especially if the amount of data to be transported is known in advance [1], [3]. In this way, bandwidth on demand services [1], [20], [21] are provided where fixed capacity is allocated for the service duration and then released by the user. Both the allocated capacity and the service duration are based on customer requirements. The capacity is available to the user

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The work described in this paper was supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (CityU 123012) and by two grants from City University of Hong Kong (No. 7002860 and No. 7003015).

exclusively for the required duration of the service, whether or not it is fully used, in accordance with circuit switching principles.

According to the 2009 book [22], an Internet model where short-lived dynamic connections are provided “is a futuristic model since lightpaths today are relatively long lived, but it is quite possible that lightpath will be provided on demand by some operators in the future.” Such possibility is justified by market pull and technology push described in the following. One recent technology push is the proposed Globally Reconfigurable Intelligent Photonic Network (GRIPhN) [1], [23] that aims to provide bandwidth on demand (BoD) service for inter-data center communication in the core network. It is motivated by market pull created by CSPs. Having a dedicated network for such inter-data center communication in the core network incurs a major cost component of the total cost of cloud computing [24] especially if they use it only occasionally. It makes economic sense for an operator to provide BoD service to a multiplicity of CSPs that share the network so that it can be more efficiently utilized, so that CSPs can enjoy cost saving.

Furthermore, not only CSPs can benefit from BoD. The potential customers can be smaller operators, enterprises, research networks, and even retail customers. The vision of Internet2 Dynamic Circuit Network (DCN) [11], [13], [25]–[28] aims to provide BoD services [1], [20], [21] where fixed capacity is allocated for the service duration and then released by the user. Both the allocated capacity and the service duration are based on customer requirements. The capacity is available to the user exclusively for the required duration of the service, whether or not it is fully used in accordance with circuit switching principles. In many cases, the BoD is provided for a large known burst of data, so the circuits are nearly fully utilized once the circuit path is established. Example of the latter is BoD service based on circuit switching provided to the above mentioned LHC burst of data generated by high energy physics experiments. In particular, such a burst uses the LHC Open Network Environment (LHCONE) which is part of Internet2 as an access network and is transported through the LHC Optical Private Network (LHCOPN) which serves as the core network [3].

Accordingly, we can expect a scenario where all these BoD service classes compete for the same pool of optical capacity available in the core network that needs to be efficiently allocated. Each of these classes can be characterized according to the arrival rate of its burst, flow or connection requests, its mean holding time and capacity requirement. For example, certain very large dynamic flows generated by the Large Hadron Collider (LHC) [3] will require more capacity and/or longer holding time than smaller retail customers flows, but far less capacity than an inter-data center flow transmitted by one of the large CSPs.

A network operator that aims to provide such a wide range of BoD services needs means to efficiently dimension the network to meet QoS requirements. To this end, there is a need for a scalable and accurate method to evaluate performance for each relevant scenario of network topology, parameter values and traffic demand. One important measure of performance

is the blocking probability. Since the end-to-end blocking probability is an important QoS measure perceived by users, having accurate blocking probability approximation will enable network designers to dimension the network resources so that the blocking probabilities for each class will not exceed the required value. Various approaches for alternative routing have been studied aiming to reduce blocking probability in circuit switched networks [12], [29]–[31]. Alternative routing also helps distribute load among the trunks improving load balancing and provides protection in case of trunk failure [32]. In a circuit switched network with deflection routing, new calls which cannot be admitted by their primary routes can overflow to other alternative routes, which are usually longer than the primary routes. The inefficiency of alternative paths may in turn increase blocking probability. To prevent the use of very long alternative paths, there needs to be a limit to the maximum number of times that a call can overflow. If we assume Poisson call arrivals for any origin destination (OD) pair and exponential call holding times, a circuit-switched network with alternative routing can be modeled as a Markovian overflow loss network and the stationary occupancy distribution can, in principle, be obtained by solving its steady-state equations. Such models usually do not admit product-form solutions [33] and are not amenable to analysis that leads to a scalable solution for realistic size networks.

In this paper we consider a model of a circuit switched multi-service network with deflection routing for which we provide accurate methods for evaluation of blocking probability. The paper is also applicable to versions of MPLS where sufficient capacity for LSP is reserved in advance to avoid significant need for buffering in the network. The labels enable the establishment of end-to-end circuits for transmissions of packets, which is fundamentally similar to CS. The term *multi-service* network (or system) refers to a network where there are multiple classes of circuit requests between each OD pair. The classes are characterized by different arrival rates, holding times and capacity requirements. Equivalent terms which are often used instead of *multi-service* in the context of multi-service networks are *multiclass* and *heterogeneous*. In [34], we consider two classes of demands with different holding times and give priority to the one with longer holding time. In this paper, the multi-classes also have different capacity requirements but fair opportunity to compete for the pool of resources.

In multi-service network models it is often assumed that the arrivals of circuit requests for each class follow a Poisson process. We will also make this assumption for tractability. It is accurate for busy hour periods when the number of sources is large and the sources are independent. We acknowledge that certain pre-arranged scheduling with advance reservation may reduce blocking probability, so in such cases our model provides conservative results. In other cases, where the sources are random and dependent, the Poisson assumption may underestimate the blocking probability. We also consider non-hierarchical deflection routing for cases where the least cost route is not available. When we use the term *deflection routing*, we also mean *alternative routing* which has been often used in the context of circuit switched telephone networks. The

focus on busy hour is important for network dimensioning to provide sufficient resources to meet the demand during this critical period.

The prevalence of multi-service systems and networks in modern telecommunications and the history and future potential of circuit switching give rise to extensive research on modeling, analyses and performance evaluation of circuit switched multi-service systems and networks. For a single trunk with multiple channels with different classes of demand, under the complete sharing policy, the steady-state probabilities can be obtained numerically. A recursive algorithm based on a product form solution is provided by [35] and [36]. Other improvements and generalizations are described in [37], [38] and [39].

For a multi-service loss network with fixed routing, the steady-state distribution has an explicit product-form solution [40]–[42]. However, obtaining the state probabilities requires computation of a normalization constant which is difficult for realistic size networks with *e.g.*, over 100 wavelengths per trunk. Such an approach is only applicable to network problems of low dimensionality, or to networks of a special topology, *e.g.*, tree networks.

Owing to the difficulties of obtaining exact solutions for realistically large networks, approximations have been developed and used. One approximation is known as the Erlang Fixed point Approximation (EFPA), the original idea of which was first first proposed in 1964 [43] for the analysis of circuit-switched networks and has remained a cornerstone of telecommunications networks and systems analysis to this day. Kelly [44] has shown that EFPA leads to exact result for the blocking probability for a multiservice network based on fixed routing, under the asymptotic conditions where trunk capacities are arbitrarily large relative to the capacity required by the most demanding traffic class. In [45] an asymptotic version of EFPA (A-EFPA) was proposed to achieve the same limiting result in significantly less computation time. Furthermore, consistency of the results of A-EFPA and EFPA can confirm that the limiting regime has been satisfied and the results are accurate. In addition, EFPA is accurate for certain large size symmetrical networks with alternative routing [46] or diverse routing [47].

Other approximations for certain special cases have been proposed in [48], which consider only one OD pair and several alternative paths, and in [49] which considered fixed routing in a multi-stage network.

Methods involving moment matching have been used to estimate blocking probability in circuit-switched networks with deflection routing [37], [43], [50]–[56]. However, they were all confined to relatively simple cases involving only a single service and where the trunks have a hierarchical structure (hierarchical networks).

Non-hierarchical deflection routing is more flexible and efficient than hierarchical routing due to the fact that it can accommodate a sudden strong increase of offered traffic in different OD pairs, which may happen at different times during the day [57]–[60]. The drawback of non-hierarchical deflection routing is that it may cause instability and collapse of throughput with heavy or overload conditions which can be

prevented by control schemes such as trunk reservation [59]. Overall, non-hierarchical deflection routing is considered an advantage and it was shown to be capable of reducing about 10% cost compared to its hierarchical counterpart [57].

However, for non-hierarchical circuit-switched networks, no robust and generic methodology is available for the approximation of blocking probability (even for cases involving only a single service) that captures networks' overflow-induced state dependencies in a scalable way, except for special cases [61]. The difficulty in obtaining accurate blocking probability is due to the effect of mutual overflows.

One simple and commonly used approach for approximation of blocking probability in non-hierarchical networks is the above mentioned EFPA. In [44], EFPA assumes that the arrivals to each trunk follow a Poisson process and the trunks are independent of each other. See [29], [44], [46], [62]–[70] and references therein for applications of EFPA. However, the accuracy of EFPA is not always satisfactory due to errors introduced by the assumptions of EFPA. EFPA assumes that the call arrivals to each trunk follow a Poisson process while, in fact, overflow traffic is more peaky than Poisson and the traffic offered to a sequence of trunks on a path is actually smoother. EFPA also assumes the trunks are independent while there can be strong dependence among their traffic loads.

There have been various attempts to refine the basic version of EFPA by addressing the errors introduced by its assumptions. In [43], the authors proposed the original idea of EFPA together with a moment matching method to reduce the error introduced by non-Poisson overflow traffic when calculating the blocking probability for a single service three-node network. In [37], a recursive scheme and the equivalent random theory are combined to estimate the blocking probability and the variance of the overflowed streams. However, the analysis of [37] is limited to a single trunk with single service and no application to realistic size network is provided. In [71] and [72], approaches to capture the dependency between trunks along a path in fixed routing networks with multiservice demands are proposed. However, they are limited only to fixed routing networks and do not consider overflow effects. In [73], a method to compute the correlation coefficients between trunks along a path has been proposed and also, it cannot be used in multiservice alternative routing networks.

Another way to categorize the errors of EFPA is to classify them into: *overflow error* and *path error* [74]. Overflow error is caused by the effect of overflow and it leads to underestimation of blocking probability because the high variance of overflow traffic and dependence between the trunks are ignored. Path error is caused by the fact that a path is composed of a sequence of trunks and it overestimates blocking probability because EFPA ignores the effect of traffic smoothing, and the positive correlation of trunk occupancy along the path both of which increase the probability to admit calls.

Another recently developed and proven to be more accurate in various circumstances method is the *Overflow Priority Classification Approximation* (OPCA) [74]–[77]. OPCA is an approximation applicable to overflow loss systems and networks. The idea of OPCA is to impose a fictitious preemptive priority structure in the given network model that yields a

surrogate network model. In the OPCA surrogate network model, preemptive priority is given to calls according to the number of times they have overflowed (seniority). Calls that have overflowed fewer times (*junior calls*) have preemptive priorities over calls that overflowed more times (*senior calls*). Then to derive the blocking probability for the surrogate model using EFPA which is expected to yield a close but somewhat different blocking probability to that of the original overflow network model and, in many cases, a better approximation to it than the one obtained by directly applying EFPA to the original model. The reason lies in the fact that in the surrogate network model, more admission opportunities are provided to the junior calls and therefore the proportion of calls that are being transmitted in its primary path increases. Since these calls do not violate the Poisson assumption, increasing their proportion can reduce the overflow error. Furthermore, by imposing preemptive priority to the surrogate model, OPCA manages to capture the dependence between trunks caused by overflows, while all the existing approximations that aim to capture the dependence between trunks only address the dependence between trunks along the same path. To the best of our knowledge, OPCA is the first method that captures the overflow dependence. Moreover, since OPCA uses an EFPA-like algorithm for its surrogate model, namely decoupling the system into independent sub-systems with Poisson arrival, OPCA is applicable to all the scenarios where EFPA is applied to, and all the enhancements of EFPA can also be implemented in OPCA to further improve the approximation. Please see [34] for more background details on applications of EFPA and OPCA for blocking probability approximation of circuit switched networks with deflection routing.

In our circuit-switched networks with non-hierarchical deflection routing we use trunk reservation to prevent low resource utilization due to large proportion of overflowed traffic, as in [77]. In our trunk reservation policy, a certain number of channels per trunk are reserved for primary path calls. In this way, primary path calls obtain advantage over alternative path calls in order to reduce long and inefficient routes that may be used by alternative calls. This is different from trunk reservation in *e.g.*, [78] where channels are reserved for certain class of traffic. Trunk reservation can also mitigate the instability caused by a large number of overflowed connections in a network. Although proof of convergence to a unique solution for a general network does not exist, it is known from experience that normally circuit switched networks with alternative routing and trunk reservation and its related EFPA solutions do converge to a unique solution. In all the numerical examples that are presented in this paper, all the algorithms of EFPA, OPCA and service-based OPCA have used trunk reservation and they all have converged to a unique solution. We have also proved in [79] that for OBS networks without trunk reservation, upper and lower bounds are produced by OPCA iterations and they approach each other as the number of iterations in each layer increases. Although this has not been proven yet for the case of OCS, it nevertheless, gives us some confidence that in OCS networks without trunk reservation, OPCA will also provide upper and lower bounds of blocking probability which approach each other with increased number

of iterations.

In this paper for a multi-service network, two versions of OPCA are considered. The first is a straightforward application of OPCA, where in the surrogate all calls have preemptive priority over more senior calls. For this OPCA version, it is appropriate to use the name OPCA, so we simply call it OPCA. According to the second approach, called *service-based OPCA*, in the surrogate network, calls have preemptive priority only over more senior calls belonging to the same class.

We compare between the results obtained by the various approximations against simulation benchmarks and explain their performance in various different scenarios and parameter value ranges. Then we discuss the insight gained into performance tradeoffs as well as design implications.

