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Bifurcations: Control and Anti-Control

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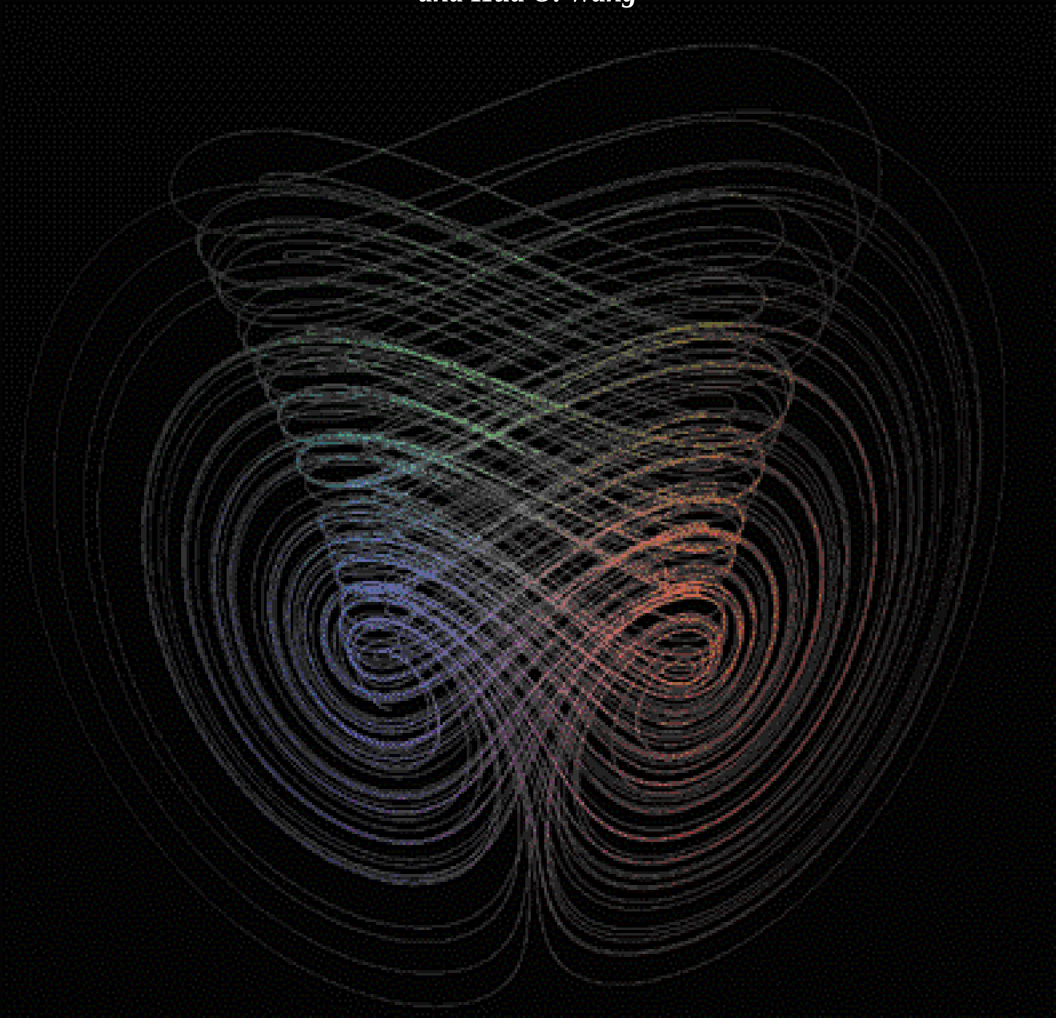

Chen's chaotic attractor: $dx=35(y-x)dt$, $dy=(-7x-xz+28y)dt$, $dz=(xy-3z)dt$

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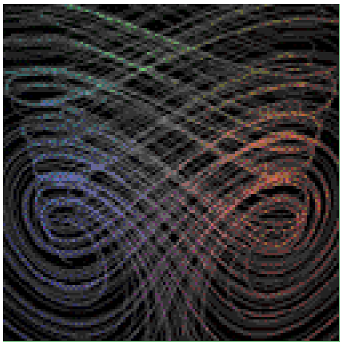


Bifurcations: Control and Anti-Control

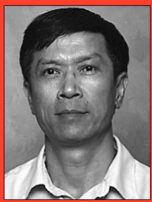
Abstract—*Various bifurcations exist in nonlinear dynamical systems such as complex circuits, networks, and devices. Bifurcations can be important and beneficial if they are under appropriate control. Bifurcation control and anti-control deal with modification of system bifurcative characteristics by a designed control input. Typical bifurcation control and anti-control objectives include delaying the onset of an inherent bifurcation, stabilizing a bifurcated solution or branch, changing the parameter value of an existing bifurcation point, modifying the shape or type of a bifurcation chain, introducing a new bifurcation at a preferable parameter value, monitoring the multiplicity, amplitude, and/or frequency of some limit cycles emerging from a bifurcation mechanism, optimizing the system performance near a bifurcation point, or a combination of some of these. This article offers a brief overview of this emerging and promising field of research, putting the main subject of bifurcations control and anti-control into perspective.*

