Analysis of Cognitive Radio Spectrum Access with Finite User Population

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***Abstract*—A new loss model for cognitive radio spectrum access with finite user population are presented, and exact solution for the model and its approximation for computation scalability are given. Our model provides the investigation of the delay performance of a cognitive radio system. We study the delay performance of a cognitive radio system under various primary traffic loads and spectrum band allocations.**

***Index Terms*—Cognitive radio networks, spectrum access, performance evaluation.**

I. INTRODUCTION

THE continuingly increasing demands for mobile and wireless communications have resulted congestion in spectrum usage. The concept of cognitive radios [1] that offer an alternative wireless channel access strategy is introduced to improve the effective use of spectrum. The technology of cognitive radios specifies a certain method for cognitive (unlicensed) wireless devices to access the licensed spectrum. This access will be transparent to primary (licensed) users such that it will not affect the operation of primary users.

Currently, there are two separate efforts in the research of cognitive radios. In one aspect, access methods for cognitive wireless devices, such as spectrum holes detections and interference avoidance, are proposed to realize cognitive radios (see [2] and references therein). In another aspect, cognitive radio performances are studied to improve our understanding of the performance characteristics of cognitive radios [3]-[5] (and references therein). This letter falls under the latter research effort of cognitive radio performances.

In [3], a continuous time Markov chain (CTMC) model for cognitive radios spectrum access is presented. The model considers two types of users, the primary users and cognitive users, accessing the same spectrum. The spectrum consists of K primary frequency bands, with each primary band forms a primary channel for each primary user to access. Each primary frequency band is divided into N sub-bands forming N secondary channels (see also Fig. 1). Each cognitive user accesses to a sub-channel for its data transmission. Any unused primary channel may serve up to N cognitive users at one time.

The primary users always have a higher channel access priority over the cognitive users. The cognitive users sense and may also predict the channel access behavior of the primary users. When an idle primary channel is detected, the cognitive users may access to the corresponding secondary channels. If a primary user decides to access the primary channel, all cognitive users using the corresponding secondary channels must relinquish their transmissions immediately. In general, these unfinished cognitive transmissions may be either handed over to other available sub-channels to continue their transmissions or simply discarded.

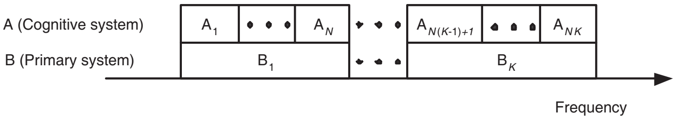

Fig. 1. Frequency band allocations for the primary and the cognitive users.

In this letter, we adopt the modeling approach in [3] for the cognitive radios spectrum access. We remodel the CTMC by considering more practical user behaviors. Precisely, the infinite user assumption with Poisson arrivals in [3] is replaced by a finite user assumption. Different from [5] that uses a truncated Poisson arrival process with blocking to model the finite user behavior, we use on-off process for packet transmissions which allows the evaluation of transmission delay. In the next section, we provide details of our system model. The analytical solution is described in Section III. Numerical results are presented in Section IV. Some important conclusions are drawn in Section V.

II. SYSTEM MODEL

We consider a cognitive radio system with M_l primary and M_c cognitive users. The licensed spectrum is partitioned equally into K primary frequency bands for the access from the primary users. Each primary frequency band is divided into N sub-bands for the access from the cognitive users (see Fig. 1).

As in [3], we assume that there is either a central controller or a suitable distributed protocol to perform sub-channel allocation for the cognitive users. This assumption implies that each arrival from a cognitive user is able to find an unused sub-channel for its transmission.

When a primary user arrives and claims a particular primary channel, all ongoing cognitive transmissions on the corresponding sub-channels are forced to terminate. We consider practical design that these forced terminated transmissions are unsuccessful. The cognitive users experiencing forced termination will perform spectrum handover and retransmissions on other available sub-channels. If the cognitive users find all sub-channels occupied, these cognitive users will be queued locally until such sub-channels become available again.

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An on-off process is associated with each user for the transmission. The on-off process is an alternating process where a user switches between on and off. The period of time used to transmit a packet is called an *on-period*, and the time between consecutive on-periods where a user remains idle is called an *off-period*. The transmission time of a packet, i.e. the on-period, is assumed to be exponentially distributed with mean $1/\mu_l$ (resp. $1/\mu_c$) for the primary (resp. cognitive) users. The off-period is assumed to be exponentially distributed with mean $1/\lambda_l$ (resp. $1/\lambda_c$) for the primary (resp. cognitive) users.