In [34], we considered two classes with the same bandwidth requirement and assumed that one class has strict priority over the other. To evaluate the blocking probability of a circuit switched network with deflection routing and these two classes of demands, we only need to consider single service traffic for the higher priority while the resource for the lower priority traffic is the leftover of the higher priority and we introduce quasi-stationary approximation to evaluate the capacity left for lower priority. Unlike [34], we consider here a general number of multiservice demands with different capacity requirements and fair opportunity to compete for the pool of resources. We develop new algorithms to capture the effect of mutual overflow among the classes and also discuss reduction of the overflow error by moment matching and the relaxation of the disjointedness assumption. To the best of our knowledge, it is the first time that the performance of multiservice demands in non-hierarchical circuit switched networks with deflection routing is studied.

The remainder of the paper is organized as follows. In Section II, we provide a detailed description of our network model and define notation and basic concepts. Next, in Section III, we describe in detail the approximations OPCA, service-based OPCA and EFPA as applied to our multi-service circuit switched network model. Then, in Section IV, we provide numerical results over a wide range of parameter values and discuss performance and design implications. We also discuss and illustrate there effects of services rates, bandwidth requirements, the number of channels per trunk, the maximum allowable number of alternative paths and trunk reservation, as well as the sensitivity of the shape of the holding time distribution. We also apply them in asymmetrical networks and the CORONET and discuss their performance. Finally, the paper is concluded in Section V.

## II. THE MODEL

We consider a circuit-switched network described by a graph  $G(N, E)$  where  $N$  is a set of  $n$  nodes and  $E$  is a set of  $e$  arcs that connect the nodes. The  $e$  arcs correspond to trunks where trunk  $i \in E$  carries  $C_i$  channels. The  $N$  nodes are designated  $1, 2, 3, \dots, N$ , each of them has circuit switching capabilities.

In the context of a hybrid TDM/WDM network, a wavelength channel is divided into multiple fixed length time slots

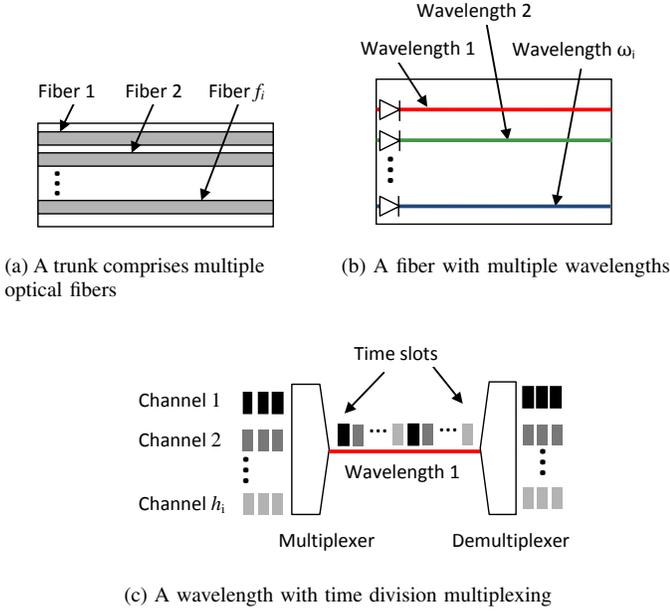


Fig. 1: Illustration of WDM trunk hierarchy.

to increase utilization. These time slots, multiplexed on the wavelength, can be viewed as channels. In this case, trunk  $i \in E$  is composed of  $f_i$  fibers, each of which supports  $w_i$  wavelengths, composed of  $h_i$  TDM time slots, as shown in Fig. 1. Accordingly, trunk  $i \in E$  carries  $C_i = f_i w_i h_i$  channels. We assume that all the nodes have full wavelength conversion capabilities and can switch traffic from any channel on one trunk to any other channel on an adjacent trunk. Note that our model and algorithms are also applicable to cases with no wavelength conversion. The number of channels on trunk  $i$  is  $C_i = f_i h_i$  with no wavelength conversion.

Let  $\Gamma$  be a set of directional OD pairs. Every directional OD pair  $m \in \Gamma$ , is defined by its end-nodes. Thus,  $m = \{i, j\} \in \Gamma$  represents the directional OD pair  $i$  to  $j$ . We will distinguish between the term *OD pair* which is an unordered set of the two endpoints: Origin and Destination, and the *directional OD pair* that refers to the ordered set: Origin-Destination.

The number of different service classes of calls offered to the network is  $P$ . For each directional OD pair  $m \in \Gamma$ , calls of class  $p$ ,  $p = 1, 2, \dots, P$ , arrive according to a Poisson process with arrival rate  $\lambda_{m,p}$ . The number of channels that a class  $p$  requires is  $v_p$ , also referred to as the bandwidth requirement of class  $p$ . The holding times of calls are assumed exponentially distributed with mean  $1/\mu_{m,p}$ . Let

$$\rho_{m,p} = \frac{\lambda_{m,p}}{\mu_{m,p}}$$

be the offered traffic (measured in erlangs) for directional OD pair  $m$ . We set

$$\rho_p = \sum_{m \in \Gamma} \rho_{m,p}.$$

A route between origin  $i$  and destination  $j$  is the sequence of trunks associated with the corresponding arcs in the path between  $i$  and  $j$  in  $G(N, E)$ .

It is very likely that for a directional OD pair  $m \in \Gamma$ , there are multiple routes between the origin and the destination that do not share a common trunk. Such routes are often called *edge-disjoint paths* or *disjoint paths* [80]–[83]. Edge-disjoint deflection routing is often used to achieve load balancing in optical and other networks [84]–[88].

For each  $m \in \Gamma$ , we designate a route with the least number of hops as the primary path  $U_m(0)$  of the directional OD pair  $m$ . If there are multiple routes with the least number of hops, the choice is made randomly with equal probabilities. Then considering a new topology, where the trunks of the primary path are excluded, the first alternative path for  $m$  is chosen to minimize the number of hops in the new topology. Again, a tie is broken randomly. Therefore, all the primary path and alternative paths for  $m$  are edge-disjoint. Let  $R_m$  be the maximum number of available alternative paths a directional OD pair  $m$  can have based on the network topology.

Furthermore, a maximal number  $D$  of overflow attempts to alternative paths are set for all directional OD pairs in  $\Gamma$ . Setting the limit  $D$  implies that a connection in the directional OD pair  $m$  can only use

$$R(m) = \min\{R_m, D\}$$

alternative paths. Therefore, before a connection is blocked, the procedure continues until all available and allowable  $R(m)$  routes are attempted.

It is convenient to maintain the entire set

$$\{U_m(0), U_m(1), \dots, U_m(R_m)\}$$

of alternative routes for the directional OD pair  $m \in \Gamma$  in which  $U_m(0)$  is the primary path and  $U_m(d)$  is the  $d$ th alternative path. This allows for cases where  $D$  do not limit the number of usable alternative path.

In our model, the ranking of alternative paths is based on the number of hops and in case of equality in the number of hops, the rank is chosen randomly. Based on our ranking, if  $d_i > d_j$  then the number of hops of  $U_m(d_i)$  is equal to or higher than the number of hops in  $U_m(d_j)$ . However, in practice, other cost functions (e.g., geographic distance) can be also used for the ranking.

If a request for a call arrives at original node  $i$  to the destination node  $j$ , and capacity is available on all trunks of the primary path  $U_{\{i,j\}}(0)$ , then this primary path will be used for the transmission of this call.

An arriving call of any class can use any free channel on any trunk. When a class  $p$  call of OD pair  $m$  arrivals, it can establish a connection if all trunks of its primary path have no less than  $v_p$  free channels. Otherwise it will overflow to its first alternative path. Then, the procedure repeats itself. If a newly arriving call is not able to obtain a lightpath in its  $R(m)$  alternative path attempts, the call is blocked and cleared of the network. Let  $\beta_p$  is the set of OD pairs that are transmitting class  $p$  calls, then a class  $p$  call can overflow in the network at most  $\max R(m), m \in \beta_p$  times, which is defined as the maximum allowable number of overflow of class  $p$  connections, referred to as  $D_p$ .

Considering the stability of the network, and recognizing that less resources are used by a call that uses its primary

path, priority is given to such calls. To facilitate such priority, a certain number of unoccupied channels are reserved for calls attempting their primary path. In particular, if the number of channels occupied on trunk  $j$  is greater than or equal to a given reservation threshold  $T_p$ , the overflowed calls of class  $p$  are not allowed to use that trunk.

### III. BLOCKING PROBABILITY APPROXIMATIONS

In this section, we describe the approximations we use for blocking probability evaluation of the multiservice model. We use the term 0-call for a call transmitted on its primary path, and the term  $d$ -call for a call transmitted on its  $d$ th alternative path, for  $d = 1, 2, \dots, \max R(m)$ . Accordingly, the term  $(d, m, p)$ -call refers to a  $d$ -call of class  $p$  from the original node towards the destination of the directional OD pair  $m$  with offered load  $a(d, m, p)$ . Assume that the arrivals of the  $(d, m, p)$ -calls at trunk  $j \in U_m(d)$  follow a Poisson process with offered load  $a(d, m, p, j)$ . Let  $b_{j,p}(d)$  be the blocking probability of any class  $p$   $d$ -call offered to trunk  $j \in \mathcal{E}$ .

The  $(d, m, p)$ -calls occur only when  $(d-1, m, p)$ -calls are blocked for directional OD pair  $m$  and for  $d = 1, 2, \dots, R(m)$ . Therefore, we have

$$a(d, m, p) = a(d-1, m, p) \left(1 - \prod_{j \in U_m(d-1)} (1 - b_{j,p}(d-1))\right) \quad (1)$$

and  $a(0, m, p) = \rho_{m,p}$ . For a particular trunk along the path  $j \in U_m(d)$ , we have

$$a(d, m, p, j) = a(d, m, p) \frac{\prod_{i \in U_m(d)} (1 - b_{i,p}(d))}{1 - b_{j,p}(d)} \quad (2)$$

for  $d = 0, 1, \dots, R(m)$ . For  $d > R(m)$  or  $j \notin U_m(d)$ ,  $a(d, m, p, j) = 0$ .

Let  $a(d, j, p)$  be the total offered load of class  $p$   $d$ -calls, on trunk  $j$ . The variables  $a(d, j, p)$  and  $a(d, m, p, j)$  are related by

$$a(d, j, p) = \sum_{m \in \Gamma} a(d, m, p, j). \quad (3)$$

Also, let  $\tilde{a}(d, j, p)$  be the total offered load of class  $p$  calls that include 0-calls, 1-calls, 2-calls ...  $d$ -calls, on trunk  $j$ . The variables  $\tilde{a}(d, j, p)$  and  $a(d, j, p)$  are related by

$$\tilde{a}(d, j, p) = \sum_{i=0}^d a(i, j, p). \quad (4)$$

#### A. EFPA

Let  $q_j(i)$  be the steady-state probability of having  $i$  channels busy in trunk  $j$ . For a single trunk loaded by multiservice traffic, where each class of calls follow a Poisson process and the attributes of all the calls are independent of each other, the steady-state probabilities have a product form solution that can be readily obtained by a recursive algorithm [89]. We evaluate the trunk state probability  $q_j(i)$ ,  $j \in \mathcal{E}$  and  $i \in \{1, \dots, C_j\}$  by

$$q_j(i) = \frac{1}{i} \sum_{r=1}^P \left( a(0, j, r) + \mathbf{1}\{T_r > i - v_r\} \sum_{n=1}^{D_r} a(n, j, r) \right) \times v_r \times q_j(i - v_r), \quad (5)$$

where  $\mathbf{1}\{\cdot\}$  is the indicator function and  $q_j(0)$  is set such that  $\sum_{i=0}^{C_j} q_j(i) = 1$  is satisfied. The blocking probability, for class  $p$  traffic with  $d$  overflows, on trunk  $j$  is estimated by

$$b_{j,p}(d) = \begin{cases} \sum_{i=c_j-v_p+1}^{C_j} q_j(i) & d = 0, \\ \sum_{i=T_p}^{C_j} q_j(i) & d \geq 1. \end{cases} \quad (6)$$

Equations (1) – (6) form a set of fixed point equations which can be solved by successive substitutions.

Having obtained the results of the fixed point equations, we calculate the blocking probabilities for class  $p$  traffic from OD pair  $m$  by

$$B_{m,p} = 1 - \sum_{d=0}^{R(m)} a(d, m, p, j) (1 - b_{j,p}(d)) / \rho_{m,p}, \quad (7)$$

where  $j$  is the last trunk in the route for the calls of OD pair  $m$  that overflow  $d$  times. Let  $B_p$  be the network blocking probability for class  $p$  traffic, which is the average of blocking probabilities of all OD pairs, weighted by their offered load.

$$B_p = \sum_{m \in \Gamma} B_{m,p} \times \rho_{m,p} / \sum_{m \in \Gamma} \rho_{m,p}. \quad (8)$$

Algorithm 1 is used to obtain the network blocking probability  $B_p$ ,  $p = 1, 2, \dots, P$  by EFPA.

---

**Algorithm 1** Compute  $B_p$  for  $p = 1, 2, \dots, P$  by EFPA

---

**Require:**  $\rho_{m,p}$  for  $m \in \Gamma$ ,  $p = 1, 2, \dots, P$

**initial:**  $b_{j,p}(d) \leftarrow 0$ ,  $\hat{b}_{j,p}(d) \leftarrow 1$  for  $j \in \mathcal{E}$ ,  $p = 1, 2, \dots, P$ ,  $d \in \{0, \dots, D_p\}$

**while**  $\sum_{r=1}^P \sum_{d \in \{0, \dots, D_r\}} \sum_{j \in \mathcal{E}} |b_{j,r}(d) - \hat{b}_{j,r}(d)| > 1e-8$  **do**

**for**  $j \in \mathcal{E}$ ,  $p = 1, 2, \dots, P$ ,  $d \in \{0, \dots, D_p\}$ ,  $m \in \Gamma$  **do**

$\hat{b}_{j,p}(d) \leftarrow b_{j,p}(d)$

    compute  $a(d, m, p)$  in Eq. (1)

    compute  $a(d, m, p, j)$  in Eq. (2)

    compute  $a(d, j, p)$  in Eq. (3)

**end for**

**for**  $j \in \mathcal{E}$ ,  $d \in \{0, \dots, D_p\}$  **do**

    compute  $q_j(i)$  in Eq. (5) for  $i \in \{1, \dots, C_j\}$

    compute  $b_{j,p}(d)$  in Eq. (6)

**end for**

**end while**

compute  $B_{m,p}$  in Eq. (7) for  $m \in \Gamma$ ,  $p = 1, 2, \dots, P$

compute  $B_p$  in Eq. (8) for  $p = 1, 2, \dots, P$ .