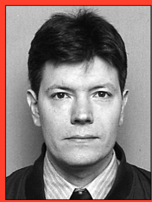
Introduction

Bifurcation control refers to the task of designing a controller to suppress or reduce some existing bifurcation dynamics of a given nonlinear system, thereby achieving some desirable dynamical behaviors. Anti-control of bifurcations, as opposed to the direct control, is to create some intended bifurcations at some preferable time or parameter values by means of various control methods [1]. Typical bifurcation control and anti-control objectives include delaying the occurrence of an inherent bifurcation, introducing a new bifurcation phenomenon at a preferable time or parameter value, changing the parameter set or values of an existing bifurcation point, modifying the shape or type of a bifurcation chain, stabilizing a bifurcated solution or branch, monitoring the multiplicity, amplitude and/or frequency of some limit cycles emerging from a bifurcation mechanism, optimizing the system performance near a bifurcation point, or a combination of some of these objectives [2].

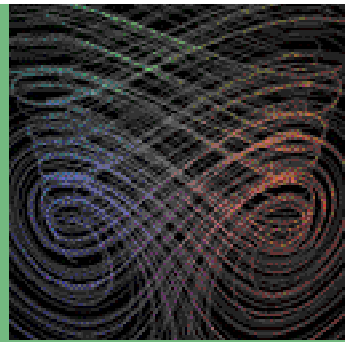
It is now known that bifurcation properties of a system can be modified via various feedback control methods. Representative approaches employ linear or nonlinear state-feedback controls, apply a washout filter-aided dynamic feedback controller, use harmonic balance approximations in (time-delayed) feedback, utilize quadratic invariants in normal forms, and so forth.

Bifurcation control and anti-control with various objectives have been implemented in some experimental systems or tested by using numerical simulations in a great number of engineering, biological, and physiochemical systems. Examples can be found in electrical, mechanical, chemical, and aeronautical engineering, as well as in biology, physics, chemistry, and meteorology, to name just a few. Bifurcation control not only is important in its own right, but also suggests a viable and effective way for chaos control [3], because bifurcation and chaos are usually “twins”; and, in particular, period-doubling bifurcation is a typical route to chaos in many nonlinear dynamical systems [1].


Guanrong Chen


Jorge L. Moiola


Hua O. Wang



Bifurcations in Nonlinear Circuits and Systems

Even very simple nonlinear circuits are rich sources of bifurcation phenomena. Chua's circuit is a typical example. In fact, there are many nonlinear circuits, which need not be very complex in structure, that can display various bifurcation properties. Coupled circuits, circuit arrays, and circuit networks are more interesting but more difficult to analyze and apply. In the study of bifurcations, circuits provide a unique paradigm that is self-unified and self-contained.

Bifurcations certainly exist almost everywhere within the realm of nonlinear dynamical systems, much beyond the territory of circuitry. For instance, power systems generally have rich bifurcation phenomena. In particular, when the consumer demand for power reaches its peaks, the dynamics of an electric power network may move to its stability margin, leading to oscillations and bifurcations. This may quickly result in voltage collapse. Such chaotic networks include some cellular neural networks, laser networks, and communication networks.

Mechanical systems provide another playground for bifurcations. A road vehicle under steering control can have Hopf bifurcation when it loses stability, which may also develop chaos and even hyperchaos. A hopping robot, even a simple two-degree-of-freedom flexible robot arm, can produce unusual vibrations and undergo period-doubling bifurcations which eventually lead to chaos. An aircraft stalling during flight, either below a critical speed or over a critical angle-of-attack, can lead to various bifurcations. Dynamics of aeroengine compressors, vehicles, ships, and so forth. can exhibit bifurcations according to vibration or wave frequencies that are close to the natural frequency of the machine, creating oscillations, bifurcations, and chaotic motions that may cause catastrophe. Many chemical reaction and fluid dynamic processes also have similar behaviors, not to mention weather dynamics and biological population dynamics.

