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# CHAOS: CONTROL AND ANTI-CONTROL

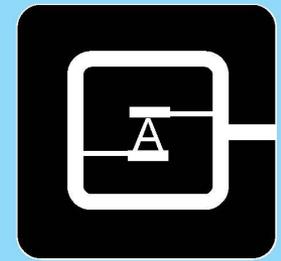
by Guanrong Chen

**Abstract:** Chaos control and anti-control technologies promise to have a major impact on many novel, time- and energy-critical applications, such as high-performance circuits and devices (e.g., delta-sigma modulators and power converters), liquid mixing, chemical reactions, biological systems (e.g., in the human brain, heart, and perceptual processes), crisis management (e.g., in power electronics), secure information processing, and critical decision-making in political, economic and military events. This new and challenging research and development area has become a scientific interdisciplinary, involving systems and control engineers, theoretical and experimental physicists, applied mathematicians, physiologists, and above all, circuits and devices specialists. Both control and anti-control of chaos can be analyzed using chaos and bifurcation theories, and can be implemented by suitable design of control and switching circuitries.

Chaos refers to one type of complex dynamical behaviors that possess some very special features such as being extremely sensitive to tiny variations of initial conditions, having bounded trajectories in the phase space but with a positive maximum Lyapunov exponent, possessing a finite Kolmogorov-Sinai entropy, a continuous power spectrum, and/or a fractional topological dimension, etc. Oftentimes, chaos coexists with some other complex dynamical phenomena like bifurcations, fractals, and strange attractors. A typical chaotic attractor generated by a power system model is shown in Figure 1.

Due to its intrinsic dynamical complexity, chaos was once believed to be neither controllable nor predictable, and, therefore, use-

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less. However, recent research advances have demonstrated that chaos not only is (long-term) controllable and (short-term) predictable, but also can be beneficial to many real-world applications. In fact, control and anti-control of chaos have become a rallying point for an important segment overlapping engineering, physics, mathematics, and biomedical science.

Chaos control refers to the situation where chaotic dynamics is weakened or eliminated by appropriate controls, while anti-control of chaos means that chaos is created, maintained, or enhanced when it is healthy and useful. Both control and anti-control of chaos can be accomplished via some conventional and nonconventional methods such as microscopic parameter perturbation, bifurcation monitoring, entropy reduction, state pinning, phase delay, and various feedback and adaptive controls [1].

There are many practical reasons for controlling or ordering chaos. First of all, chaotic (messy, irregular, or disordered) system response with little meaningful information content is unlikely to be of use as chaos can lead systems to harmful or even catastrophic situations. In these troublesome cases, chaos should be reduced as much as possible, or totally suppressed. For instance, stabilizing chaos can avoid fatal voltage collapse in power networks and deadly heart arrhythmias, can guide disordered circuit arrays (e.g., multi-coupled oscillators and cellular neural networks) to reach a certain level of desirable pattern formation, can regulate dynamical responses of mechanical and electronic devices (e.g., diodes, laser machines, and machine tools), can help well-organize an otherwise mismanaged multi-agency corporation to reach a stable equilibrium state whereby achieving optimal agent performance, etc.

Ironically, recent research has shown that chaos can actually be useful under certain circumstances, and there is growing interest in utilizing the very nature of chaos, particularly in some novel time- and/or energy-critical applications. The most motivative reason is the observation that chaos permits a system to explore its every dynamical possibility: when chaos is under control, it provides the designer with an exciting variety of properties, richness of flexibility, and a cornucopia of opportunities. Figure 2 visualizes how by varying a constant feedback control gain within a simple quadratic map, period-doubling bifurcation and chaos can be created and then be stabilized to a variety of equilibria of different periods. Traditional engineering design always tries to reduce irregular dynamical behaviors of a system and, therefore, completely eliminates chaos. However, such overdesign is usually accomplished at the price of losing great flexibilities in achieving high perfor-

mance near the stability boundaries, or at the expense of radically modifying the original system dynamics. In many occasions, this proves to be unnecessary.

It has been shown that the sensitivity of chaotic systems to small perturbations can be used to direct system trajectories to a desired target quickly with very low and ideally minimum control energy. This can be crucial for navigation in the multi-planetary space system. A suitable modification of chaotic dynamics such as stability conversion or bifurcation delay not only can significantly extend the operational range of machine tools and jet engines, but also may enhance the artificial intelligence of neural networks, as well as increase coding/decoding efficiency in signal and image communications. Other application examples of chaos control and anti-control

technologies include designing high-performance circuits and devices (e.g., delta-sigma modulators, automatic gain control loops, and power converters), achieving chaos synchronization for information processing, pattern recognition, and secure communications, forming various wave patterns and self-organized behaviors in oscillator arrays and neural networks, delaying bifurcations in electric power systems and energy convection loops, and performing crisis management and critical decision-making in political and economic, as well as military events.