III. PERFORMANCE ANALYSIS

A. The Exact Model

Let $\pi_{i,j}$ $(0 \le i \le K, 0 \le j \le M_c)$ be the state probability representing i primary packets being transmitted and j cognitive packets being either transmitted or queued for transmission. The number of idle users is given by $M_l - i$ (resp. $M_c - j$) for primary (resp. cognitive) users. The model can be described by the following steady state equations. Precisely, for $iN + j < KN$, we have

$$\begin{aligned} & \pi_{i,j}\left[(M_l - i)\lambda_l + (M_c - j)\lambda_c + i\mu_l + j\mu_c\right] \\ = \; & \pi_{i-1,j}(M_l - i + 1)\lambda_l + \pi_{i,j-1}(M_c - j + 1)\lambda_c \\ & + \pi_{i+1,j}(i+1)\mu_l + \pi_{i,j+1}(j+1)\mu_c \end{aligned} \tag{1}$$

and for $iN + j \ge KN$, we have

$$\begin{aligned} & \pi_{i,j}\left[(M_l - i)f_i\lambda_l + (M_c - j)\lambda_c \right. \\ & \left. + i\mu_l + (K - i)N\mu_c\right] \\ = \; & \pi_{i-1,j}(M_l - i + 1)\lambda_l + \pi_{i,j-1}(M_c - j + 1)\lambda_c \\ & + \pi_{i+1,j}(i+1)\mu_l + \pi_{i,j+1}(K - i)N\mu_c \end{aligned} \tag{2}$$

where $f_i = 1$ if $i < K$ and 0 otherwise. In (1) and (2), $\pi_{i,j} = 0$ for $(i,j) \notin \{(i,j) : i = 0, \ldots, K; j = 0, \ldots, M_c\}$. Introducing the normalization equation $\sum_{\forall i,j} \pi_{i,j} = 1$ gives rise to a set of equations, which can be used to solve for $\pi_{i,j}$. The total load offered by primary packets is given by

$$O_l = \textstyle\sum_{\forall i,j}(M_l - i)\left(\frac{\lambda_l}{\mu_l}\right)\pi_{i,j}.$$

The total load carried by primary packets is given by

$$C_l = \textstyle\sum_{\forall i<K,j}(M_l - i)\left(\frac{\lambda_l}{\mu_l}\right)\pi_{i,j}.$$

The average packet blocking probability, P_B, for primary users can be determined by $P_B = 1 - \frac{C_l}{O_l}$.

The average arrival rate of cognitive packets, A_c, and the average number of cognitive packets in the system, $\overline{N_c}$, are given by

$$\begin{cases} A_c = \sum_{\forall i,j}(M_c - j)\lambda_c\pi_{i,j}, \\ \overline{N_c} = \sum_{\forall i,j} j\pi_{i,j}. \end{cases}$$

By Little's formula, the mean packet transmission delay (including queueing) is given by $D = \frac{\overline{N_c}}{A_c}$.

Given the bi-dimensional state-space of size $K + 1$ by $M_c + 1$, solving (1)-(2) may not be scalable for large values of K and M_c. We shall present an approximation that provides scalable numerical computation as follows.

B. Approximation

The approximation applies decoupling of the two types of users to analyze their performances. Since primary users utilize channel resources regardless of the existence of the cognitive users, the performance analysis for the primary users can be evaluated independently which remains exact. Then, based on the computed resource usage, the performance of cognitive users can be approximated.

The approximation procedure consists of two stages. The first stage yields the exact blocking probability and state probability distribution for the primary users, i.e. $\{p_i : i = 0, \ldots, K\}$, where p_i is the probability that i primary packets are in transmission. The second stage is based on the *quasi stationary* approach which calculates the cognitive packet blocking probability by conditioning on the state distribution for the primary users.

We first express the primary packet blocking probability by

$$\text{Eng}(\lambda_l, \mu_l, M_l, K) \triangleq \frac{\binom{M_l-1}{K}(\frac{\lambda_l}{\mu_l})^K}{\sum_{i=0}^{K}\binom{M_l-1}{i}(\frac{\lambda_l}{\mu_l})^i},$$

which is the standard Engset formula with the corresponding state probabilities where

$$p_i = \frac{\binom{M_l}{i}\left(\frac{\lambda_l}{\mu_l}\right)^i}{\sum_{j=0}^{K}\binom{M_l}{j}\left(\frac{\lambda_l}{\mu_l}\right)^j}, \qquad i = 0, \ldots, K.$$

The second stage involves approximating the cognitive blocking probability by conditioning on $\{p_i : i = 0 \ldots, K - 1\}$. Note that we ignore the case of $i = K$ since the system becomes unstable in this case.