Bifurcations are ubiquitous in physical systems, even subject to controls. It is now known that various bifurcations can occur in many nonlinear systems including, perhaps unexpectedly, some closed-loop systems under feedback or adaptive controls. This seems to be counterintuitive; however, local instability and complex dynamical behavior can indeed result from such controlled systems. Chances are, in these systems, one or more poles of the closed-loop transfer function of the linearized system may move to cross over the stability boundary when feedback or adaptation mechanisms are not robust enough, potentially leading to signal divergence as the control process continues. This sometimes may not lead to a global unboundedness in a complex nonlinear system, but rather, to some local self-excited oscillations, bifurcations, and even chaos. Examples include the popular automatic gain control loops and various controlled or uncontrolled pendula. Adaptive control systems, on the other hand, are more likely to produce bifurcations due to stability changes of some system components such as the estimator and the adaptor.

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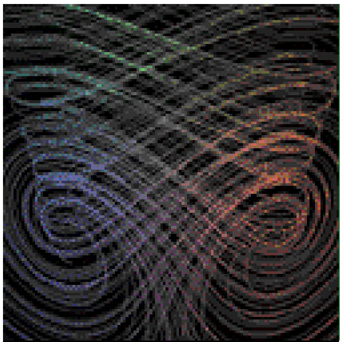

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Challenges from Bifurcation Control — Two Examples

Controlling and anti-controlling bifurcations have foreseen a tremendous impact on real-world applications; and their significance in both dynamics analysis and systems control will not only be enormous, but actually be both profound and far-reaching.

Before getting into more technical details, it is illuminating to discuss some control problems of two representative examples — the discrete-time logistic map and a continuous-time model of an electric power system — to appreciate the challenge of bifurcation control and anti-control.

The logistic map

The well-known logistic map is described by

$$x_{k+1} = f(x_k, p) := p x_k (1 - x_k),$$

where $p > 0$ is a real variable parameter. This map has two equilibria, $x^* = 0$ and $x^* = (p-1)/p$.

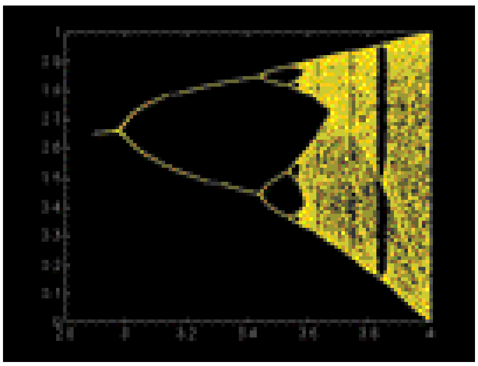
With $0 < p < 1$, the point $x^* = 0$ is stable. However, it is interesting to observe that, for $1 < p < 3$, all initial points of the map converge to $x^* = (p-1)/p$ in the limit. The dynamical evolution of the system behavior, as p is gradually increased from 3.0 to 4.0 by small steps, is shown in Fig. 1. This figure (the bifurcation diagram) shows that at $p = 3$, a stable period-two orbit is born out of x^*, which becomes unstable at the moment, so that in addition to 0 there emerge two more stable equilibria:

$$x^{*1,*2} = \left(1 + p \pm \sqrt{p^2 - 2p - 3}\right) \Big/ (2p)$$

When p increases to the value of $1 + \sqrt{6} = 3.44948\ldots$, each of these two points bifurcates into two new points, as can be seen from the figure. These four points together constitute a period-four solution of the map (at $p = 1 + \sqrt{6}$). As p moves through a sequence of values: $3.54409\ldots$, $3.5644\ldots$, $\cdots$, an infinite series of bifurcations is created by such *period-doubling*, which eventually leads to chaos:

$$\text{period } 1 \rightarrow \text{period } 2 \rightarrow \text{period } 4 \rightarrow \cdots \rightarrow \text{period } 2^k \rightarrow \cdots \rightarrow \text{chaos}$$

Figure 1. Period-doubling of the logistic map.



At this point, several control oriented problems may be asked: Is it possible (and, if so, how) to find a simple (say, linear) control sequence, $\{u_k\}$, to be added to the right-hand side of the logistic map, such that

(i) the limiting chaotic behavior of the period-doubling bifurcation process is suppressed?

(ii) the first bifurcation is delayed, or this and the subsequent bifurcations are changed either in form or in stability?

(iii) the asymptotic behavior of the system becomes chaotic (if chaos is beneficial), for a parameter value of p that is not in the chaotic region without control?

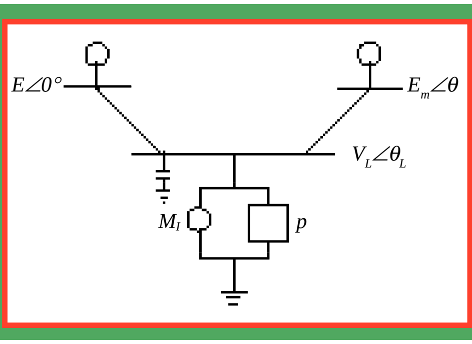
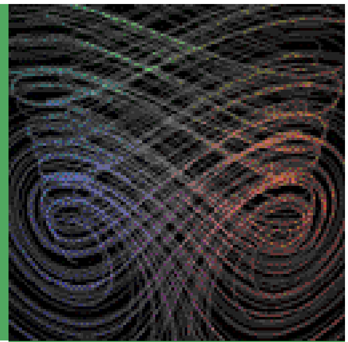


Figure 2. A simple electric power system.