Fluid mixing is another good example in which chaos is not only useful but actually very desirable,

where two fluids are to be thoroughly mixed while the required energy is minimized. For this purpose, it turns out to be much easier if the dynamics of the particle motion of the two fluids are strongly chaotic, because it is otherwise difficult to obtain rigorous mixing properties due to the possibility of invariant two-tori in the flow. This has been one of the main subjects in fluid mixing, known as chaotic advection. Chaotic mixing is also momentous in applications involving heating, such as in plasma heating for a nuclear fusion reactor. In this process, heat waves are injected into the reactor, for which the best result is obtained when the heat convection inside the reactor is chaotic.

Within the context of biological systems, controlled biological chaos appears to be important to the way a human brain executes its tasks. There have been some suggestions that the human brain can process massive information instantly, in which case the ability of human beings in controlling brain chaos could be a fundamental reason. The idea of anti-control of chaos has been proposed for solving the problem of driving responses of a human brain model away from the saddle-type of equilibrium, so that undesirable periodic behaviors of

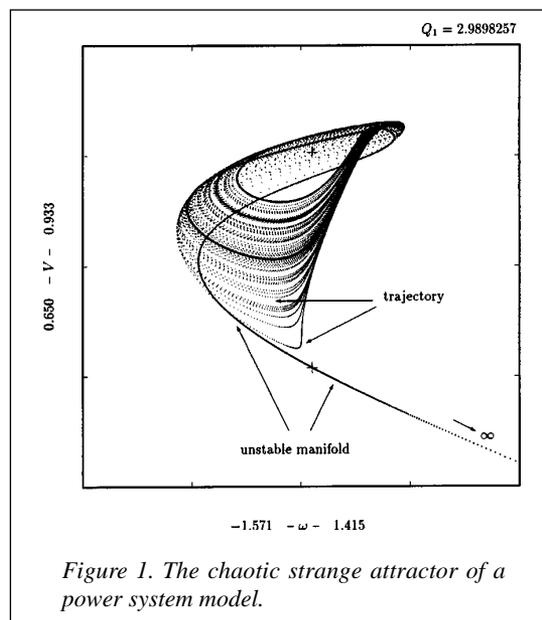


Figure 1. The chaotic strange attractor of a power system model.

neuronal population bursting can be prevented. Also, some recent laboratory studies reveal that the complex variability of healthy dynamics in a variety of physiological systems has features reminiscent of chaos. For example, in the human heart, the amount of intracellular  $\text{Ca}^{2+}$  is closely regulated by a coupled process in a way similar to a system of coupled oscillators. Medical evidence reveals that controlling the chaotic arrhythmia in an appropriate way can be a new, safe, and auspicious approach to the design of a smart pacemaker for regulating heartbeats. Figure 3 shows a self-tuned delayed-feedback control simulation of a chaotic human-heart model to a period-three equilibrium state.

Motivated by many such potential real-world applications, current research on control and anti-control of chaos has become intensive. In the theoretical aspect, chaos control and anti-control are posing a new challenge to both system analysts and control engineers. This is due to the extreme complexity and sensitivity of chaotic dynamics, which can cause many unusual difficulties in long-term predictability and short-term controllability of chaos. A controlled chaotic system is inherently nonautonomous, and cannot be converted to an autonomous system in most cases since the controller as a time function is yet to be designed. Possible time-delay, noise, and coupling effects often make a controlled chaotic system Lyapunov-irregular and topologically extremely complex. As a result, many existing theories and methodologies for autonomous systems are no longer applicable. On the other hand, at the technical level, chaos control and anti-control have also posed new challenges to circuit designers and instrument specialists. A successful circuit implementation in a chaotic environment is generally difficult, due to the extreme sensitivity of chaos to parameter variations and noise perturbations, and the nonrobustness of chaos to structural stability, within the

