Maximum Free Circuit Routing in Circuit-Switched Networks

Eric W. M. Wong and Tak-Shing Yum

Department of Information Engineering
The Chinese University of Hong Kong
Shatin, Hong Kong.

Abstract

In nonhierarchical circuit-switched networks, calls should always be routed to the most direct path if possible. If the direct path is blocked, some alternate path should be tried. The selection of alternate paths has a significant effect on the network throughput. In this paper, we analyze an alternate-path routing rule called the *Maximum Free Circuit Routing* (*MFCR*). In the use of *MFCR*, a call is routed to the alternate path that has the maximum number of free circuits when the direct path is blocked. Analytical results show that in conjunction with trunk reservation, this routing rule can offer a stable throughput at high traffic conditions and can increase the call carrying capacity by about 20% (compared to direct path routing) under a blocking requirement of 10^{-2} on a fully connected symmetrical nonhierarchical network.

I Introduction

Network management is "the supervision of the telecommunication network to assure the maximum flow of traffic under all conditions" [1]. When an overload occurs, various network management functions must be performed to control the flow of traffic to minimize network congestion. These control functions include the reduction of operator traffic, recorded announcements, alternate route cancellation, traffic rerouting etc. With the use of common channel signaling and stored-program control, more sophisticated control functions can be used in network management. Among these control functions, re-routing of traffic to less congested routes should always be done first, as it affects neither the customers nor the other network management functions.

Previous analytical studies in this area include the work of Krupp [2] on *Random Alternate Routing* with and without trunk reservation on symmetrical networks, the extension by Akinpelu [3] on general non-symmetrical networks and the incorporation of external blocking by Yum and Schwartz [4].

Recently, a variety of approaches in alternate routing networks have been developed. AT&T uses a decentralized nonhierarchical routing strategy, called *Dynamic Nonhierarchical Routing* (*DNHR*) [5]. *DNHR* is a time-dependent routing scheme that increases network efficiency by taking advantage of the noncoincidence of busy hours in a large toll network. The second approach, which is currently being implemented in the British Telecom main network, is called *Dynamic Alternate Routing* (*DAR*) [6]. The *DAR* scheme has the advantages of (1) distributed control, (2) no need for detailed information passing between nodes and (3) no need for a pre-planning of routing patterns. The *Dynamically Controlled Routing* (*DCR*) [7] proposed by Northern Telecom is a centralized routing rule. A central routing processor receives information every 10 seconds from all the switches and update their *DCR* tables accordingly. The choice of alternate routes is based on the number of idle trunks and the exchange utilization level and is therefore a state-dependent rule.

In this paper, we attempt to analyze the performance of a state-dependent routing procedure called *Maximum Free Circuit Routing* (*MFCR*) on a symmetrical fully connected network. The *MFCR* directs a call to the alternate path that has the maximum number of free circuits. It was reported in [6,8] as the *Least Busy Alternate Routing*. We choose to call it *MFCR* because it is more descriptive. It will also not be confused with another rule that we are still studying called *Maximum Free Circuit Routing with Minimum Occupied Channels* (*MFCR/MOC*). We shall show that the use of this routing procedure together with the trunk reservation technique can indeed give a higher network carrying capacity compared to the use of direct path routing. Due to analytical difficulties, we shall use the same fully connected, symmetrical, uniformly loaded, nonhierarchical network model used in [2] and [4]. We shall also use the same set of simplifying assumptions in [2-4], namely that the traffic statistics are assumed to be independent at each link and that the alternately routed (or the overflowed) traffic is assumed to be Poisson.

Recently, Garzia and Lockhart [9] applied Compartmental Modeling to non-hierarchical communications networks. This modeling is much more complicated than ours, but it allows the formulation of network dynamics. Our approach is to derive the steady state performance via three nonlinear algebraic equations.

II Maximum Free Circuit Routing

Consider a fully connected and uniformly loaded network where all links consist of N channels. Let P_n be the probability that there are n calls on a link (or that n channels are occupied). Then P_N is the probability of blocking on that link. Let D be the direct-route offered load to a link. Then DP_N is the overflowed load to the alternate paths. In [2-4], the alternate path load is randomly distributed on the set of m alternate paths. We shall call this procedure *Random Alternate Routing*.

For the fully connected network we are considering, we shall restrict our choice of alternate paths consisting of only two links. Consider a particular alternate path. Let the number of occupied channels on the first link be i and that on the second link be j. Then the number of occupied circuits k in that path is $k = \max(i, j)$. When the direct path is full, the *MFCR* will direct the call to the alternate path with the maximum number of free circuits or with minimum k. When there are more than one such paths, choose one at random.