An electric power model

A simple yet representative electric power system is shown in Fig.2, where θ is the rotational angle of the power generator. In this power system, the load is represented by an induction motor, M_I, in parallel with a constant PQ (active-reactive) load. The variable reactive power demand, p, at the load bus is used as the primary system parameter. Also in the power system, the load voltage is $V_L\angle\theta_L$, with magnitude V_L and angle θ_L, the slack bus has terminal voltage $E\angle 0^o$ (a phasor), and the generator has terminal voltage denoted $E_m\angle\theta$.

When the system parameter p is gradually increased or decreased, with appropriate values of the other system parameters, very complex dynamical phenomena can be observed [4]. These are shown in Fig. 3, where

on the left-hand side:

- p=10.818, a turning point of periodic orbit occurs;
- p=10.873, first period-doubling bifurcation occurs;
- p=10.882, second period-doubling bifurcation occurs;
- p=10.946, a subcritical Hopf bifurcation occurs;

on the right-hand side:

- p=11.410, a saddle-node bifurcation occurs;
- p=11.407, a supercritical Hopf bifurcation occurs;
- p=11.389, first period-doubling bifurcation occurs;
- p=10.384, second period-doubling bifurcation occurs.

In this figure, (1) denotes stable equilibria, (2) stable limit cycles, (3) and (4) different types of unstable equilibria, and (5) and (6) different types of unstable limit cycles. The dynamics of this system, with a varying second parameter (machine damping), have shown the connection of the two Hopf bifurcation points with a degenerate Hopf bifurcation and the disappearance of the chaotic behavior.

Similar to the logistic map discussed above, a few interesting control problems are:

(i) can the limiting chaotic behavior of the period-doubling bifurcation process be suppressed?

(ii) can the first bifurcation be delayed in occurrence, or this and the subsequent bifurcations be changed either in form or in stability?

(iii) can the voltage collapse be avoided or delayed through bifurcation or chaos control?

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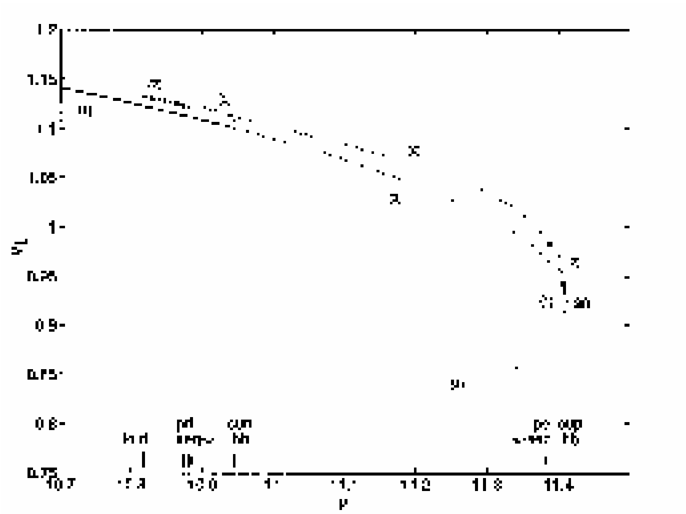
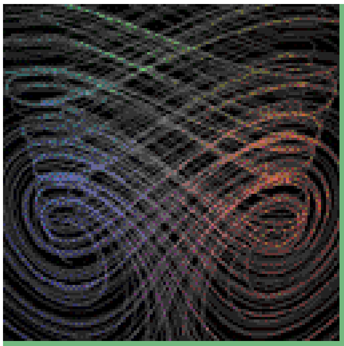


Figure 3. Bifurcation diagram of the power system. Bifurcation positions are marked on the x-axis. Solution types: (1) Stable equil. (2) Stable limit cycle (period-1.) (3) Type-2 unstable equil. (4) Type-1 unstable equil. (5) Unstable limit cycle from cyclic fold. (6) Unstable limit cycle from p.d.



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Nonconventional control problems like these have posed a real challenge to dynamics analysts, control engineers, and circuit specialists.

Various Bifurcation Control Methods

As mentioned above, bifurcations can be modified (controlled or anti-controlled) via various feedback control methods. Representative approaches employ linear or nonlinear state-feedback controls, apply a washout filter-aided dynamic feedback controller, use harmonic balance approximations in (time-delayed) feedback, utilize quadratic invariants in normal forms, and so forth.