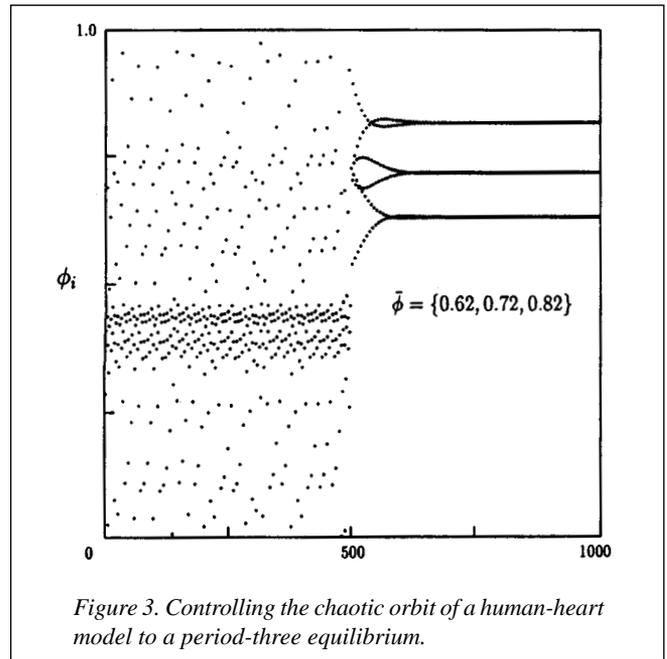


Figure 3. Controlling the chaotic orbit of a human-heart model to a period-three equilibrium.

physical devices. Notwithstanding many technical obstacles, both theoretical and technical developments in this area have gained remarkable progress in the last few years. For instance, some unified control methods have been developed under the nonautonomous Lyapunov stabilization theory; some rigorous anti-control techniques, even for spatiotemporal systems, have been initiated; some novel chaos-based encryption approaches have been advanced; and some chips of chaotic circuits have been made toward commercialization [2].

In summary, the emerging field of chaos control and anti-control is very stimulating and full of promise; it is expected to have far-reaching impacts with enormous opportunities in industrial and commercial applications. New theories for dynamics analysis, new methodologies for control, and new circuitry design for implementation altogether are calling for new efforts and endeavors from the communities of nonlinear dynamics, controls, and circuits and systems. The IEEE Circuits and Systems Society has been very active in this field in the past decade, and should continuously maintain its leadership in the field in the future.

[1] G. Chen, "Chaos, Bifurcation, and Their Control," *Wiley Encyclopedia of Electrical and Electronics Engineering*, 1998.

[2] G. Chen and X. Dong, *From Chaos to Order—Perspectives, Methodologies, and Applications*. World Scientific Pub. Co.: Singapore, 1998.



Guanrong (Ron) Chen received the M.S. degree in computer science from the Sun Yatsen University, China, in 1981, and the Ph.D. degree in applied mathematics from Texas A&M University in 1987. His research interest is within the broad area of nonlinear systems, on both dynamics and controls. He is the (co)author of about a hundred journal papers and several research monographs and advanced textbooks including *Nonlinear Feedback Control Systems* (with Rui J. P. de Figueiredo, 1993) and *Hopf Bifurcation Analysis* (with Jorge L. Moiola, 1996). He served as associate editor for the *IEEE Transactions on Circuits and Systems—Part I* from 1993 to 1995.

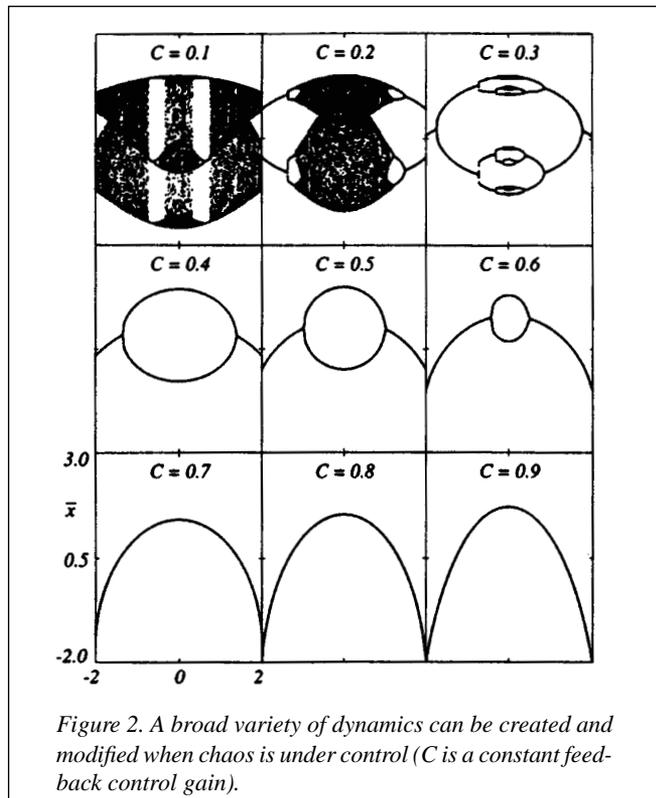


Figure 2. A broad variety of dynamics can be created and modified when chaos is under control ( $C$  is a constant feedback control gain).