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Consider a particular path AC. If link AC is full, the overflowed AC calls of rate DP_N will be routed randomly to one of the *Maximum-Free-Circuit* (*MFC*) paths. Let there be a total of α such *MFC* paths. Then, the alternate path load of AC that falls on a particular *MFC* path, say path ABC, is DP_N/α. Let Z_k be the probability that a two link alternate path has k or more occupied circuits. Then,

$$Z_k = 1 - \left\{\text{Prob}\begin{bmatrix}\text{a link has less than k}\\ \text{occupied channels}\end{bmatrix}\right\}^2$$

$$= \begin{cases} 1 & k = 0 \\ 1 - \left(\sum_{n=0}^{k-1} P_n\right)^2 & 0 < k \le N. \end{cases} \tag{1}$$

Given that path ABC has k occupied circuits, the probability that

(i) the $\alpha - 1$ other alternate paths also have k occupied circuits each *and*

(ii) each of the remaining $m - \alpha$ alternate paths has $k + 1$ or more occupied circuits,

is given by

$$f(\alpha \mid k) = \binom{m-1}{\alpha-1}(Z_k - Z_{k+1})^{\alpha-1} Z_{k+1}^{m-\alpha}$$

$$= \frac{\alpha}{m}\binom{m}{\alpha}(Z_k - Z_{k+1})^{\alpha-1} Z_{k+1}^{m-\alpha}$$

where $Z_k - Z_{k+1}$ is the probability that an alternate path has k occupied circuits. Therefore, given that path ABC has k occupied circuits, the amount of traffic $y(k)$ that gets routed from AC to alternate path ABC is

$$y(k) = \sum_{\alpha=1}^{m} \frac{DP_N}{\alpha} f(\alpha \mid k)$$

$$= DP_N \frac{Z_k^m - Z_{k+1}^m}{m(Z_k - Z_{k+1})}. \tag{2}$$

Therefore, given that links AB and BC have i and j busy channels respectively, the overflowed traffic a_i from link AC to link AB is

$$a_i = \sum_{j=0}^{N-1} y(\max(i,j))P_j \qquad 0 \le i \le N-1. \tag{3}$$

Since link AB carries the alternate traffic from $2m$ alternate paths, when link AB has i busy channels, the total alternate-route traffic A_i on link AB is

$$A_i = 2ma_i. \tag{4}$$

Let the call holding time be exponentially distributed with mean $1/\mu$. The call completion rate and the call arrival rate in state i, denoted as d_i and b_i respectively, are

$$\begin{aligned} d_i &= i\mu \qquad & i &= 1,2,\ldots,N \\ b_i &= (D + A_i)\mu \qquad & i &= 0,1,\ldots,N-1 \end{aligned} \tag{5}$$

where D and A_i are in Erlangs. The state probabilities $\{P_i\}$ can be determined from the birth-death process with birth and death rate b_i and d_i respectively. However, since the birth rates are functions of the state probabilities, the process can only be solved by iteration. The following iteration procedure requires only three equations in three unknowns: P_0, P_1 and P_N.

From the balance equation $b_i P_i = d_{i+1} P_{i+1}$, we have

$$b_{i+1} - b_i = \frac{(i+2)\mu P_{i+2}}{P_{i+1}} - \frac{(i+1)\mu P_{i+1}}{P_i} \tag{6}$$

But from (5), we also have

$$b_{i+1} - b_i = 2\mu DP_N\left(\sum_{j=0}^{i} P_j\right)\left[\frac{Z_{i+1}^m - Z_{i+2}^m}{Z_{i+1} - Z_{i+2}} - \frac{Z_i^m - Z_{i+1}^m}{Z_i - Z_{i+1}}\right]$$

$$i = 0,1,\ldots,N-2. \tag{7}$$

Equating (6) and (7), P_{i+2} can be expressed as a function of P_0, P_1,..., P_{i+1} and P_N. Therefore, starting from $i = 0$ and proceeding by induction, we can express P_i as

$$P_i = \Phi_i(P_0, P_1, P_N) \qquad i = 2,3,\ldots,N. \tag{8}$$

Setting $i = N$, we have the first equation for iteration

$$P_N = \Phi_N(P_0, P_1, P_N). \tag{9}$$

Substituting (8) into the normalization equation, the second equation for iteration is obtained as

$$P_0 + P_1 + \sum_{i=2}^{N} \Phi_i(P_0, P_1, P_N) = 1. \tag{10}$$

Next, we note that the average link occupancy is simply the average direct-route traffic plus the average alternate-route traffic on a link, or

$$\sum_{i=1}^{N} iP_i = (1 - P_N)D + 2P_N D\left\{1 - [1 - (1 - P_N)^2]^m\right\}. \tag{11}$$