Conditioning on having i primary users in the system, the distribution of having j cognitive users in the system is given by the following set of one-dimensional state equations, where

$$p_{i,j} = \begin{cases} (M_c - j + 1)\frac{\lambda_c}{j\mu_c}p_{i,j-1}, 0 < j \le KN - iN, \\ (M_c - j + 1)\frac{\lambda_c}{(KN - iN)\mu_c}p_{i,j-1}, \text{otherwise}. \end{cases}$$

Together with $\sum_{\forall i,j} p_{i,j} = 1$, we can compute $p_{i,j}$. The average arrival rate of cognitive packets, A_c, and the average number of cognitive packets in the system, $\overline{N_c}$, are given by

$$\begin{cases} A_c = \sum_{\forall i<K,j}(M_c - j)\lambda_c p_{i,j}p_i, \\ \overline{N_c} = \frac{1}{1-p_K}\sum_{\forall i<K,j} jp_{i,j}p_i. \end{cases}$$

Note that the normalization term (i.e. $1 - p_K$) compensates for the missing case where $i = K$. By Little's formula, we then have $D = \frac{\overline{N_c}}{A_c}$.

IV. NUMERICAL RESULT DISCUSSION

We consider an existing wireless communication network characterized by M_l and K. We study the performance of a cognitive radio design that will operate concurrently with the existing system. Knowing the channel bandwidth consumed by primary users measured by C_l, our model can predict the delay performance of a particular cognitive radio design. The computation requires solving for $\frac{\lambda_l}{\mu_l}$ given a particular C_l. Then, the quantities $\pi_{i,j}, \forall i, j$, can be solved and used to compute D. It is obvious that the approximation that reduces the state probabilities from bi-dimension to one-dimension offers scalable computation for this problem.

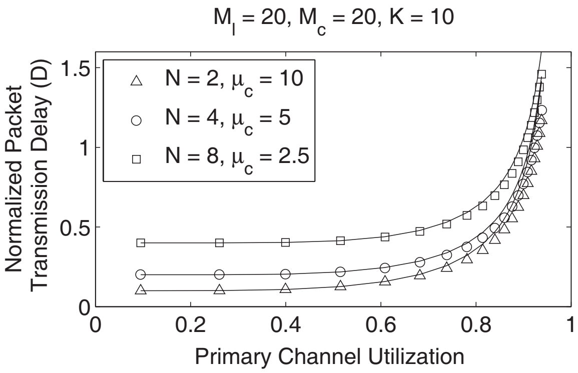

Fig. 2. Normalized packet transmission delay versus channel utilization of primary channel with various numbers of sub-channels.

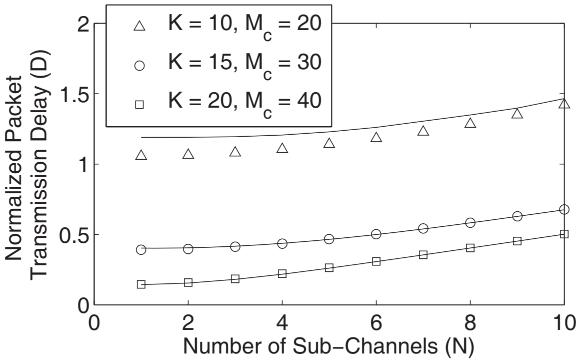

Fig. 3. Normalized packet transmission delay with various N, K.

In Fig. 2, we plot the cognitive delay performance given a range of C_l with various N. The exact numerical (shown with lines) and approximated (shown with symbols) results are also compared. Time unit is normalized to $1/\mu_l$. The service rate for a cognitive packet μ_c is also adjusted accordingly with different N. The accuracy of our approximation is indicated by the excellent agreement between the exact numerical and approximated results. We see that various design parameters maintain relatively steady delay before the primary channel utilization reaching beyond 70%.

In Fig. 3, we extend our previous experiment to show the cognitive delay performance with various N and K which are related to spectrum band allocation. While some discrepancies are found between the exact numerical (lines) and approximated (symbols) results for the case of $K = 10$, other cases show good agreement. The delay performance results show that a cognitive radio system may perform better when it is implemented on an existing system with an adequately large number of primary bands and maintains a low number of the sub-bands within each primary band in the design.

V. CONCLUSION

We have presented a new loss model for cognitive radio spectrum access with finite user population and its approximation for computation scalability. Using the models, we study the impact of primary channel loads and spectrum band allocations on the delay performance of cognitive radio users.

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