---

#### B. OPCA

Although in our multiservice network model, no service class traffic has priority over another service class traffic, OPCA works by using a hierarchical surrogate second system in which junior calls have preemptive priority over senior calls and estimating the blocking probability in the second system by applying an EFPA-like algorithm.

For multiservice networks, the preemptive priority of junior calls in OPCA can be operated over more senior calls belonging to any class, referred to as OPCA, or over more senior calls belonging to the same class, referred to as service-based OPCA.

1) *OPCA in multiservice networks*: In the following we provide detailed information on how to apply OPCA to the present problem of approximating blocking probability of circuit switched networks with different classes of calls.

We begin by evaluating the trunk state probability  $t_{d,j}(i)$  for each trunk  $j \in \mathcal{E}$ , for  $d \in \{0, \dots, \max R(m)\}$  deflections and each state  $i \in \{1, \dots, C_j\}$  using

$$t_{d,j}(i) = \frac{1}{i} \sum_{r=1}^P (a(0, j, r) + \mathbf{1}\{T_r > i - v_r\} \sum_{n=1}^d a(n, j, r)) \times v_r \times t_{d,j}(i - v_r), \quad (9)$$

where  $t_{d,j}(0)$  is set such that  $\sum_{i=0}^{C_j} t_{d,j}(i) = 1$  is satisfied [89].

The average blocking probability  $\bar{b}_{j,p}(d)$  on trunk  $j \in \mathcal{E}$ , for class  $p$  calls with up to and including  $d$  overflows, is estimated by

$$\bar{b}_{j,p}(d) = \sum_{i=T_p}^{C_j} t_{d,j}(i). \quad (10)$$

The average blocking probability  $\bar{b}_{j,p}(0)$  for class  $p$  primary calls is estimated by

$$\bar{b}_{j,p}(0) = \sum_{i=C_j-v_p+1}^{C_j} t_{0,j}(i). \quad (11)$$

The term *average blocking probability* is referred to blocking probability in a non-priority system where the junior and senior calls have the equal opportunities. Equivalently, the term *average blocked traffic* is the blocked traffic in a non-priority system which is obtained by multiplying offered load.

The actual blocking probability of class  $p$  calls, for  $d$ -calls ( $d \geq 1$ ) on trunk  $j$  is estimated by

$$b_{j,p}(d) = \bar{b}_{j,p}(d) + \frac{\sum_{r=1}^P (\sum_{n=0}^{d-1} a(n, j, r) \times (\bar{b}_{j,r}(n) - b_{j,r}(n))) \times (1 - \bar{b}_{j,p}(d))}{\sum_{r=1}^P a(d, j, r) \times (1 - \bar{b}_{j,r}(d))} \quad (12)$$

where the first term  $\bar{b}_{j,p}(d)$  is the average blocking probability of  $d$ -calls ( $d \geq 1$ ) of class  $p$  on trunk  $j$ . Note that in the surrogate model of OPCA, the junior calls have preemptive priority over senior calls belonging to any class. To consider this preemptive priority, the term  $\sum_{r=1}^P (\sum_{n=0}^{d-1} a(n, j, r) \times (\bar{b}_{j,r}(n) - b_{j,r}(n)))$  is the difference between the average blocked traffic of junior calls and its actual blocked traffic (in the preemptive priority system of the OPCA surrogate). This term becomes the total blocked traffic of  $d$ -calls of all the service classes. Then we estimate the part of blocked  $d$ -call traffic of class  $p$  according to its proportion of the total  $d$ -call carried traffic, which is  $a(d, j, p)(1 - \bar{b}_{j,p}(d)) / \sum_{r=1}^P a(d, j, r) \times (1 - \bar{b}_{j,r}(d))$ . Dividing the latter by the offered load  $a(d, j, r)$  yields the difference between the blocking probability of  $d$ -calls of class  $p$  and the average blocking probability which is the second term in (12).

For class  $p$  calls that are transmitted on their primary paths ( $d = 0$ ),

$$b_{j,p}(0) = \bar{b}_{j,p}(0). \quad (13)$$

Algorithm 2 is used to obtain the network blocking probability  $B_p$ ,  $p = 1, 2, \dots, P$  by OPCA.

---

**Algorithm 2** Compute  $B_p$  for  $p = 1, 2, \dots, P$  by OPCA

---

**Require:**  $\rho_{m,p}$  for  $m \in \Gamma$ ,  $p = 1, 2, \dots, P$

**initial:**  $b_{j,p}(d) \leftarrow 0$ ,  $\hat{b}_{j,p}(d) \leftarrow 1$  for  $j \in \mathcal{E}$ ,  $p = 1, 2, \dots, P$ ,  $d \in \{0, \dots, D_p\}$

**for**  $d \in \{0, \dots, \max R(m)\}$  **do**

**while**  $\sum_{r=1}^P \sum_{j \in \mathcal{E}} |b_{j,r}(d) - \hat{b}_{j,r}(d)| > 1e-8$  **do**

**for**  $j \in \mathcal{E}$ ,  $p = 1, 2, \dots, P$ ,  $m \in \Gamma$  **do**

$\hat{b}_{j,p}(d) \leftarrow b_{j,p}(d)$

      compute  $a(d, m, p)$  in Eq. (1)

      compute  $a(d, m, p, j)$  in Eq. (2)

      compute  $a(d, j, p)$  in Eq. (3)

**end for**

**for**  $j \in \mathcal{E}$  **do**

**if**  $d == 0$  **then**

      compute  $t_{0,j}(i)$  in Eq. (9) for  $i \in \{1, \dots, C_j\}$

      compute  $\bar{b}_{j,p}(0)$  in Eq. (11)

      compute  $b_{j,p}(0)$  in Eq. (13)

**else**

      compute  $t_{d,j}(i)$  in Eq. (9) for  $i \in \{1, \dots, C_j\}$

      compute  $\bar{b}_{j,p}(d)$  in Eq. (10)

      compute  $b_{j,p}(d)$  in Eq. (12)

**end if**

**end for**

**end while**

**end for**

compute  $B_{m,p}$  in Eq. (7) for  $m \in \Gamma$ ,  $p = 1, 2, \dots, P$

compute  $B_p$  in Eq. (8) for  $p = 1, 2, \dots, P$ .

---

2) *Service-based OPCA*: For multiservice networks, results of OPCA for the surrogate model may be biased relative to the real model in some classes. It is more difficult for high-bandwidth required calls to enter the network than low-bandwidth required ones and therefore they require more overflows to establish a connection. The priorities operated by OPCA, however, worsen the acceptance of these high-bandwidth required calls which have overflowed many times and can be preempted by the junior calls of the low-bandwidth required class. This effect additionally brought by the priority of OPCA will increase the blocking probability of the high-bandwidth required traffic to a large extent when the difference of the bandwidth requirement of the classes is large or the offered load of the low-bandwidth required class is much more than the high-bandwidth required class. Taking this into consideration, we operate this kind of priority within but not across the classes, which means the junior calls have priority over the senior calls of the same class but not the ones of different classes.

We remind the reader that the prioritization introduced in the surrogate system of service-based OPCA is also artificially introduced to obtain a more accurate approximation and it is not a feature of the real network.

In the following we provide detailed information on how to apply service-based OPCA to the present problem of approximating blocking probability of circuit switched networks with

different classes of calls.

We begin by evaluating the trunk state probability  $t_{d,j,p}(i)$  of class  $p$  for each trunk  $j \in \mathcal{E}$ , for  $d \in \{0, \dots, D_p\}$  deflections and each state  $i \in \{1, \dots, C_j\}$  using

$$\begin{aligned} t_{d,j,p}(i) &= \frac{1}{i} \sum_{r \in \{1, \dots, P\}, r \neq p} a(0, j, r) \times v_r \times t_{d,j,p}(i - v_r) + \\ &\frac{1}{i} \sum_{r \in \{1, \dots, P\}, r \neq p} (\mathbf{1}\{T_r > i - v_r\} \sum_{n=1}^{D_r} a(n, j, r)) \times v_r \times t_{d,j,p}(i - v_r) + \\ &\frac{v_p}{i} (a(0, j, p) + \mathbf{1}\{T_p > i - v_p\} \sum_{n=1}^d a(n, j, p)) \times t_{d,j,p}(i - v_p), \end{aligned} \quad (14)$$

where  $t_{d,j,p}(0)$  is set such that  $\sum_{i=0}^{C_j} t_{d,j,p}(i) = 1$  is satisfied [89].

The average blocking probability  $\bar{b}_{j,p}(d)$  on trunk  $j \in \mathcal{E}$ , for class  $p$  calls with up to and including  $d \in \{0, \dots, D_p\}$  overflows, is estimated by

$$\begin{aligned} \bar{b}_{j,p}(d) &= \frac{\sum_{n=1}^d (a(n, j, p) \sum_{i=T_p}^{C_j} t_{d,j,p}(i))}{\bar{a}(d, j, p)} + \\ &\frac{a(0, j, p) \sum_{i=C_j - v_p + 1}^{C_j} t_{d,j,p}(i)}{\bar{a}(d, j, p)}. \end{aligned} \quad (15)$$

The blocking probability of class  $p$  calls, for  $d$ -overflows calls,  $d \in \{0, \dots, D_p\}$ , on trunk  $j$  is estimated by

$$b_{j,p}(d) = \begin{cases} \bar{b}_{j,p}(0) & d = 0, \\ \frac{\bar{b}_{j,p}(d) \bar{a}(d, j, p) - \bar{b}_{j,p}(d-1) \bar{a}(d-1, j, p)}{\bar{a}(d, j, p)} & 1 \leq d \leq D_p. \end{cases} \quad (16)$$

Algorithm 3 is used to obtain the network blocking probability  $B_p$ ,  $p = 1, 2, \dots, P$  by service-based OPCA.

---

**Algorithm 3** Compute  $B_p$  for  $p = 1, 2, \dots, P$  by service-based OPCA

---

**Require:**  $\rho_{m,p}$  for  $m \in \Gamma$ ,  $p = 1, 2, \dots, P$

**initial:**  $b_{j,p}(d) \leftarrow 0$ ,  $\hat{b}_{j,p}(d) \leftarrow 1$  for  $j \in \mathcal{E}$ ,  $p = 1, 2, \dots, P$ ,  $d \in \{0, \dots, D_p\}$

**while**  $\sum_{r=1}^P \sum_{d \in \{0, \dots, D_r\}} \sum_{j \in \mathcal{E}} |b_{j,r}(d) - \hat{b}_{j,r}(d)| > 1e - 8$  **do**

**for**  $j \in \mathcal{E}$ ,  $p = 1, 2, \dots, P$ ,  $m \in \Gamma$ ,  $d \in \{0, \dots, D_r\}$  **do**

$\hat{b}_{j,p}(d) \leftarrow b_{j,p}(d)$

    compute  $a(d, m, p)$  in Eq. (1)

    compute  $a(d, m, p, j)$  in Eq. (2)

    compute  $a(d, j, p)$  in Eq. (3)

    compute  $\bar{a}(d, j, p)$  in Eq. (4)

**end for**

**for**  $j \in \mathcal{E}$ ,  $d \in \{0, \dots, D_p\}$ ,  $p = 1, 2, \dots, P$  **do**

    compute  $t_{d,j,p}(i)$  in Eq. (14) for  $i \in \{1, \dots, C_j\}$

    compute  $\bar{b}_{j,p}(d)$  in Eq. (15)

    compute  $b_{j,p}(d)$  in Eq. (16)

**end for**

**end while**

compute  $B_{m,p}$  in Eq. (7) for  $m \in \Gamma$ ,  $p = 1, 2, \dots, P$

compute  $B_p$  in Eq. (8) for  $p = 1, 2, \dots, P$ .

---

All the three approximation methods are based on fixed point iterations. The relative error criterion is a parameter, set to measure the difference of the substitution results and the iteration will stop when

$$\sum_{j \in \mathcal{E}} |b(d, j, 1) - \hat{b}(d, j, 1)| < \text{relative error criterion.}$$

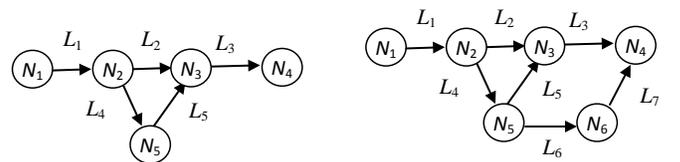
The numbers of iterations until convergence can be affected by initial values, relative error criterion and all the parameters of the model.

### C. The trunk offered load in non-disjoint path cases

We now consider cases where the primary and the alternative paths of each OD pair are not necessarily disjoint, i.e., they may contain some common trunks. If the non-disjoint paths can also be included as alternative paths, calls can have more opportunities to be served, especially when the traffic load is not evenly distributed in the network. In this case, the heavily loaded trunks cause path congestion while other trunks on the paths may not be fully utilized, and can be used for non-disjoint alternative paths. To calculate the blocking probability in such cases, notice that all we need is to calculate the traffic offered of each trunk. After that, the remaining procedure can be completed exactly as in the previous cases with disjoint paths for all the algorithms: EFPA, OPCA and service-based OPCA.