State feedback controls

State feedback can be used for determining the one-dimensional transcritical, pitchfork, and saddle-node types of bifurcations, as well as the stabilities of the equilibria. The period-doubling bifurcation can also be controlled in a similar way.

Hopf bifurcation exists in higher-dimensional (≥ 2) systems, but can also be controlled by state feedback. A Hopf bifurcation corresponds to the situation where, as the parameter p is varied to pass a critical value p_0, the system Jacobian has one pair of complex conjugate eigenvalues moving from the left-half plane to the right, crossing the imaginary axis, while all the other eigenvalues remain stable. At that moment of crossing, the real parts of the two eigenvalues become zero, and the stability of the existing equilibrium changes from being stable to unsable. Also, at the moment of crossing, a limit cycle is born. These phenomena are completely characterized by the classical result of the Hopf bifurcation theorem.

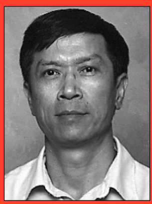
To design controllers for a bifurcation modification purpose, Taylor expansion, and sometimes linearization, of the given nonlinear dynamical system is a common approach. Because bifurcations are closely related by the eigenvalues of the linearized model, controlling the behaviors of these eigenvalues in an appropriate way is key to many bifurcation control objectives.

Controllers designed based on normal forms

The general theory of bifurcations in nonlinear dynamical systems is built on the basis of normal forms. Systems with the same normal form have equivalent bifurcations. Therefore, bifurcations can be classified according to equivalent systems in normal forms. Thus, development of a systematic design technique for bifurcation control requires a unified basis — a set of normal forms for control systems.

A set of normal forms is a family of simple nonlinear control systems, such that many systems in a general form can be transformed into a unique system in that family. For dynamical systems without control, Poincaré developed a framework of normal forms for autonomous systems. The normal form theory for control systems differs from the theory of Poincaré in the following two aspects:

(i) In a dynamical system without control, a single vector field is involved. However, there are two vector fields (the nonlinear system **f** and the nonlinear control gain **g**) in a controlled system to be simplified simultaneously.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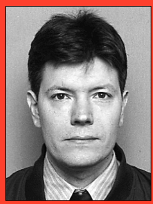
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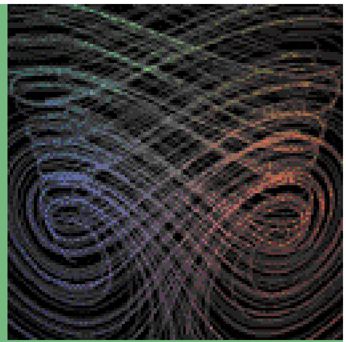

(ii) In the Poincaré theory of normal forms, the transformations used are changes of coordinates. The transformation group for control systems consists of both changes of coordinates and state feedbacks.

Because of these two differences, the study of bifurcations for control systems requires a set of normal forms for both functions **f** and **g**, under the transformation group consisting of changes of coordinates as well as state feedbacks. This poses some real challenges for further studies.

Controls via harmonic balance approximations

As is well known, limit cycles are associated with bifurcations. In fact, one type of degenerate (or singular) Hopf bifurcations determines the appearance of multiple limit cycles under system parameter variation. Therefore, the birth and the amplitudes of multiple limit cycles can be controlled by monitoring the corresponding degenerate Hopf bifurcations. This task can be accomplished in the frequency domain setting [5].

For continuous-time systems, limit cycles generally do not have analytic forms, and so have to be approximated in applications. For this purpose, the harmonic balance approximation technique is very efficient. This technique is useful in controlling bifurcations, such as for delaying and stabilizing the onset of period-doubling bifurcations.

As an example, consider again the electric power model shown in Fig. 2. The amplitudes of the system limit cycles can be controlled (e.g., to zero) by using a state-feedback controller designed based on this frequency domain approach employing the harmonic balance approximation technique.

To show a simple application of changing the multiplicities of limit cycles, consider the following system studied in [6]:

$$\begin{aligned}\dot{x}_1 &= -x_2 + \lambda x_1 + (a - w - \theta)x_1^3 + (3\mu - \eta)x_1^2 x_2 \\ &\quad + (3\theta + \xi - 3w - 2a)x_1 x_2^2 + (\nu - \mu)x_2^3, \\ \dot{x}_2 &= x_1 + \lambda x_2 + (\nu + \mu)x_1^3 + (3w + 3\theta + 2a)x_1^2 x_2 \\ &\quad + (\eta - 3\mu)x_1 x_2^2 + (w - \theta - a)x_2^3,\end{aligned}$$

