On Teletraffic Applications to OBS

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***Abstract*—We provide teletraffic models for loss probability evaluation of optical burst switching (OBS). We show that the popular Engset formula is not exact for OBS modeling and demonstrate that in certain cases it is not appropriate. A new exact model is provided. The various models are compared using numerical results for various OBS alternatives with and without Burst Segmentation.**

***Index Terms*—Blocking probability, Engset loss formula, optical burst switching (OBS), optical Internet, teletraffic, traffic modeling.**

I. INTRODUCTION

THE future optical Internet will be based on the Internet Protocol (IP) with wavelength division multiplexing (WDM) technologies. Optical burst switching (OBS) [3], [6] has been considered a viable option to support IP over WDM. In OBS, IP packets with a common destination arriving at an edge router (ER) are aggregated into large bursts, each of which is switched and routed as a single unit.

A large subset of OBS proposals are loss-based. The main loss-based proposals may be classified into two groups which we designate: 1) OBS/JET and 2) OBS/BS. In OBS/JET (e.g., Just Enough Time (JET) [6]), the ER sends bursts to their destination without having the entire route reserved. A single control packet is associated with each burst. The control packet precedes the burst payload and attempts to reserve switching and transmission resources at each optical cross connect (OXC) along the route. If the control packet arrives at an OXC and a suitable wavelength channel in the next output link of the route is not available, the burst is lost.

OBS with Burst Segmentation (OBS/BS) [2], [8] is similar to OBS/JET except that a burst may be segmented with only part of it blocked so that the overall *packet* blocking probability is reduced. We consider here a variant of OBS/BS whereby the burst continues to be dumped as long as there is no free wavelength channel. As soon as a channel becomes available, one of the dumped bursts is immediately directed toward that free channel and the remainder of the burst is transmitted to its destination. An updated control packet may be generated by the OXC to reserve capacity for the shorter burst in subsequent OXCs.

For first cut performance results for an OBS system, one may consider the burst process to follow a Poisson process [3], [9]. This assumption gives rise to the well known M/M/k/k queueing system. However, in many cases, bursts contending for a group of wavelength channels at the output of an OXC may not be using a large enough number of wavelength channels at the input to justify the Poisson assumption. Therefore, there is a scope for resurrection of classical teletraffic models for blocking probability evaluation and network dimensioning.

II. SINGLE LINK LOSS-BASED OBS MODELLING

Consider an output link of an OXC. Such an output link has F optical fibers and each optical fiber can carry W wavelengths. If full wavelength conversion is available, this output link has $C = FW$ wavelength channels, otherwise, an arriving burst that uses a given wavelength, must use the same wavelength at the output, thus, only F wavelength channels are available to it. All input wavelengths that carry bursts directed to our output link can be considered sources or customers. On all these input wavelengths (sources), we consider traffic to arrive as on/off processes. For each source, time periods during which bursts are transmitted are on-periods and time periods between bursts are off-periods. For simplicity, we ignore here effects related to the use of the control packets and only consider the streams of payload-bursts in our model. Although our scope here is somewhat limited as we consider a single link model, this single link model can be used as a module in the analysis of [7] to provide blocking probability evaluation in a case of a general network.

Let the on and off periods of each source have means of $1/\mu$ and $1/\lambda$, respectively. Let $\rho = \lambda/\mu$. Let the number of the relevant input wavelength channels be M. (Without wavelength conversion, there is an independent system for each wavelength, so M will include only input channels of a specific wavelength.) We also assume that the sources (input channels) are homogenous. Such assumption can be supported by the efforts made by network designers to balance the load.

One may consider the well-known Engset loss formula [1], [4], [5] to be an appropriate means to evaluate OBS/JET blocking probability for a single OXC. In particular,λ can be the Engset arrival rate per free customer, μ the service rate, M the limited number of customers, and the number of servers, denoted K, can take the values of C or F for the cases of

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with or without wavelength conversion, respectively. Engset loss formula is also appealing as it is insensitive to the on-time as well as off-time distributions [5]. Unfortunately, it is not always suitable for loss-based OBS systems. Consider a case of full wavelength conversion with $F = 3$ and $W = 40$ (thus, $K = C = 40 \times 3 = 120$). Also, let $M = 121$ and $\rho = 10\,000$. This may apply to heavy traffic cases where bursts are efficiently synchronized. Applying Engset loss formula gives burst blocking probability of 0.988. The true burst blocking probability in this case is approximately 1/121 because the traffic from 120 wavelengths can be served without any loss.

The failure of Engset model in such situations is due to the fact that under the Engset model, ρ represents the intensity per free customer and, more importantly, a blocked customer stays free and keeps attempting at the same intensity. In our example, while the 120 wavelength channels are processing, one burst each, from 120 sources, the remaining 121st customer (wavelength) has its burst lost. In OBS, when a burst is lost, it is still arriving for the entire duration of the burst at the input wavelength and it is dumped at the OXC. However, according to the Engset model, during the duration of that lost burst, the source is making, on average, $\rho = 10\,000$ further attempts and is losing 10 000 additional "bursts" instead of losing a single burst. In OBS when a burst is lost, the source does not become free immediately as in the Engset model. Instead, in OBS/JET, the blocked burst behaves as if it is served by a "dummy" server and does not become free (in the Engset sense) until the entire burst is dumped. In OBS/BS, again, at first, a blocked burst dumps its packets until perhaps a free wavelength channel is available to serve it.