Using (8) in the left hand side of (11), a third equation involving only P_0, P_1 and P_N is obtained. As a check, we note that the carried load on a link can be expressed as

$$\sum_{i=0}^{N-1} b_i P_i = \mu D(1 - P_N) + 2\mu DP_N \sum_{i=0}^{N-1} P_i \sum_{j=0}^{N-1} P_j \frac{Z_{\max(i,j)}^m - Z_{\max(i,j)+1}^m}{Z_{\max(i,j)} - Z_{\max(i,j)+1}}$$

$$= \mu D(1 - P_N) + 2\mu DP_N \sum_{i=0}^{N-1}\left(2P_i \sum_{j=0}^{i-1} P_j + P_i^2\right)\frac{Z_i^m - Z_{i+1}^m}{2P_i \sum_{k=0}^{i-1} P_k + P_i^2}$$

$$= \mu D(1 - P_N) + 2\mu DP_N \sum_{i=0}^{N-1} [Z_i^m - Z_{i+1}^m]$$

$$= \mu D(1 - P_N) + 2\mu DP_N\left\{1 - [1 - (1 - P_N)^2]^m\right\} \tag{12}$$

But (12) is just μ multiplied by the average load in Erlang on a link.

The above approach results in a tremendous saving of computation effort when compared to the direct iteration of N balance equations plus one normalization equation. With P_N known, the end-to-end blocking probability for a fully connected network with m alternate paths using *MFCR*, denoted as $\mathrm{EEBP_M}$, is

$$\mathrm{EEBP_M} = \mathrm{Prob}\begin{bmatrix}\text{Blocking on}\\ \text{the direct path}\end{bmatrix} \mathrm{Prob}\begin{bmatrix}\text{Blocking on all}\\ \text{m alternate paths}\end{bmatrix}$$

$$= P_N[1-(1-P_N)^2]^m$$

III MFC Routing with Trunk Reservation

With trunk reservation, the last r free channels on a link are always reserved for direct route traffic. Hence the call completion rate and the call arrival rate on a particular link become

$$d_i = i\mu \qquad i = 1,2,\ldots,N$$

$$b_i = \begin{cases}(D + A_i)\mu & i = 0,1,\ldots,N-r-1 \\ D\mu & i = N-r, N-r+1,\ldots,N-1.\end{cases}$$

Following the same derivation, (7) becomes

$$b_{i+1} - b_i = \begin{cases} 2\mu D P_N\left(\sum_{j=0}^{i} P_j\right)\left[\frac{Z_{i+1}^m - Z_{i+2}^m}{Z_{i+1}-Z_{i+2}} - \frac{Z_i^m - Z_{i+1}^m}{Z_i - Z_{i+1}}\right] & i = 0,1,\ldots,N-r-2 \\ 0 & i = N-r-1,\ldots,N-2 \end{cases} \quad (13)$$

and (8) becomes

$$P_i = \Phi_i(P_0, P_1, P_N) \qquad i = 2,3,\ldots,N-r. \quad (14)$$

For $i \geq N - r$, we can solve the balance equation directly to obtain

$$P_i = \frac{N!P_N}{i!D^{N-i}} \qquad i = N-r, N-r+1,\ldots,N. \quad (15)$$

As in section II, we derive three equations to be solved numerically for the three unknowns P_0, P_1 and P_N. Setting $i = N - r$ in equations (14) and (15), we obtain the first equation as

$$P_N = \frac{(N-r)!D^r}{N!}\Phi_{N-r}(P_0, P_1, P_N). \quad (16)$$

Adding the P_i's derived in (14) and (15), we obtain the second equation as

$$P_0 + P_1 + \sum_{i=2}^{N-r-1}\Phi_i(P_0,P_1,P_N) + \frac{N!P_N}{D^N}\sum_{i=N-r}^{N}\frac{D^i}{i!} = 1. \quad (17)$$

Similar to (11), the third equation can be obtained as

$$P_1 + \sum_{i=2}^{N-r-1} i\Phi_i(P_0,P_1,P_N) + \frac{N!P_N}{D^N}\sum_{i=N-r}^{N}\frac{D^i}{(i-1)!} =$$

$$D(1-P_N) + 2DP_N\left\{1 - \left[1 - \left(1 - \frac{N!P_N}{D^N}\sum_{i=N-r}^{N}\frac{D^i}{i!}\right)^2\right]^m\right\} \quad (18)$$

where $(\cdot)^2$ is the probability that a particular alternate path is not blocked. With P_N known, the end-to-end blocking probability for *MFCR* with Trunk Reservation, denoted as $\mathrm{EEBP_T}$, is

$$\mathrm{EEBP_T} = P_N\left[1 - \left(1 - \frac{N!P_N}{D^N}\sum_{i=N-r}^{N}\frac{D^i}{i!}\right)^2\right]^m.$$