In the case of disjoint paths, the offered load to an alternative path is simply a function of the probability of one or more trunks in the primary path being congested. However, in the case of non-disjoint path, a blocked trunk in one path immediately implied that all alternative paths using this trunk are blocked, so the offered load contribution to an alternative path from another path that shares a blocked common trunk with it is zero. Therefore, derivation of the total offered traffic to an alternative path, from a previous path requires conditioning that all the trunks common to both paths are not blocked. This creates a large number of overflow events that need to be considered when deriving the offered traffic to each alternative path. This introduces a significant complexity in writing different equations for the offered load of each trunk. This complexity significantly increases with the size of the network and  $D$ . There is no fundamental difficulty in evaluating blocking probability for these cases but the difficulty is caused by the complexity due to the large number of cases that must be considered.

This is illustrated in the following example.



(a) Two paths

(b) Three paths

Fig. 2: Examples of non-disjoint paths.

We take the scenario of Fig. 2(a) for example when we transmit data from node  $N_1$  to node  $N_4$  along two paths. The primary path contains trunks  $L_1$ ,  $L_2$  and  $L_3$  and the first alternative path contains trunks  $L_1$ ,  $L_4$ ,  $L_5$  and  $L_3$ . They contain common trunks  $L_1$  and  $L_3$ . We first transmit data along the primary path and the first alternative will be used only in the case that  $L_2$  is congested while  $L_1$  and  $L_3$  still have free channels. For the cases that at least one of the trunks  $L_1$  and  $L_3$  congested, both the primary path and the first alternative path fail. Therefore, the offered load to the first alternative path is

$$a(1, m, p) = \rho_{m,p} b_{2,p}(0) (1 - b_{1,p}(0)) (1 - b_{3,p}(0)). \quad (17)$$

Then we consider the scenario of Fig. 2(b), in which a second alternative containing trunks  $L_1$ ,  $L_4$ ,  $L_6$  and  $L_7$  is added to the case of Fig. 2(a). The second alternative path has a common trunk  $L_1$  with the primary path and two common trunks  $L_1$  and  $L_4$  with the first alternative path. The offered load to the second alternative path can be introduced by the congestion of trunk  $L_3$  or/and the simultaneous congestion of  $L_2$  and  $L_5$ . In this case, we have

$$a(2, m, p) = \rho_{m,p} (1 - b_{1,p}(0)) (1 - b_{4,p}(1)) \times ((b_{3,p}(0)) + (1 - b_{3,p}(0)) b_{2,p}(0) b_{5,p}(1)). \quad (18)$$

Equation (18) demonstrates how the offered load on alternative path is derived conditioning on the congestion state of the common trunks. For a realistic network such equations need to be individually written for each alternative path. By comparison, equation (1) applies to all alternative paths in the network. Nevertheless, the calculation procedure of trunk blocking probability and network blocking probability are the same with disjoint paths. A numerical example with non-disjoint paths is presented in Section IV-L. We assume disjoint paths in default scenario for simplicity. Those who are interested in the application to the cases with non-disjoint paths can calculate the offered load on each trunk as we do in the examples and follow the remaining steps of the algorithms.

We have defined the set of routes of  $m \in \Gamma$  as

$$\{U_m(0), U_m(1), \dots, U_m(R_m)\}.$$

One of the routes  $U_m(i)$  in which the number of trunks is  $n_i$  has all together  $2^{n_i} - 1$  non-empty subsets  $v_m(i, r)$ ,  $r = 1, 2, \dots, 2^{n_i} - 1$ .

If these paths are non-disjoint, define  $u_m(i) = U_m(i) - \cup_{k=0}^{i-1} U_m(k)$  and  $u_m(0) = U_m(0)$ , then the routes of the set

$$\{u_m(0), u_m(1), \dots, u_m(R_m)\}$$

are disjoint.

Let the indicator function  $H(i, m, j)$  for trunk  $j \in U_m(i)$  be

$$H(i, m, j) = \begin{cases} 0, & j \in \cup_{k=0}^{i-1} v_m(k, r_k), \\ 1, & \text{otherwise.} \end{cases} \quad (19)$$

Algorithms 4, 5 and 6 are used to obtain the network blocking probability  $B_p$ ,  $p = 1, 2, \dots, P$  by EFPA, OPCA and service-based OPCA, respectively, where the primary and the alternative paths of each OD pair are not necessarily disjoint.

---

**Algorithm 4** Compute  $B_p$  for  $p = 1, 2, \dots, P$  by EFPA for non-disjoint cases

---

**Require:**  $\rho_{m,p}$  for  $m \in \Gamma$ ,  $p = 1, 2, \dots, P$   
**initial:**  $b_{j,p}(d) \leftarrow 0$ ,  $\hat{b}_{j,p}(d) \leftarrow 1$  for  $j \in \mathcal{E}$ ,  $p = 1, 2, \dots, P$ ,  $d \in \{0, \dots, D_p\}$   
**while**  $\sum_{r=1}^P \sum_{d \in \{0, \dots, D_r\}} \sum_{j \in \mathcal{E}} |b_{j,r}(d) - \hat{b}_{j,r}(d)| > 1e-8$  **do**  
  **for**  $j \in \mathcal{E}$ ,  $p = 1, 2, \dots, P$ ,  $d \in \{0, \dots, D_p\}$ ,  $m \in \Gamma$  **do**  
     $\hat{b}_{j,p}(d) \leftarrow b_{j,p}(d)$   
    **for**  $i \in \{0, \dots, d-1\}$  **do**  
      **for**  $r_i \in \{1, \dots, 2^{n_i} - 1\}$  **do**  
        **if**  $\cup_{i=0}^{d-1} v_m(i, r_i) \cap U_m(d) == \emptyset \& \cup_{i=0}^{d-1} v_m(i, r_i) \cap (\cup_{i=0}^{d-1} (u_m(i) - v_m(i, r_i))) == \emptyset$  **then**  
           $F(i) = \prod_{j \in v_m(i, r_i)} b_{j,p}(i)^{H(i, m, j)}$   
           $G(i) = \prod_{j \in u_m(i) - v_m(i, r_i)} (1 - b_{j,p}(i))$   
           $a(d, m, p) = a(d, m, p) + \prod_{i=0}^{d-1} F(i)G(i)$   
        **end if**  
      **end for**  
    **end for**  
  **end for**  
  compute  $a(d, m, p, j)$  in Eq. (2)  
  compute  $a(d, j, p)$  in Eq. (3)  
**end for**  
**for**  $j \in \mathcal{E}$ ,  $d \in \{0, \dots, D_p\}$  **do**  
  compute  $q_j(i)$  in Eq. (5) for  $i \in \{1, \dots, C_j\}$   
  compute  $b_{j,p}(d)$  in Eq. (6)  
**end for**  
**end while**  
compute  $B_{m,p}$  in Eq. (7) for  $m \in \Gamma$ ,  $p = 1, 2, \dots, P$   
compute  $B_p$  in Eq. (8) for  $p = 1, 2, \dots, P$ .

---

#### D. The max(EFPA, service-based OPCA) approximation

The accuracy of the above three approximations depends on the combined effect of overflow error and path error introduced, in which overflow error causes underestimation and path error causes overestimation. Both of the errors can be affected by different network parameter values, which therefore affect the accuracy of the approximations and make different approximations the most accurate under different scenarios. The difference in behavior of EFPA versus service-based OPCA under different scenarios give rise to a new approximation based on choosing the maximal value of the EFPA and service-based OPCA blocking probability approximations designated. As demonstrated empirically in Section IV, it can lead to an accurate approximation in most scenarios, and almost always it seems to be a conservative approximation.

## IV. NUMERICAL RESULTS

In this section, we compare the performance of OPCA, service-based OPCA and EFPA in approximating the network blocking probabilities of multiservice classes. To this end, we will consider a wide range of scenarios. However, one scenario, which we call a *default scenario* where the offered load for each OD pair is the same and has a symmetrical network topology will receive much attention. In all cases considered, we also provide intuitive explanations to the discrepancies between the approximations and simulation results for the network blocking probabilities as they vary according

---

**Algorithm 5** Compute  $B_p$  for  $p = 1, 2, \dots, P$  by OPCA for non-disjoint cases

---

**Require:**  $\rho_{m,p}$  for  $m \in \Gamma$ ,  $p = 1, 2, \dots, P$   
**initial:**  $b_{j,p}(d) \leftarrow 0$ ,  $\hat{b}_{j,p}(d) \leftarrow 1$  for  $j \in \mathcal{E}$ ,  $p = 1, 2, \dots, P$ ,  $d \in \{0, \dots, D_p\}$   
**for**  $d \in \{0, \dots, \max R(m)\}$  **do**  
  **while**  $\sum_{r=1}^P \sum_{j \in \mathcal{E}} |b_{j,r}(d) - \hat{b}_{j,r}(d)| > 1e-8$  **do**  
    **for**  $j \in \mathcal{E}$ ,  $p = 1, 2, \dots, P$ ,  $m \in \Gamma$  **do**  
       $\hat{b}_{j,p}(d) \leftarrow b_{j,p}(d)$   
      **for**  $i \in \{0, \dots, d-1\}$  **do**  
        **for**  $r_i \in \{1, \dots, 2^{n_i} - 1\}$  **do**  
          **if**  $\cup_{i=0}^{d-1} v_m(i, r_i) \cap U_m(d) == \emptyset \& \cup_{i=0}^{d-1} v_m(i, r_i) \cap (\cup_{i=0}^{d-1} (u_m(i) - v_m(i, r_i))) == \emptyset$  **then**  
             $F(i) = \prod_{j \in v_m(i, r_i)} b_{j,p}(i)^{H(i,m,j)}$   
             $G(i) = \prod_{j \in u_m(i) - v_m(i, r_i)} (1 - b_{j,p}(i))$   
             $a(d, m, p) = a(d, m, p) + \prod_{i=0}^{d-1} F(i)G(i)$   
          **end if**  
        **end for**  
      **end for**  
      compute  $a(d, m, p, j)$  in Eq. (2)  
      compute  $a(d, j, p)$  in Eq. (3)  
    **end for**  
  **for**  $j \in \mathcal{E}$  **do**  
    **if**  $d == 0$  **then**  
      compute  $t_{0,j}(i)$  in Eq. (9) for  $i \in \{1, \dots, C_j\}$   
      compute  $\hat{b}_{j,p}(0)$  in Eq. (11)  
      compute  $b_{j,p}(0)$  in Eq. (13)  
    **else**  
      compute  $t_{d,j}(i)$  in Eq. (9) for  $i \in \{1, \dots, C_j\}$   
      compute  $\hat{b}_{j,p}(d)$  in Eq. (10)  
      compute  $b_{j,p}(d)$  in Eq. (12)  
    **end if**  
  **end for**  
  **end while**  
**end for**  
compute  $B_{m,p}$  in Eq. (7) for  $m \in \Gamma$ ,  $p = 1, 2, \dots, P$   
compute  $B_p$  in Eq. (8) for  $p = 1, 2, \dots, P$ .

---

to various effects. In particular, we consider effects such as the effect of the service rates and bandwidth requirements of both classes. We then consider design factors such as: the number of channels per trunk, the maximum allowable number of alternative paths, and the effect of trunk reservation. We also discuss the robustness of the approximations to the shape of the holding time. We will consider symmetric and asymmetric scenarios, and networks of various topologies, including the Next Generation Core Optical Network (CORONET).

The performance results are compared based on simulations unless the running times are prohibitive. Error bars for the 95% confidence intervals based on Student's t-distribution are provided for all the simulation results although in many cases the intervals are too small to be clearly visible. In any case, the length of the confidence interval is always less than 10% of the mean value measured.

---

**Algorithm 6** Compute  $B_p$  for  $p = 1, 2, \dots, P$  by service-based OPCA for non-disjoint cases

---

**Require:**  $\rho_{m,p}$  for  $m \in \Gamma$ ,  $p = 1, 2, \dots, P$   
**initial:**  $b_{j,p}(d) \leftarrow 0$ ,  $\hat{b}_{j,p}(d) \leftarrow 1$  for  $j \in \mathcal{E}$ ,  $p = 1, 2, \dots, P$ ,  $d \in \{0, \dots, D_p\}$   
**while**  $\sum_{r=1}^P \sum_{d \in \{0, \dots, D_r\}} \sum_{j \in \mathcal{E}} |b_{j,r}(d) - \hat{b}_{j,r}(d)| > 1e-8$  **do**  
  **for**  $j \in \mathcal{E}$ ,  $p = 1, 2, \dots, P$ ,  $m \in \Gamma$ ,  $d \in \{0, \dots, D_r\}$  **do**  
     $\hat{b}_{j,p}(d) \leftarrow b_{j,p}(d)$   
    **for**  $i \in \{0, \dots, d-1\}$  **do**  
      **for**  $r_i \in \{1, \dots, 2^{n_i} - 1\}$  **do**  
        **if**  $\cup_{i=0}^{d-1} v_m(i, r_i) \cap U_m(d) == \emptyset \& \cup_{i=0}^{d-1} v_m(i, r_i) \cap (\cup_{i=0}^{d-1} (u_m(i) - v_m(i, r_i))) == \emptyset$  **then**  
           $F(i) = \prod_{j \in v_m(i, r_i)} b_{j,p}(i)^{H(i,m,j)}$   
           $G(i) = \prod_{j \in u_m(i) - v_m(i, r_i)} (1 - b_{j,p}(i))$   
           $a(d, m, p) = a(d, m, p) + \prod_{i=0}^{d-1} F(i)G(i)$   
        **end if**  
      **end for**  
    **end for**  
    compute  $a(d, m, p, j)$  in Eq. (2)  
    compute  $a(d, j, p)$  in Eq. (3)  
    compute  $\tilde{a}(d, j, p)$  in Eq. (4)  
  **end for**  
  **for**  $j \in \mathcal{E}$ ,  $d \in \{0, \dots, D_p\}$ ,  $p = 1, 2, \dots, P$  **do**  
    compute  $t_{d,j,p}(i)$  in Eq. (14) for  $i \in \{1, \dots, C_j\}$   
    compute  $\hat{b}_{j,p}(d)$  in Eq. (15)  
    compute  $b_{j,p}(d)$  in Eq. (16)  
  **end for**  
**end while**  
compute  $B_{m,p}$  in Eq. (7) for  $m \in \Gamma$ ,  $p = 1, 2, \dots, P$   
compute  $B_p$  in Eq. (8) for  $p = 1, 2, \dots, P$ .