III. The Solution

A. OBS/BS

OBS/BS is much easier to analyze and we will consider it first. To evaluate its blocking probability, we use the binomial distribution. What we really have here are K servers plus $M-K$ dummy servers. Accordingly, the burst blocking probability for OBS/BS is obtained by

$$B = \frac{E[X-K]^+}{E[X]} \tag{1}$$

where Z^+ is defined by $Z^+ = Z$ if $Z \geq 0$, and $Z^+ = 0$ if $Z < 0$, and X is a random variable representing the number of wavelengths that would have been busy if the number of available wavelengths would have been M or higher; X is a binomial random variable with parameters ($\hat{\rho}$, M) where

$$\hat{\rho} = \frac{\rho}{(1+\rho)} = \frac{\lambda}{\lambda+\mu}.$$

Notice that if we consider an arbitrary point in time, we realize each of the sources will be on with probability $\hat{\rho}$ and off with probability $1-\hat{\rho}$. Thus,

$$E[X-K]^+ = \sum_{j=K+1}^{M} (j-K)\binom{M}{j}\hat{\rho}^j(1-\hat{\rho})^{M-j}$$

and $E[X] = \hat{\rho}M$. Note that $E[X]$ is called the *intended load* [1]. By [5], this Binomial type solution is insensitive to the off period as well as to the on period distributions.

B. OBS/JET

For OBS/JET, we consider a two dimensional Markov chain assuming exponential on and off times. As mentioned above, there are three types of "customers": (1) busy (bursts that are being transmitted), (2) free (empty wavelength at the input), and (3) blocked (bursts that being dumped). The sum of the three types is always M, thus, the number of the free "customers" is always M minus the other two. Accordingly, let $\pi_{i,j}$ be the steady state probability where $i(0 \leq i \leq K)$ is the number of busy customers and $j(0 \leq j \leq M-K)$ is the number of *frozen* customers (sources who transmit blocked bursts). We have the following steady state equations:

For $i = 0, 1, 2, \ldots, K-1$ we have

$$\begin{aligned}
&[(M-i-j)\lambda + (i+j)\mu]\,\pi_{i,j} \\
&\quad = (M-i+1-j)\lambda\pi_{i-1,j} \\
&\qquad + (j+1)\mu\pi_{i,j+1} + (i+1)\mu\pi_{i+1,j}
\end{aligned} \tag{2}$$

$$\begin{aligned}
&\{(M-K-j)\lambda + (K+j)\mu\}\,\pi_{K,j} \\
&\quad = (M-K+1-j)\lambda\pi_{K-1,j} + (j+1)\mu\pi_{K,j+1} \\
&\qquad + (M-K+1-j)\lambda\pi_{K,j-1}.
\end{aligned} \tag{3}$$

For brevity, in (2) and (3) $\pi_{i,j}$ values out of the range $0 \leq i \leq K$ and $0 \leq j \leq M-K$ take the value zero.

Then we also have the normalization equation:

$$\sum_{i=0}^{K}\sum_{j=0}^{M-K}\pi_{i,j} = 1. \tag{4}$$

Since the number of frozen customers cannot be more than $M-K$, as a customer cannot become frozen if there are less than K busy customers, the offered load is given by

$$T_o = \sum_{i=0}^{K}\sum_{j=0}^{M-K}(M-i-j)\rho\pi_{i,j} \tag{5}$$

the carried load is given by

$$T_c = \sum_{i=0}^{K}\sum_{j=0}^{M-K} i\pi_{i,j} \tag{6}$$

and the blocking probability is obtained by

$$B = \frac{T_o - T_c}{T_o}. \tag{7}$$

Notice that for the case $M = K$, $T_o = T_c$ and $B = 0$. In this case, (5) and (6) reduce to those of an Engset system.

IV. Numerical Results

We will now present several numerical results to demonstrate the error introduced by the Erlang and Engset loss formulae approximations to OBS/JET, the sensitivity of our OBS/JET solution to the on and the off time distributions and the benefit of OBS/BS over OBS/JET. For the Erlang approximation we use the *intended traffic load* [1] given by $\hat{\rho}M$ as the traffic load.