where λ plays the role of the main bifurcation parameter, and a, w, θ μ, η, ξ and ν are auxiliary parameters. Notice that the Hopf bifurcation conditions are met when $\lambda = 0$ at the origin. Take the following parameter set: $a = 2.0$, $w = 1.0$, $\theta = -1.0$, $\mu = 1.0$, $\eta = 1.0$, $\xi = -0.5$ and $\nu = -1.0$. When the main bifurcation control parameter $\lambda = 0.0003$, $\lambda = -0.002$ and $\lambda = 0.002$ there are three, two and one limit cycles, as depicted in Figs. 4, 5, and 6, respectively, which were generated by software available in the literature [7]. The complete picture of the periodic branch of limit cycles, when the main bifurcation parameter λ is varied, is shown in Fig. 7. In this figure, two fold points of periodic solutions (also called saddle-node bifurcations of cycles) are depicted, showing the coalescence of stable and unstable limit cycles. The limit cycles are colored yellow (stable) and red (unstable), respectively, for clarity.

Potential Applications of Bifurcation Control

Bifurcation control is useful in many engineering applications. Due to the vast

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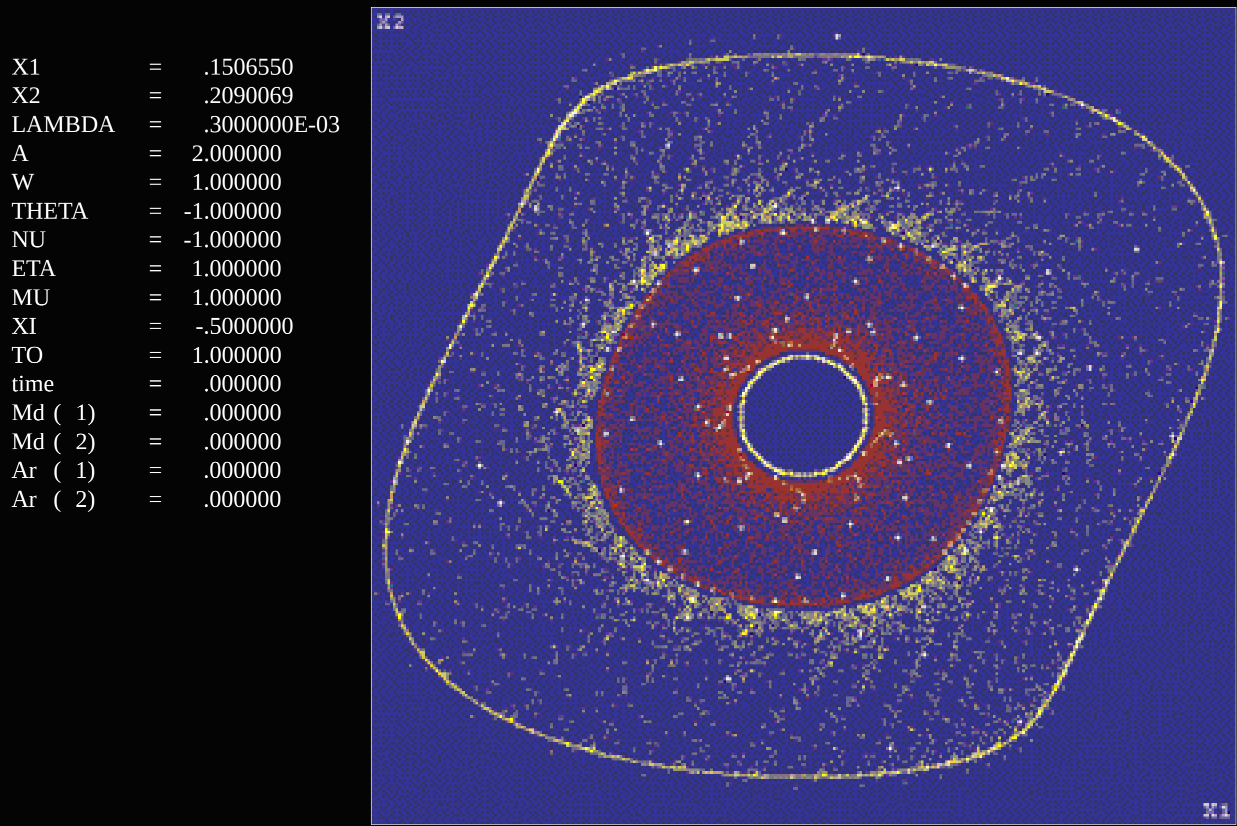


Figure 4. Three limit cycles in the Sibirskii example. The large and the small cycles (in yellow) are stable, while the "in-between" cycle (in red) is unstable. The equilibrium point at the origin is an unstable focus ($-0.5 \leq x_1 \leq 0.5$; $-0.5 \leq x_2 \leq 0.5$).