---

#### A. Default scenario

There is one network scenario that we repeatedly use in many experiments with the same set of parameter values, or possibly with small variations. It is convenient to present it once in this subsection as a default scenario and throughout the section only to point out the deviations from this default scenario.

Our default scenario is a 2-class 6-node fully meshed network where each trunk has 50 channels. Its traffic load is characterized by call arrivals of both classes following Poisson processes, and the holding time of both classes are exponentially distributed with mean holding time equal to 1. In our 6-node fully meshed network, there are in total 15 different OD pairs (or equivalently 30 directional OD pairs). The trunk reservation threshold of class 1 traffic is 38 channels (76% of trunk capacity) and the trunk reservation threshold of the class 2 traffic is 40 channels (80% of trunk capacity). The maximum allowable number of alternative paths is set to 4 for both classes. The bandwidth requirements are 2 and 5 for class 1 and class 2 calls, respectively.

In our default scenario, the traffic for all directional OD pairs is the same, in which case for each directional OD pair, the offered arrival rates of class 1 and class 2 are  $\rho_1$  and  $\rho_2$ , respectively. Then, the total arrival rate in the network is  $30(\rho_1 + \rho_2)$ .

## B. Network Blocking probabilities for the classes

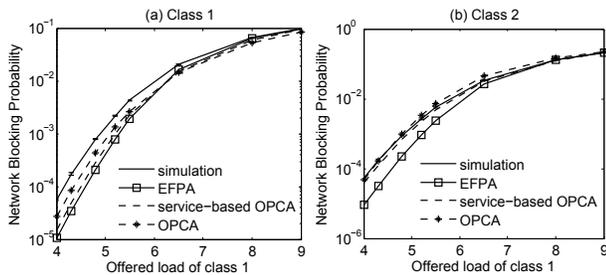


Fig. 3: Network blocking probabilities for (a) class 1 calls and (b) class 2 calls. The offered load of class 2 is 1 erlang.

We first consider the default scenario. In Fig. 3(a), we present results for the network blocking probabilities obtained by OPCA, service-based OPCA, EFPA and simulation for class 1, as a function of class 1 offered load. We observe in the figure that all the three approximations tend to underestimate the network blocking probability when the offered load is low. This is due to the fact that in a fully meshed network with low traffic load, and therefore less overflows, long paths will be very rare. Note that in a fully mesh network the primary path contains only one trunk and hence does not introduce any path error. Accordingly, overflow error will dominate path errors causing underestimation of network blocking probability.

In the surrogate model of OPCA, where the maximum allowable number of overflow is  $D$ , when a junior call, which has overflowed  $d_1$  times, encounters and preempts a senior call, which has overflowed  $d_2$  times and  $d_1 < d_2$ , the senior call is overflowed as a result of the contention, but its remaining number of allowable overflows is limited to no more than  $D - d_2$ . In the real model under the same circumstances, the junior call will overflow and its remaining number of allowable overflows is  $D - d_1$  times, which is more than the allowable number of the overflowed call in the surrogate model of OPCA. The preemptive priority of junior calls over senior calls in OPCA and service-based OPCA implies smaller number of allowable overflows and therefore less proportion of overflowed traffic in the total offered load in the network [74]. Since the surrogate model of OPCA gives preemptive priority to new calls over overflowed calls of any class, while service-based OPCA gives preemptive priority to new calls over overflowed calls of the same class, new calls in the surrogate model of OPCA obtain higher level of priority than those in the surrogate model of service-based OPCA. Therefore, the surrogate model of OPCA exhibits lower proportion of overflow traffic than the service-based OPCA, and the service-based OPCA exhibits lower proportion of overflow traffic than the original (real) model where no priority is given to junior calls. This leads to lower overflow error and therefore lower underestimation of network blocking probability in this case for class 1 traffic of OPCA than service-based OPCA and of service-based OPCA than EFPA. This explains higher estimation of network blocking probability by OPCA than by service-based OPCA, and the lowest estimation of EFPA.

Furthermore, we observe that as the traffic load increases,

the underestimation by all the network blocking probability approximation methods is reduced. This is consistent with the fact that in high load, overflow probability increases. Then, more and more overflows imply the use of longer and longer alternative paths, and therefore path error increases. As observed, the path error in cases of high traffic load may cancel out the overflow error and in this way may improve the approximations. Because EFPA exhibits higher path error than OPCA and service-based OPCA, EFPA may outperform them as the offered load increases, as shown in Fig. 3(a). Since service-based OPCA is more accurate than EFPA when the traffic load is small and EFPA is generally more accurate than service-based OPCA when the traffic load is heavy, we conservatively consider  $\max(\text{EFPA}, \text{service-based OPCA})$  as our approximation of choice rather than EFPA or service-based OPCA over the whole range of traffic load.

In Fig. 3(b), we present results for the network blocking probability obtained by OPCA, service-based OPCA, EFPA and simulations for class 2 traffic and we observe certain similar performance behaviors and trends of the approximations as observed for the class 1 traffic. One noticeable difference is that the network blocking probability obtained by service-based OPCA is more accurate for class 2 traffic than for that of class 1. This is because the bandwidth requirement of class 2 overflow traffic is larger than that of class 1 overflow traffic, and therefore it is more difficult for overflowed traffic of class 2 to find free channels to transmit once it is preempted. Therefore, in the surrogate model of service-based OPCA, network blocking probability of class 2 is higher, closer to the results obtained by simulation in this case.

We also observe that the network blocking probability by OPCA exceeds the simulation result as the traffic increases. In the surrogate model of OPCA, where the large overflowed traffic of class 2 can be preempted by the class 1 calls that require lower-bandwidth, the performance of the more bandwidth hungry class will be lower and the network blocking probability of class 2 traffic predicted by OPCA is further increased to more than the result obtained by simulation. This inaccuracy is more severe when the offered load of class 1 traffic that requires lower bandwidth in this case is larger than that of class 2 traffic that requires higher bandwidth, as shown in Fig. 4(b). We observe that when the ratio of the offered load of class 1 and that of class 2 is 10:1, the result of OPCA is significantly more than that of the simulation. This increase in network blocking probability evaluation of OPCA also happens if the bandwidth requirement of class 2 far exceeds that of class 1, as shown in Fig. 5(b). As demonstrated, OPCA can significantly overestimate the network blocking probability under the scenarios when the offered load of the class that require low-bandwidth far exceeds that of the class that requires high-bandwidth, or when the difference of bandwidth requirements by the two classes is large. The high sensitivity of network blocking performance of OPCA to these parameters adversely affects its robustness in the case of multiservice circuit switched networks and therefore OPCA will not be further considered after this subsection.

Here we consider an NSF network with 13 nodes and 16 bidirectional trunks, as shown in Fig. 6 and the maximum

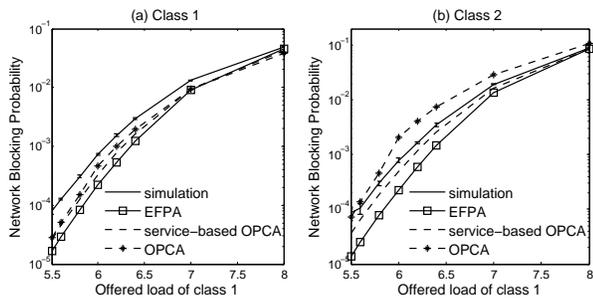


Fig. 4: Network blocking probabilities for (a) class 1 calls and (b) class 2 calls. The ratio of the offered load of class 1 and that of class 2 is 10:1.

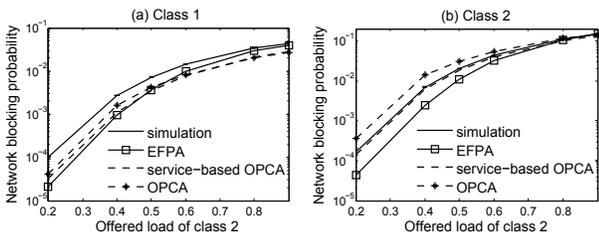


Fig. 5: Network blocking probabilities for (a) class 1 calls and (b) class 2 calls. The bandwidth requirement of class 2 is 8.

allowable number of alternative paths  $D = 1$  for each OD pair in this network. In Fig. 7, we present the network blocking probabilities for class 1 and class 2 traffic in the NSFNet while maintaining all the other parameter values as in the default scenario. Although very close to each other, we observe that service-based OPCA still outperforms EFPA a little for both class 1 and class 2 traffic. For the blocking probability for class 2 traffic, OPCA also exceeds the simulation results with the increased offered load of class 1, which is similar to their

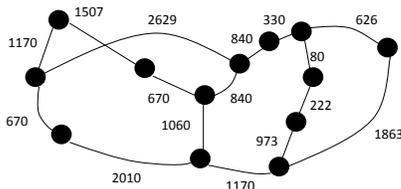


Fig. 6: 13-node NSFNet topology.

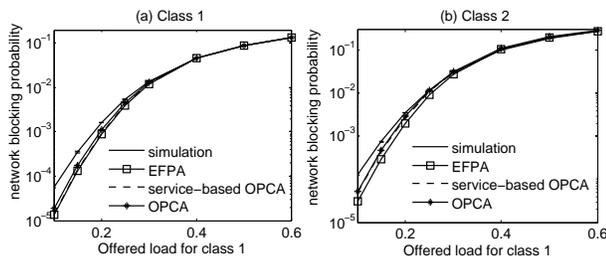


Fig. 7: Network blocking probabilities for (a) class 1 calls and (b) class 2 calls in NSFNet. The offered load of class 2 is 0.05 erlang.

behaviors in the 6-node fully meshed network.

### C. Effect of multi-service rate

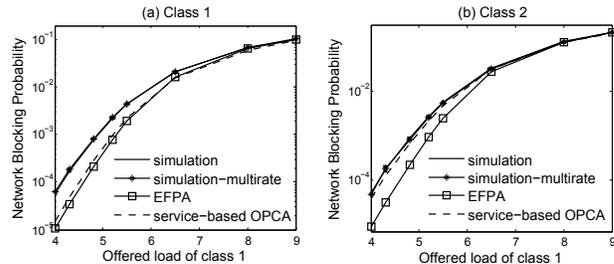


Fig. 8: Network blocking probabilities for (a) class 1 calls and (b) class 2 calls. The offered load of class 2 is 1 erlang. The service rate of class 1 and class 2 are 3 and 1, respectively.

We illustrate here the effect of multi-service rate on network blocking probabilities. By increasing both the arrival rate and service rate of class 1 three times while keeping other parameter values unchanged in the default scenario, Fig. 8 shows the network blocking probabilities of both classes obtained by service-based OPCA, EFPA and simulation for the multi-service rate scenario, compared to the simulation results of the default scenario in Section IV-B. We observe that in this case, the simulation results are very close to each other and their confidence intervals are overlapped which shows that the network blocking probabilities of the classes are insensitive to the service rate as long as the offered load remains the same. For service-based OPCA and EFPA, according to the state probability equations 5 and 14 in Section III, the results only depend on the offered load on the trunk and therefore, we can ignore the effect of service rate on the network blocking probabilities.

### D. Effect of bandwidth requirement of both classes

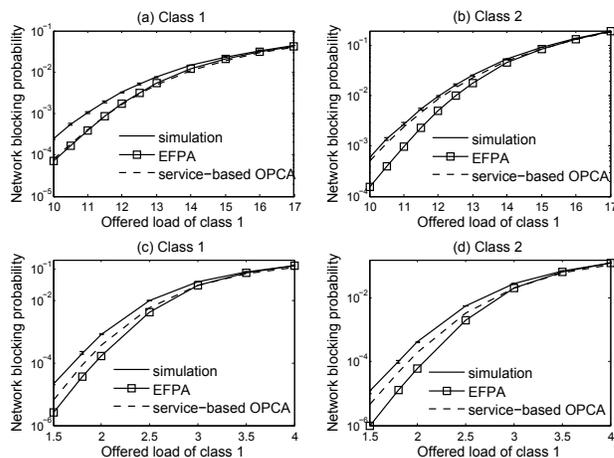


Fig. 9: Network blocking probabilities for both classes. The offered load for class 2 is 1 erlang. The bandwidth requirements of class 1 are 1 for (a) and (b), 4 for (c) and (d). The bandwidth requirement of class 2 remains 5.

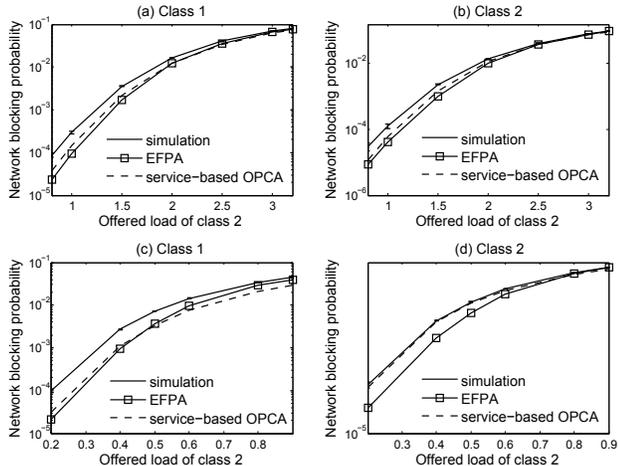


Fig. 10: Network blocking probabilities for both classes. The offered load for class 1 is 6 erlangs. The bandwidth requirements of class 2 are 3 for (a) and (b), 8 for (c) and (d). The bandwidth requirement of class 1 remains 2.