IV Performance Comparisons

Fig.1 shows EEBP versus the direct route offered load D for various routing strategies on a fully connected symmetrical network. The blocking probability of direct routing is given by the Erlang B formula. The *Random Alternate Routing* and its trunk reservation version are from [4]. $N = 30$, $m = 2$ and $r = 3$ are assumed. We see that *MFCR* always gives a lower EEBP than the *Random Alternate Routing*. This is also true with the trunk reservation option added. Their difference, however, diminishes as $D \rightarrow N$. Similar unstable behaviour is observed for *MFCR* as reflected by its blocking probability rising above that of direct-route routing when channel utilization exceeds 0.83. Fortunately *MFCR* augmented with trunk reservation offers a stable throughput at heavy traffic, similar to *Random Alternate Routing* with trunk reservation.

Fig.2 shows the case for $m = 3$. We see that having one additional alternate path to divert overflowed traffic does in fact reduce the blocking probability. Assuming a blocking requirement of 10^{-2}, Figures 1 and 2 show that using *MFCR* with trunk reservation the throughput can be increased by about 20% over direct path routing.

As the above analysis invokes the simplifying assumptions in [10], computer simulation is needed to establish the validity of the analytical results. We are currently working on this as well as the generalization of the analysis for a refined *MFCR* rule.

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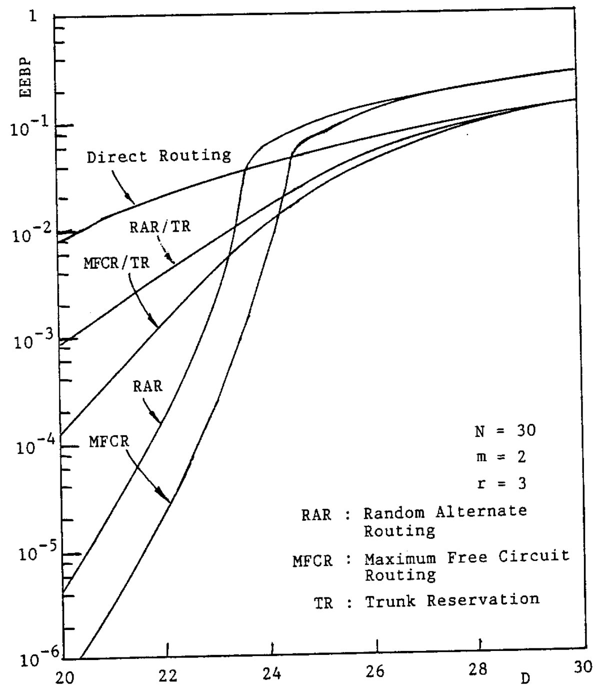

Fig.1. Comparisons of Alternate Routing Strategies (m=2)

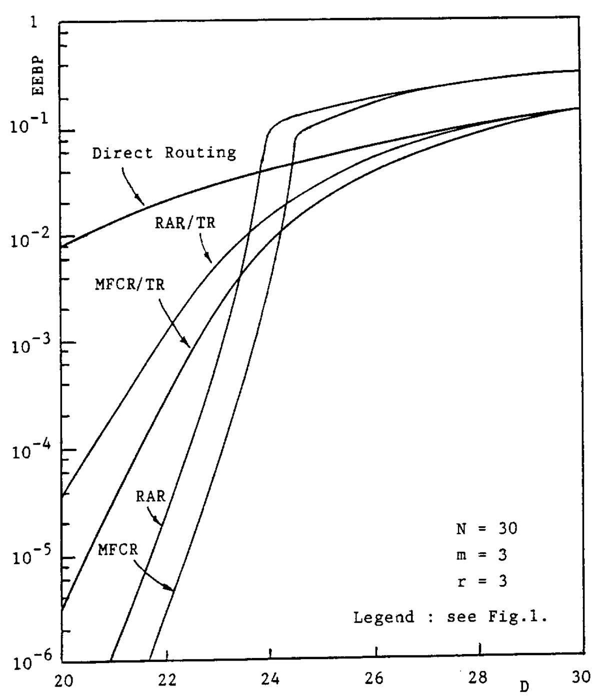

Fig.2. Comparisons of Aternate Routing Strategies (m=3)