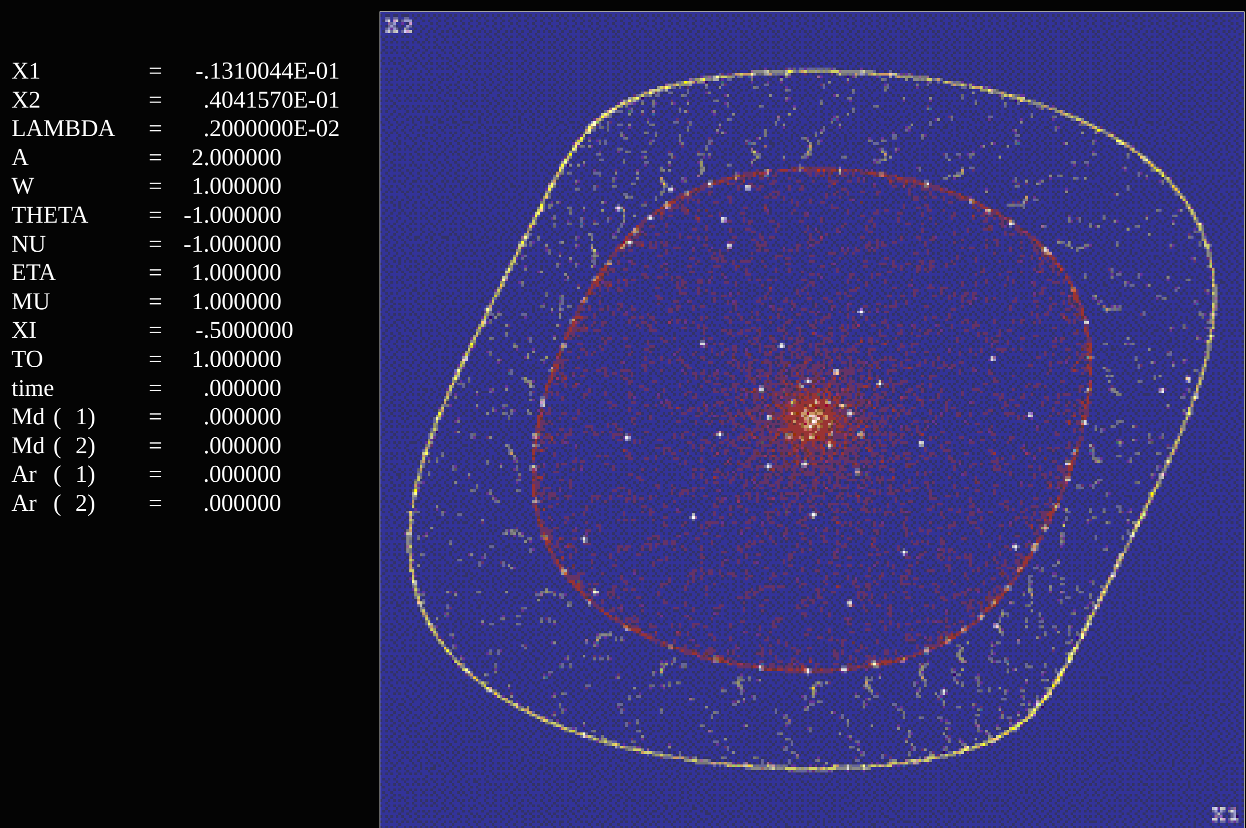
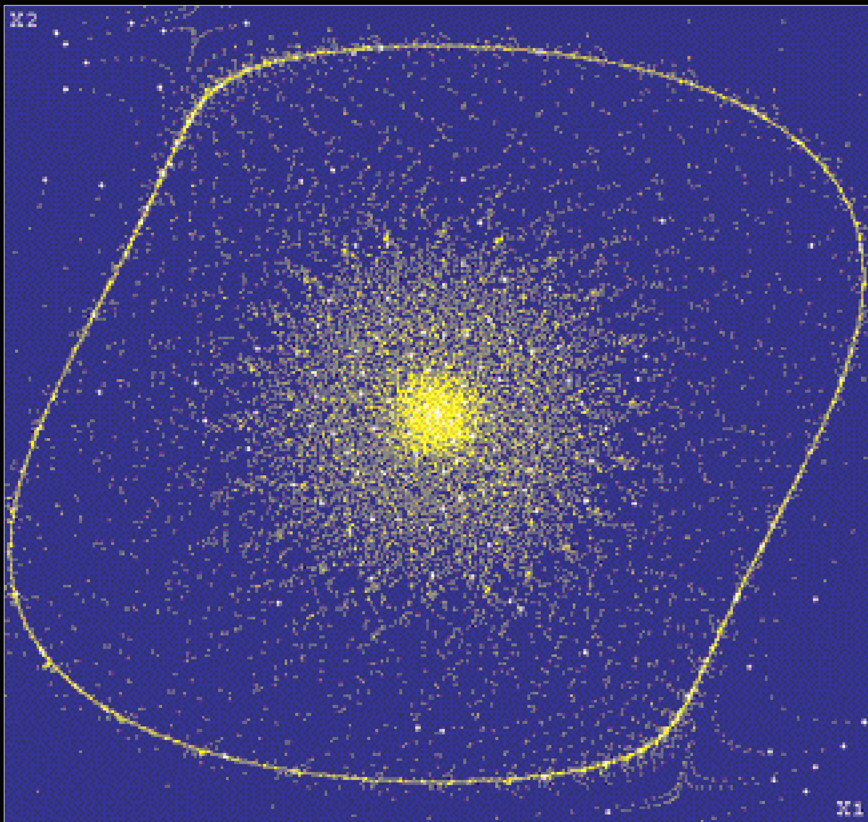
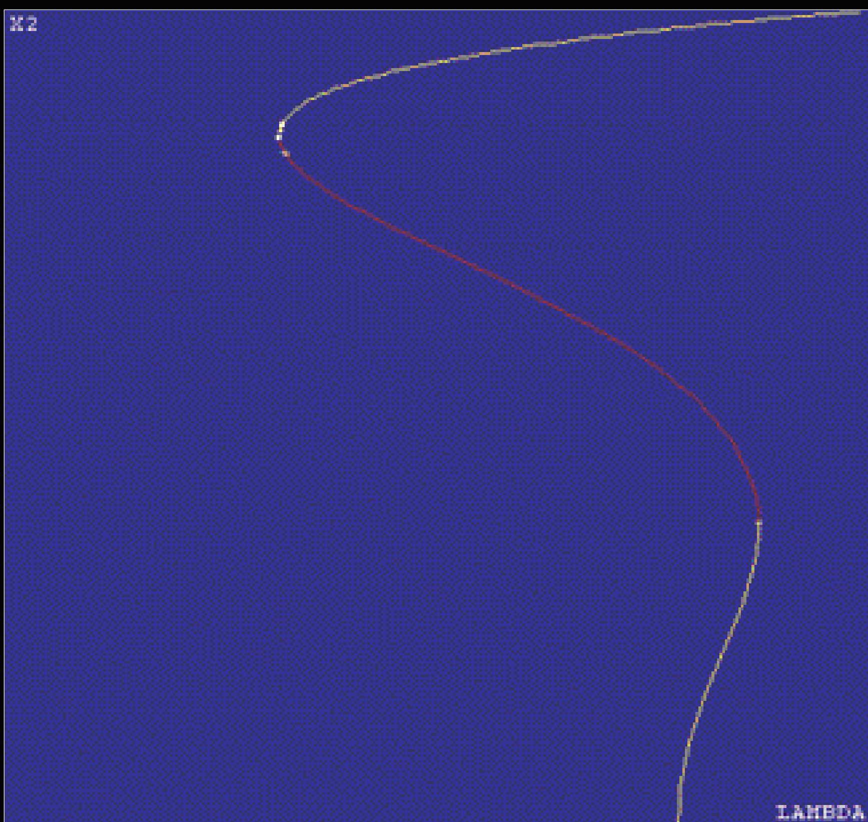