Fig. 9 shows the network blocking probabilities of both classes when the bandwidth requirements of class 1 connections are equal to 1 for (a) and (b) and to 4 for (c) and (d), respectively, while all the other parameter values are kept the same as in the default scenario.

Comparing Figs. 9(a) and (c), we observe that because of the preemptive priority (within a class) of service-based OPCA, overflowed traffic receives fewer opportunities to be admitted by service-based OPCA than by EFPA, and therefore the network blocking probability estimated by service-based OPCA is higher than by EFPA when the offered load is light and the network blocking probability is realistically acceptable (less than 0.001). When the offered load is sufficiently heavy, the network blocking probabilities by EFPA increases due to the increasing path error effect and can be higher than that of service-based OPCA. However, this effect normally occurs in the range when the network blocking probability is so high that it is beyond our region of interest.

When the bandwidth requirement of class 1 increases, the network blocking probability of class 1 estimated by service-based OPCA will further increase because it will be more difficult for the overflowed traffic of class 1 to find an available alternative path to complete service after it is preempted in the surrogate model of service-based OPCA. This in turn leads to the reduction of class 2 network blocking probability estimated by service-based OPCA since the two traffic classes compete for the same pool of capacity, as shown in Figs. 9(b) and (d). We also observe that with no priority in the original model and in EFPA, network blocking probabilities by EFPA are not so sensitive to bandwidth requirement changes as service-based OPCA.

Fig. 10 shows the network blocking probabilities of both classes when the bandwidth requirements of class 2 traffic are 3 for (a) and (b) and 8 for (c) and (d) while all the other parameter values are kept the same as in the default scenario in Section IV-A. We observe that with the increased bandwidth

requirement of class 2, network blocking probability of class 2 traffic by service-based OPCA will increase and that of class 1 traffic will decrease, which is consistent with the figures in Fig. 9.

#### E. Effect of the number of channels per trunk

To examine the effect of the number of channels (wavelength channels) per trunk on network blocking probabilities and on the accuracy of EFPA and service-based OPCA, we increase now the number of channels per trunk to 100 in the default scenario we consider above. Accordingly, we set the trunk reservations 76 (76%) and 80 (80%), for class 1 and class 2, respectively, while all the other parameter values are kept the same as in the default scenario.

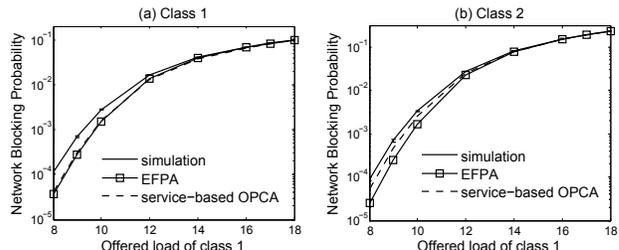


Fig. 11: Network blocking probabilities for the classes in the default scenario with 100 channels each trunk. The offered load of Class 2 is 3 erlangs.

In Fig. 11, we provide the results obtained for the network blocking probabilities of the two traffic classes in the default scenario with 100 channels in each trunk. We observe that the accuracy of both service-based OPCA and EFPA are improved compared to the case of 50 channels per trunk shown in Fig. 3. The improvement in accuracy is achieved because of the following reasons.

- 1) When the number of channels per trunk increases, the variance of the overflow traffic decreases, leading to a lower Poisson error.
- 2) The increase of the number of channels per trunk also reduces the proportion of overflowed traffic and therefore reduces the overflow error, which also increases the accuracy of EFPA.

We also observe that, in general, service-based OPCA is superior to EFPA and it is sandwiched between EFPA and the simulation results.

Notice also that for the network blocking probabilities evaluation of both classes, service-based OPCA still outperforms EFPA in the case of 100 channels per trunk when the traffic is light. This together with the improved accuracy of EFPA as the number of channels per trunk increases from 50 to 100, provide some evidence that service-based OPCA can be accurate as the network capacity scales upwards and performs even better than for networks with lower capacity.

#### F. Effect of maximum allowable number of alternative paths

Here we examine how the network blocking probabilities are affected by the maximum allowable number of alternative

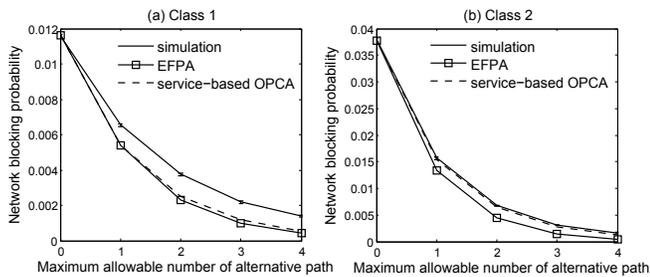


Fig. 12: Network blocking probabilities of both classes in the default scenario with offered load 5 erlangs and 1 erlang for class 1 and class 2, respectively.

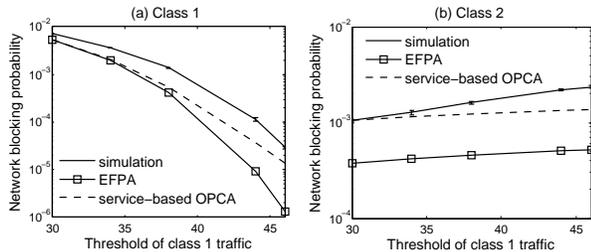


Fig. 13: Network blocking probabilities of both classes in the default scenario. The threshold of class 2  $T_2$  remains 40 (80%).

paths  $D$  which limits how many times traffic can overflow. Traffic that already overflowed  $D$  times is not allowed to overflow again and will be blocked and cleared from the network. For single class networks with light traffic, on one hand, increasing  $D$  means more opportunities to overflow which may reduce the network blocking probabilities, but on the other hand, increasing  $D$  implies that calls use longer paths in alternative routes which leads to inefficiency which in turn may even increase the network blocking probabilities especially when the network is congested. In general, the maximum number of allowable alternative paths  $D$  should be set appropriately to reserve channels for the primary path traffic and prevent the network from being congested by overflowed calls that take long routes.

Fig. 12 (a and b) demonstrates the effect of maximum allowable number of alternative paths on the network blocking probabilities of both classes obtained by simulation, EFPA and service-based OPCA. The offered traffic load of class 1 and class 2 are 5 erlangs and 1 erlang, respectively. We change the maximum allowable number of alternative paths, while keeping all the other parameter values the same as in the default scenario.

We observe that there is a clear benefit for both classes, in the present scenario case, to increase  $D$  to at least 3. After that, the rate of decrease in the network blocking probabilities of both classes slow down as  $D$  increases, due to the inefficiency caused by the long alternative paths.

#### G. Effect of trunk reservation

We again consider the default scenario with the offered traffic load equal to 5 erlangs and 1 erlang for class 1 and class 2, respectively. We change the trunk reservation threshold of

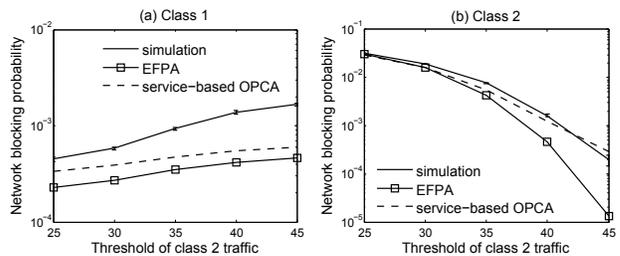


Fig. 14: Network blocking probabilities of both classes in the default scenario. The threshold of class 1  $T_1$  remains 38 (76%).

class 1,  $T_1$ , while keeping all the other parameter values the same as in the default scenario.

For this case, Fig. 13(a) illustrates the effect of  $T_1$  on the network blocking probability of class 1 traffic.

We observe that increasing  $T_1$ , in the present case, reduces the network blocking probability of class 1 traffic by allowing overflowed traffic of class 1 to use more resources, which in turn increases the network blocking probability of class 2 because they compete for the same pool of capacity, as shown in Fig. 13(b). Fig. 14 shows the network blocking probabilities when as we vary  $T_2$ , with the similar trends and behaviors as the Fig. 13.

#### H. Effect of the shape of the holding time distribution

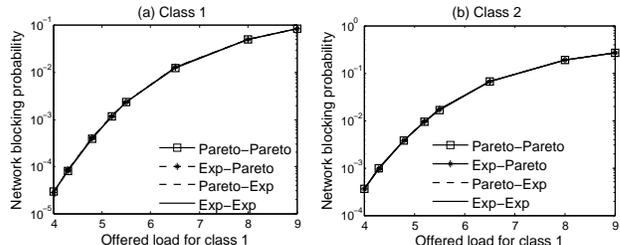


Fig. 15: Network blocking probabilities of both classes, considering different service time distributions in the default scenario. The offered load for class 2 is 1 erlang.

The results presented above are based on the assumption that the holding times of the traffic of both classes are exponentially distributed. It is therefore important to examine the robustness of the approximations to the shape of the holding time distribution. To this end, we compare the results obtained under the exponential assumptions versus results obtained under heavy-tailed holding time distribution, where we maintain the same mean for the two alternatives. The use of heavy-tailed holding times are justified because such connections may represent traffic demands associated with individual application flows, and it has been established that Internet flow size distributions are heavy tailed [90], [91].

In particular, we consider our heavy-tailed holding times, denoted  $h$ , to follow a Pareto distribution with a complementary distribution function (CDF) that takes the form:

$$Prob(h > x) = \begin{cases} (\delta/x)^\gamma, & x \geq \delta \\ 1, & \text{otherwise.} \end{cases} \quad (20)$$

where  $\delta$  (seconds) is the scale parameter (minimum holding time) and  $\gamma$  is the shape parameter of the Pareto distribution. The mean of  $h$  is given by

$$E(h) = \begin{cases} \infty, & 0 < \gamma \leq 1 \\ \delta\gamma/(\gamma-1), & \text{otherwise.} \end{cases} \quad (21)$$

For  $0 < \gamma \leq 2$ , the variance  $Var(h) = \infty$ . In our simulation we set  $\delta = 0.5$  and  $\gamma = 2$  for both classes. All the other parameter values are kept the same as in the default scenario.

Fig. 15 (a and b) shows the simulation results for network blocking probabilities of both classes traffic for the default scenario with the following four cases of service distribution:

- 1) Exp-Exp – holding times of both classes are exponentially distributed
- 2) Exp-Pareto – holding time of class 1 is exponentially distributed while that of class 2 is Pareto distributed
- 3) Pareto-Pareto – holding times of both classes are Pareto distributed
- 4) Pareto-Exp – holding time of class 1 is Pareto distributed while that of class 2 is exponentially distributed.

The four curves are very close to each other and their confidence interval overlap, which shows that network blocking probabilities of both classes are not very sensitive to the shape of the holding time distribution in the present case.

### I. Asymmetrical cases

All the results we have presented are for the symmetrical models, where for each OD pair traffic is sent for both classes, the offered loads for all OD pairs are identical, and the network topology is symmetrical as well. However, in reality, core network topologies are normally not symmetrical and the traffic between some OD pairs have a very different profile than others, because the OD pairs can be very different, *e.g.*, data-centers, core routers, or LHCOPN node, with different traffic profiles.

Therefore, in this subsection, we study the performance of EFPA and service-based OPCA in asymmetrical cases.

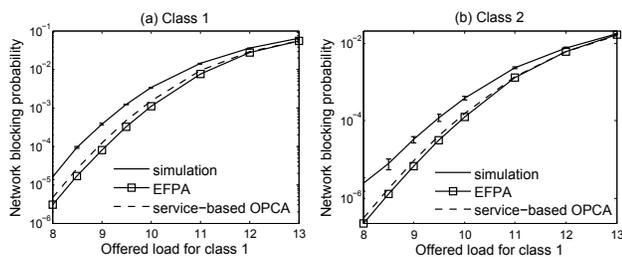


Fig. 16: Network blocking probabilities of both classes, while 20 OD pairs will only send the class 1 traffic and the rest 10 OD pairs will only transmit class 2 traffic in a 6-node fully meshed network with 50 channels in each trunk. The offered load for class 2 is 1 erlang.

For the 6-node fully meshed network, the total 30 OD pairs are divided into two groups, in which 20 OD pairs only transmit class 1 traffic, and the remaining 10 OD pairs will only transmit class 2 traffic. All other parameter values are the same as in the default scenario.

We observe that in this case, the network blocking probabilities estimated by EFPA and service-based OPCA, as shown in Fig. 16, are closer to each other than in the symmetrical case shown in Fig. 3. This is because the service-based OPCA can benefit more from the congestion information exchanged when senior calls are preempted, in the symmetrical case than in the asymmetrical case. This benefit is more prominent in symmetrical network because with evenly distributed offered load, all the trunks have overflowed calls from all the other trunks and the congestion information of all trunks spreads efficiently to all the other trunks in the network. The asymmetry reduces the advantage of the service-based OPCA and makes its results closer to those of EFPA. Nevertheless, we still observe that service-based OPCA gives more accurate results than EFPA also in this asymmetrical case.

We further study the performance of EFPA and service-based OPCA in a 13-node NSFNet (shown in Fig. 6) that has both asymmetrical offered load and asymmetrical topology. We choose all possible OD pairs with shortest path routing, where a tie is broken randomly. There are  $12 \times 13 = 156$  OD pairs in the 13-node NSFNet and we set randomly chosen 104 OD pairs out of the total of 156 to only send class 1 traffic, and the remaining 52 send only class 2 traffic, while keeping all the other parameter values the same as in the default scenario.