In Fig. 1 we present results for the proportion of work lost versus the normalized intended traffic load (per channel), defined by $\hat{\rho}M/K$, for the case of $K = 30$ and $M = 33$. We compare here between OBS/BS, OBS/JET, Engset and Erlang models. For OBS/BS, the result in Fig. 1 is based on (1). For OBS/JET, the blocking probability (proportion of work lost) is calculated by (7). We observe that the Engset Loss Formula does not provide an accurate approximation for OBS/JET for high normalized load (over 0.85). This is consistent with our example

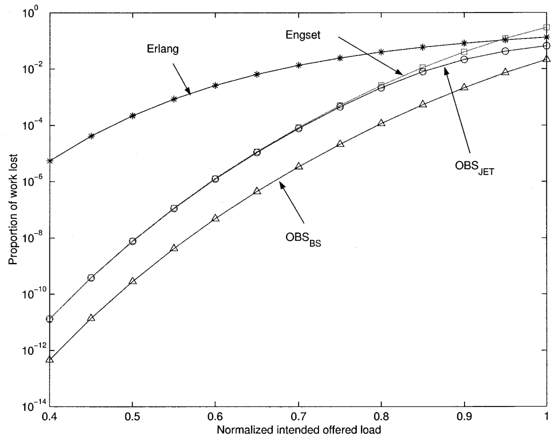

Fig. 1. Proportion of work lost vs. normalized intended traffic load for $K = 30$ and $M = 33$.

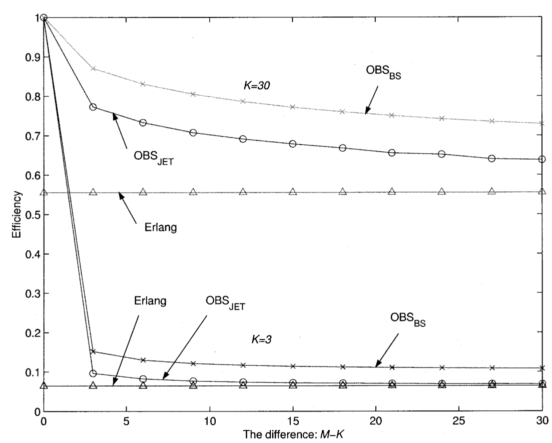

Fig. 2. Efficiency versus number of sources for $B = 0.001$.

above for high ρ. On the other hand, when the normalized load is below 0.8, Engset Formula is accurate. We also observe, as expected, that Erlang loss formula over-estimates the blocking probability of OBS/JET, and OBS/BS reduces the proportion of work lost.

Fig. 2 focuses on efficiency. We set the blocking probability (or proportion of work lost) at $B = 0.001$, and we vary the number of sources. We define efficiency as the maximal carried traffic per server (wavelength) that maintains blocking of no more than 0.1%. We present results for $K = 3$ and for $K = 30$. As expected, we see a clear reduction in efficiency in the case of $K = 3$ (the bottom three curves). We also see that Erlang under-estimate the efficiency versus the other approaches that involves limited sources, and OBS/BS is shown to be more efficient than OBS/JET.

Next, we verify by simulations the OBS/JET model, i.e. (2), (3) and (4), and the procedure used to solve these equations. Using simulations, we also examine the sensitivity of the model to on and off time distributions. Fig. 3 presents simulation results for blocking probability versus λ/μ for the case of $M = 6$ and $K = 3$. We consider cases with the on and the off time distributions being exponential and Gaussian, respectively. In the Gaussian case the standard deviation is equal to 4% of the mean, both for on and off distributions. The vertical bars represent 95% confidence intervals based on Student-t distribution.

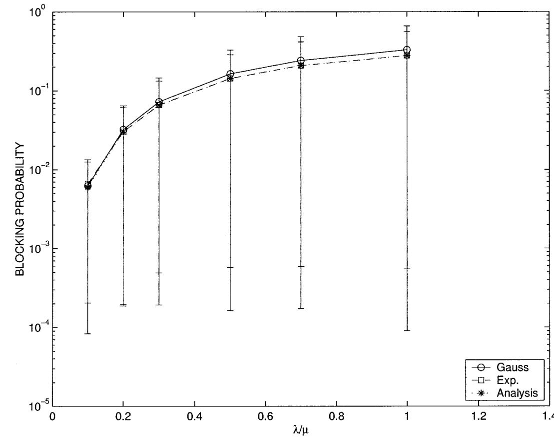

Fig. 3. Blocking probability vs. λ/μ for $K = 3$ and $M = 6$.

From Fig. 3, we observe that the results obtained for the OBS/JET model and for the simulation based on exponential on and off distributions are indistinguishable when plotted. This verifies the correctness of the OBS/JET model and that of the procedure to solve it. We also observe that the results based on Gaussian on and off distributions are very close to their exponential counterparts indicating that the sensitivity of the OBS/JET blocking probability results to the on and off distributions may not be too significant.

V. Conclusions

Demonstrating gross inaccuracy of Engset formula, in a particular case, for blocking probability evaluation for OBS/JET, we have proposed a more accurate alternative for a single OXC loaded by on-off sources. We have demonstrated by simulation that the proposed method is not too sensitive to on and off time distributions. We have also provided a simple formula to evaluate blocking probability for OBS/BS. Discrepancies introduced by Engset and Poisson modeling have also been demonstrated.

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