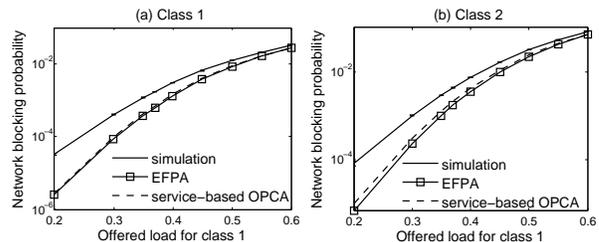


Fig. 17: Network blocking probabilities of both classes, while 104 OD pairs will only send the class 1 traffic and the rest 52 OD pairs will only transmit class 2 traffic in 13-node NSFNet with 50 channels in each trunk. The offered load for class 2 is 0.1 erlang.

We observe similar results in Fig. 17, where EFPA and service-based OPCA are very close to each other but service-based OPCA still outperforms EFPA slightly.

### J. Effect of network size on simulation running time

In this subsection, we examine the effect of the number of nodes in the network on simulation running time required to achieve accuracy within a given confidence interval. We consider fully meshed networks with default setting and increase the number of nodes.

Fig. 18 shows the simulation running time when we increase the number of nodes in fully meshed networks and maintain the 95% confidence intervals less than 3% of the average value. The resulting blocking probabilities in all cases are around 0.001. We observe the increase of the simulation running time (which typically grows exponentially in the number of nodes), that already reaches several hours when the number of nodes is 15.

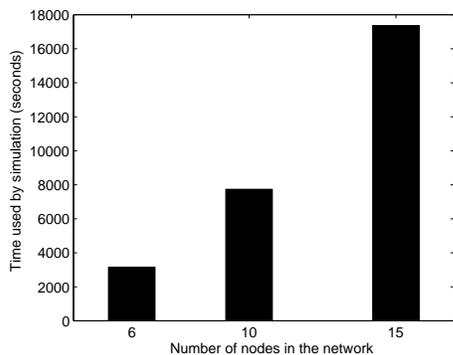


Fig. 18: Running times used by simulation in the fully meshed networks.

### K. The CORONET

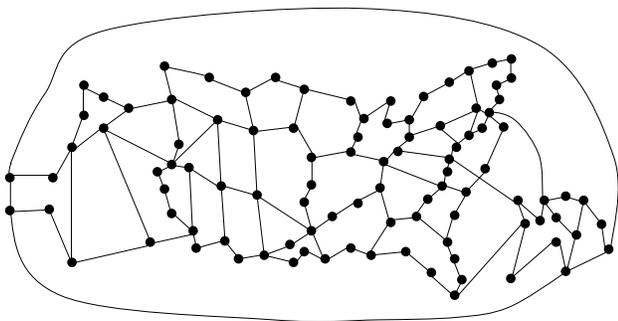


Fig. 19: The CORONET topology.

We demonstrate here that  $\max(\text{EFPA}, \text{service-based OPCA})$  is applicable to large scale networks such as the CORONET, shown in Fig. 19. Given the running time results presented in Fig. 18, and considering the 100 nodes and 9900 OD pairs of the CORONET, clearly, simulations are computationally prohibitive. Fortunately, the network blocking probabilities for both classes traffic can be obtained by service-based OPCA and EFPA within reasonable running times. The results are shown in Fig. 20. The parameters used to obtain these results were set as in the default setting except that  $D = 1$  for each OD pair in CORONET. The running times used to calculate the network blocking probabilities in the CORONET were about 33.31 seconds and 41.73 seconds by EFPA and service-based OPCA, respectively, obtained using MATLAB 7.6.0 executed on a desktop PC with IntelR CoreTM 2 Quad @ 3 GHz CPU, 4 GHz RAM and 32-bit operating system. We observe in Fig. 20 that the results for the CORONET based on EFPA and service-based OPCA are close to each other. These together with our experiments for small networks provide some confidence in the accuracy of  $\max(\text{EFPA}, \text{service-based OPCA})$  also for the CORONET. However, we note that the results of [46], [47] related to large networks do not apply for a general asymmetric network, so unfortunately no conclusive statement about accuracy can be made in this case of the CORONET with alternate routing.

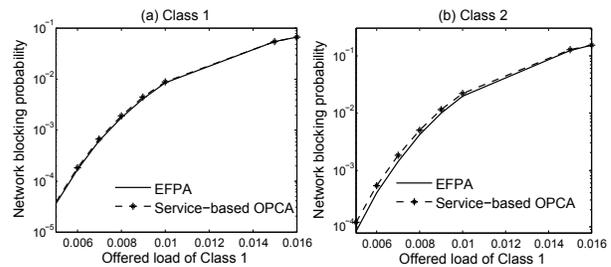


Fig. 20: Network blocking probabilities for (a) class 1 calls and (b) class 2 calls in the CORONET. The offered load of class 2 per OD pair is always 0.001 erlang.

### L. Three service classes

It is difficult to predict the number of service classes in future networks. For example, the proposed service model in [92] considered three service classes, while the Cisco MGX 8000 Series multiservice switch can support 16 service classes. All the results we have presented so far are for the cases with two service classes in different scenarios. Here we present cases with three service classes.

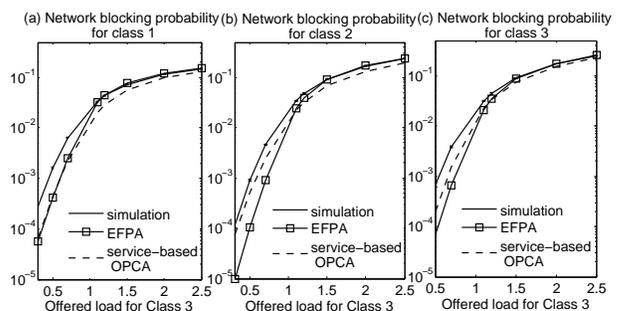


Fig. 21: Network blocking probabilities for (a) class 1 calls, (b) class 2 calls and (c) class 3 calls. The offered load of both class 1 and class 2 are 2 erlangs.

We again consider a 6-node fully meshed network where each trunk has 50 channels. The bandwidth requirements are 2, 4 and 5 for class 1, class 2 and class 3 calls, respectively. The trunk reservation thresholds are 38, 42 and 43 for class 1, class 2 and class 3 traffic, respectively. The maximum allowable number of alternative paths is set to 4 for all classes. We observe in Fig. 21 that blocking probabilities for class 2 and class 3 calls obtained by service-based OPCA are much higher than those obtained by EFPA and relatively close to the simulation results. This is because the bandwidth requirements for class 2 and class 3 calls are much larger and the overflowed class 2 and class 3 calls will hardly be served again in the surrogate model of service-based OPCA, which lead to the high blocking probabilities by service-based OPCA. As a result, many resources are left for class 1 calls, which causes relatively low blocking probability of class 1 calls by service-based OPCA. Nevertheless, service-based OPCA still outperforms EFPA in general.

### M. Non-disjoint paths

Here we present the results of our algorithms when we relax the disjointedness assumption and the paths of a same OD pair contain some common trunks. As discussed, the only difference between evaluating blocking probability in this case and in the cases based on disjoint paths is the computation of the trunk offered load, which are affected by the common trunks and their positions along the paths and therefore need to be calculated case by case.

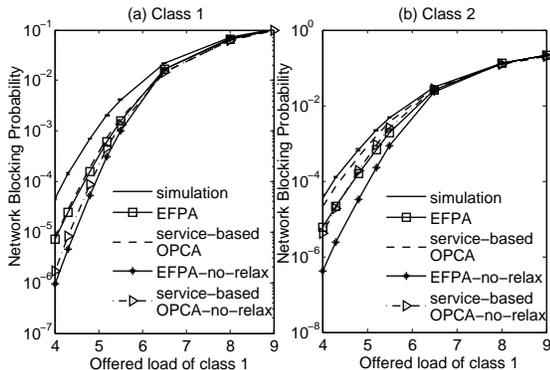


Fig. 22: Network blocking probabilities for (a) class 1 and (b) class 2 calls. The offered load of class 2 is 1 erlang.

In the default scenario described in Section IV-A, we assign five disjoint paths for each OD pair. To relax the disjointedness assumption, we add two alternative paths to each OD pair and they both have two common trunks which are also contained in the five paths. Fig. 22 shows that in this case, when we relax the disjointedness assumption, the results obtained by service-based OPCA are still more accurate than by EFPA most of time, and  $\max(\text{EFPA}, \text{service-based OPCA})$  is still applicable to this case involving non-disjoint paths. If we consider an approximation, in which the non-disjoint paths of a same OD pair are treated as if they were disjoint, the results of both EFPA and service-based OPCA will be lower, as shown in Fig. 22. This is because we ignore the dependency and give the alternative paths more overflowed offered load that in fact should be blocked, and in this way, we underestimate the network blocking probabilities. However, the growth of the complexity when calculating the trunk offered load does not necessarily cause longer computation time because, although many conditions are considered, the numbers of terms in the equations for trunk offered load are not necessarily more than in the equivalent disjoint path case.

### N. Moment Matching

As discussed, traffic offered by an overflow stream is known to have higher peakedness than a Poisson process. The error introduced by assuming them to be Poisson processes can be reduced by moment matching [37], [43], [55], which resort to processes the moments of which match those of the overflow streams. Here we implement one of the moment matching approaches in [55] in both EFPA and service-based OPCA.

Tables I and II show the results obtained by EFPA and service-based OPCA with moment matching with increase of

TABLE I: Network blocking probability for class 1 in 6-node fully mesh network with moment matching implemented in EFPA and service-based OPCA.

Offered load of class 1	EFPA	EFPA with moment matching	service-based OPCA	service-based OPCA with moment matching
4.3	0.0000341	0.0000366	0.0000490	0.0000527
4.8	0.0002106	0.0002361	0.0002889	0.0003149
5.2	0.0007877	0.0008884	0.0009953	0.0010929
5.5	0.0019289	0.0020963	0.0022383	0.0024608
6.5	0.0167839	0.0185148	0.0155129	0.0163552
8	0.0650663	0.0670232	0.0603895	0.0618303
9	0.1010129	0.1018540	0.0963064	0.0980423

TABLE II: Network blocking probability for class 2 in 6-node fully mesh network with moment matching implemented in EFPA and service-based OPCA.

Offered load of class 1	EFPA	EFPA with moment matching	service-based OPCA	service-based OPCA with moment matching
4.3	0.0000315	0.0000341	0.0001243	0.0001344
4.8	0.0002201	0.0002499	0.0006679	0.0007335
5.2	0.0009122	0.0010449	0.0021566	0.0023869
5.5	0.0024201	0.0026637	0.0046690	0.0051738
6.5	0.0270943	0.0304664	0.0309264	0.0327577
8	0.1295776	0.1343897	0.1259148	0.1292402
9	0.2130164	0.2159933	0.2060681	0.2103931

class 1 offered load while the offered load of class 2 remains 1 erlang. Comparing with the results obtained by the original EFPA and service-based OPCA, we observe small increase of accuracy in both EFPA and service-based OPCA by moment matching. However, the benefit is not so obvious due to the other assumptions that cause errors in the approximations, which is consistent with results in [74]. Nevertheless, the time complexity is not largely increased by moment matching. In the example of Tables I and II, when the offered load of class 1 is 5.5 erlangs, the computation time used by EFPA is 0.139794 sec. and is increased to 0.276348 sec. by moment matching; the computation time used by service-based OPCA is 0.717002 sec. and is increased to 0.835406 sec. by moment matching.

### O. Effect of setup delay

Here we aim to investigate the effect of setup delay on the network blocking probability results of  $\max(\text{EFPA}, \text{service-based OPCA})$ . The dependence of this effect on  $D$  is also studied because deflected paths are often longer, so the setup delay becomes more significant as  $D$  increases. Setup delay in circuit switched networks also depends on the end-to-end propagation delay and the handshaking algorithm during setup. The effect of setup delay on network blocking probability can be significant if the ratio of mean service duration per connection to the propagation delay is small because during part of the setup time, capacity is already reserved for the connection even though it is not yet used and cannot be used by other connections. In this subsection, we assume that for

a given connection, a setup delay of twice the propagation delay plus twice the processing delay to each node in a path is a conservative upper bound for the period that the entire path is reserved for the connection, it is neither used by this connection, nor by any other connections. This is a conservative assumption because it includes the time that a control packet travels to make reservations before actual reservations (and confirmations) are made. Our approach is to evaluate the network blocking probabilities twice, we use max(EFPA, service-based OPCA) to approximate the scenarios once the setup delay is added and once where it is ignored. The former gives us an upper bound for the blocking probability and the latter a lower bound. For each of the scenarios discussed below, for the cases where the setup delay is not included, we increase the mean service duration and reduce the arrival rate at the same rate so that the offered traffic and the blocking probability remain constant. Then for each of the above cases we also provide the blocking probability where setup delay is included.

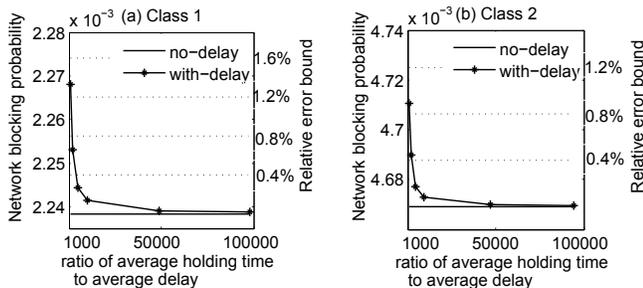


Fig. 23: Network blocking probabilities for (a) class 1 and (b) class 2 calls. The offered load are 5.5 erlangs and 1 erlang of class 1 and class 2, respectively. The propagation delay is set 0.001 second on every trunk and the processing delay is 0.0001 second in each node.

Fig. 23 provides the upper and lower bounds of network blocking probabilities for class 1 and class 2 calls based on max(EFPA, service-based OPCA) for the default scenario when the maximum number of alternative paths  $D = 4$ . The propagation delay is set 0.001 second on every trunk and the processing delay is 0.0001 second in each node. As the mean service duration and therefore the ratio of average holding time to average delay increases, the upper and lower bounds approach each other.

In Fig. 24, we provide the network blocking probabilities for class 1 and class 2 calls when the maximum allowable number of alternative path is 0. As discussed, the average setup delay and its effect increases with the increased use of alternative paths. This can be demonstrated by comparing Fig. 23 and Fig. 24. In Fig. 24, the maximum number of alternative paths is 0 and the number of trunks along each primary path is always equal to 1 while in Fig. 23, the maximum number of alternative paths is 4 and the number of trunks along alternative path is 2. We observe that when the ratio of average holding time to average delay is 1000, the relative error is about 1.3% in Fig. 23 when  $D$  is 4, while 0.6% in Fig. 24 when  $D$  is 0.

In Fig. 25, we demonstrate the effect of the setup delay in

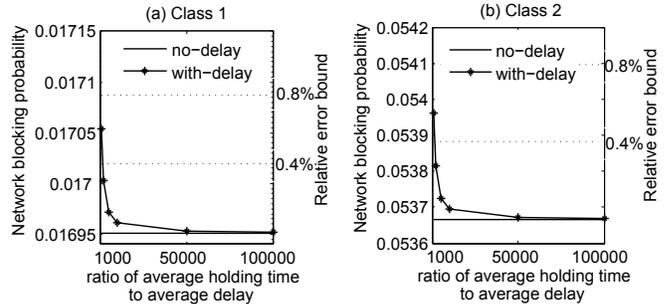


Fig. 24: Network blocking probabilities for (a) class 1 and (b) class 2 calls. The offered load are 5.5 erlangs and 1 erlang of class 1 and class 2, respectively. The propagation delay is set 0.001 second on every trunk and the processing delay is 0.0001 second in each node.

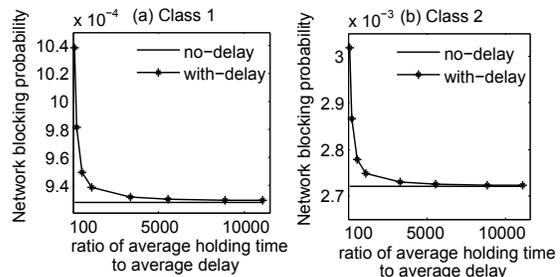


Fig. 25: Network blocking probabilities for (a) class 1 and (b) class 2 calls in NSFNet. The offered load are 0.2 erlang and 0.05 erlang of class 1 and class 2, respectively.

the NSFNet, where the lengths of the trunks are different and therefore the propagation delay on them are different. We set the length of each trunk as shown in Fig. 6, where some of them are obtained from [93] and others are approximated by the distances between the capitals of the states. In Fig. 25, we observe similar behavior of the upper and lower bounds to those in the default scenario.

#### P. Dimensioning

As discussed, blocking probability estimations are applied for dimensioning purposes for acceptable blocking probabilities such as  $10^{-3}$  or  $10^{-4}$ . Here we illustrate that the error introduced by max(EFPA, service-based OPCA) is small in terms of error in dimensioning, even for the most inaccurate scenario of the 3-class case, discussed in Section IV-L.

In Fig. 26 we consider the 3-class case discussed in Section IV-L. We keep the ratio of the arrival rate 2, 1 and 1.5 for class 1, class 2 and class 3, respectively and increase the total offered load to the network. We dimension the network to find the number of channels per trunk required to keep the biggest blocking probability of the three classes below 0.001. Fig. 26 illustrates that the number of channels per trunk required approximated by max(EFPA, service-based OPCA) is very close to those obtained by simulation. The relative errors of the approximation are less than 4% which is an acceptable error especially given the much larger errors in traffic prediction.

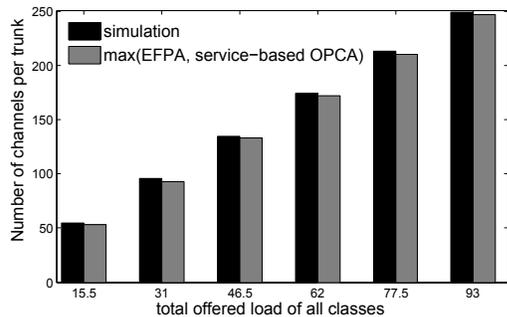


Fig. 26: Number of channels per trunk required to keep the biggest blocking probability of the three classes below 0.001.

### Q. Benefit of full wavelength conversion

The major benefit of having full wavelength conversion is improved efficiency. It is well known for an  $M/M/k/k$  queueing system that when the ratio of the offered load  $\rho$  and the number of servers  $k$  remains constant while  $k$  (or  $\rho$ ) increases, the variability (standard deviation to mean ratio) of the link occupancy decreases and this improves efficiency. This also applies to multiservice systems and networks. One simple approach to compare the case of no wavelength conversion with the case of full wavelength conversion (which also provides an optimistic bound to the benefit of any limited wavelength conversion) is by comparing the blocking probability of an  $M/M/k/k$  system (using the Erlang B formula) with a given offered traffic load  $\rho$  and number of servers  $k = fw$  representing the case of full wavelength conversion, versus a system where the offered load is  $\rho/w$  and the number of servers  $k = f$  representing the case of no wavelength conversion. In this way, the case of no wavelength conversion is modeled by  $w$  identical  $M/M/k/k$  systems each loaded by traffic  $\rho/w$ . In the case of full wavelength conversion, the entire traffic may utilize all the available channels while in the case of no wavelength conversion the traffic is divided among  $w$  independent networks each of which is associated with one wavelength (color). The analogy to the simple  $M/M/k/k$  model explains the improved efficiency of full wavelength conversion. In other words, for the same traffic load and the same trunk capacities full wavelength conversion will reduce the blocking probability relative to the case of no wavelength conversion [94], [95]. This implies that less capacity can be used for the same traffic which meets the same blocking probability requirements and hence better efficiency is achieved under full wavelength conversion. This is illustrated in Fig. 27 which shows the network blocking probabilities by simulations when each node in the network has full and no wavelength conversion. In Fig. 27, the number of fibers in each trunk is  $f = 10$  and each fiber has  $w = 10$  wavelengths while all the parameters are the same as in the *default setting* and the offered load of Class 2 is 3 erlangs. We observe that when the offered loads are the same, due to the reduction of link occupancy variability, the blocking probabilities for the case with no wavelength conversion are much higher than with full wavelength conversion. Note that with no wavelength conversion, the traffic is divided to  $w = 10$  networks, in each of which the number of channels is  $f = 10$

while with full wavelength conversion, the total traffic can use the full capacity in each trunk, which is  $fw = 100$ .

While the above performance comparisons are useful, a more important question is: what is the benefit of wavelength conversion in terms of saving achieved considering the capacity required to meet a given GoS level? Such benefit is bounded by how much capacity can be saved by using full wavelength conversion versus no wavelength conversion. This can be simply evaluated by considering two  $M/M/k/k$  systems (as described above) that model the case of without and with full wavelength conversion fed by equivalent traffic load achieving the same blocking probability.

While the  $M/M/k/k$  can provide a first approximation for the benefit of wavelength conversion, it is of value to know the accuracy of such an approximation for a given network. In Fig. 27, when the offered load are  $\rho_1 = 9.1$  erlangs and  $\rho_2 = 3$  erlangs for class 1 and class 2, respectively, the network blocking probability for both of the classes can be less than  $10^{-3}$  with full wavelength conversion while with no wavelength conversion, the number of fibers  $f$  should be increased from 10 to 18, which means that full wavelength conversion save the capacity by 44%. More results are presented in Table III in which scenarios 1–4 are consistent with what we have in IV-A, scenarios 5–7 are the example of Fig. 7 when  $D = 1$  and scenarios 8–10 are when  $D = 0$ . Scenario 11 is the CORONET in Fig. 19 with single service and fixed routing. We have used both EFPA and A-EFPA to calculate the benefit of full wavelength conversion in scenario 11. A-EFPA provides sufficiently close prediction which indicate that this is the region of capacity that EFPA is accurate. The benefit of full wavelength conversion in Scenario 1 is obtained by simulation while those of all the other scenarios are obtained by max(EFPA, service-based OPCA). We observe that in all the cases full wavelength conversion can save capacities and the amount of savings is affected by many factors in different scenarios. However, the Erlang B results can be inaccurate which means that we should not rely on Erlang B for accurate estimation of benefit of full wavelength conversion and a more detailed analysis of the particular network scenario as provided in this paper is required.

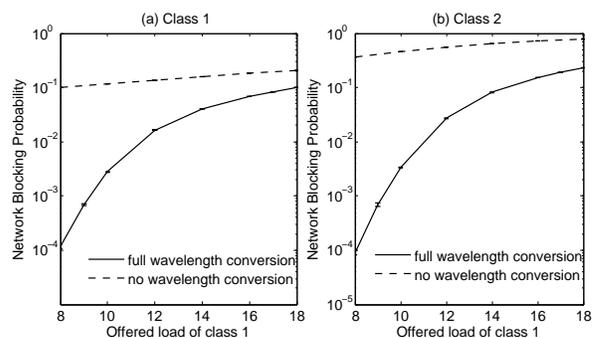


Fig. 27: Network blocking probabilities for the classes in the default scenario with full wavelength conversion and no wavelength conversion. The offered load of Class 2 is 3 erlangs.

TABLE III: Efficiency improved by full wavelength conversion with the same traffic and meets the same blocking probability requirements

Scenario			Benefit of full wavelength conversion	Erlang B benefit estimation
No.	Network	$(f, w)$		
1.	6-node fully meshed	(10, 10)	44.44%	28.57%
2.		(10, 20)	52.38%	28.57%
3.		(20, 10)	35.48%	16.67%
4.		(20, 20)	41.18%	16.67%
5.	NSF with deflection routing	(100, 100)	24.24%	20.63%
6.		(10, 10)	54.55%	28.57%
7.		(20, 20)	45.95%	16.67%
8.	NSF with fixed routing	(100, 100)	33.78%	20.63%
9.		(10, 10)	61.53%	28.57%
10.		(20, 20)	54.55%	16.67%
11.	CORONET	(100, 100)	17.36%	20.63%

### R. Number of iterations until convergence

TABLE IV: Number of iterations required until convergence by the approximations

Scenario		number of iterations required until convergence		
		EFPA	service-based OPCA	OPCA
A	6-node fully meshed	16	12	9
B		16	10	14
C		22	16	10
D		18	13	12
E		17	10	15
F	CORONET	7	8	/
G		11	16	/
H		16	17	/
I		12	12	/
J		19	19	/

Table IV shows the number of iterations required until convergence by the approximations in different scenarios. Scenario A is the example of Fig. 3 in which the offered load of class 1 and class 2 are 6 erlangs and 1 erlang, respectively. The resulting blocking probabilities are around 0.01, the initial values of all trunk blocking probabilities are 0.1 and the relative error criterion is  $10^{-7}$ . All of Scenarios B, C and D are similar to Scenario A except that for Scenario B offered load of class 1 is 4.5 erlangs and the resulting blocking probabilities are around  $10^{-4}$  and for Scenario C the relative error criterion is  $10^{-10}$  and for Scenario D the initial values of all trunk blocking probabilities are 0.00001. Scenario E is the example of Fig. 11 when the number of channels per trunk is 100 and the offered load of class 1 and class 2 are 10 erlangs and 3 erlangs, respectively. Scenarios F - J are the large scale CORONET example of Fig. 20 while in Scenario F offered load of class 1 and class 2 are both 0.0001 erlang and the number of channels per trunk is 10. In Scenario G the offered load of class 1 and class 2 are 0.0075 erlang and 0.001 erlang, respectively and the number of channels per trunk is 50. In Scenario H the offered load of class 1 and class 2 are both 0.007 erlang and the number of channels per trunk is 100. In Scenario I the offered load of class 1 and class 2 are both 0.04 erlang and the number of channels per trunk is 500. In

Scenario J the offered load of class 1 and class 2 are both 0.09 erlang and the number of channels per trunk is 1000.

We observe that these different factors can affect the number of iterations the approximations need to converge but in general, all the approximations can converge within several iterations.

## V. CONCLUSIONS

We have considered a circuit-switched multiservice multi-rate network with deflection routing and trunk reservation, and introduced two new approximations, OPCA and service-based OPCA, for the estimation of the network blocking probabilities of various traffic classes. We have explained the causes of the errors of the approximations and provided intuitive insights of their accuracy as compared to EFPA. Numerical results under a wide range of scenarios and parameter values have demonstrated that in most cases that we studied, service-based OPCA can estimate the network blocking probabilities reasonably well and is generally more accurate and more conservative than EFPA. We have also observed that OPCA can significantly overestimate the network blocking probabilities under certain scenarios and the performance of OPCA is not as robust as EFPA and service-based OPCA. Furthermore, we have proposed the more conservative max(EFPA, service-based OPCA), which is more accurate than EFPA and service-based OPCA and more robust than OPCA. The results have also demonstrated the robustness of the approximations to the shape of the holding time distribution. Furthermore, we have shown that max(EFPA, service-based OPCA) is applicable to the network blocking probabilities estimation in large networks such as the CORONET, for which the simulation results are computationally prohibitive. Finally, we have shown that the relative error of max(EFPA, service-based OPCA) is acceptable in the case we studied when it is applied for the purpose of network dimensioning.

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