Figure 5. Two limit cycles in the Sibirskii example. The large cycle (in yellow) is stable, while the small cycle (in red) is unstable. The equilibrium point at the origin is a stable focus. Compared to the previous figure, the inner stable limit cycle has disappeared under the Hopf bifurcation mechanism ($-0.5 \leq x_1 \leq 0.5$; $-0.5 \leq x_2 \leq 0.5$).



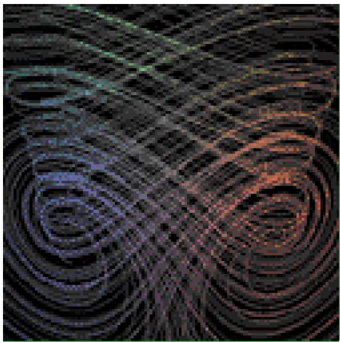
X1	=	.4061135
X2	=	.4307159
LAMBDA	=	.2000000E-02
A	=	2.000000
W	=	1.000000
THETA	=	-1.000000
NU	=	-1.000000
ETA	=	1.000000
MU	=	1.000000
XI	=	-.5000000
TO	=	1.000000
time	=	.000000
Md (1)	=	.000000
Md (2)	=	.000000
Ar (1)	=	.000000
Ar (2)	=	.000000

Figure 6. One stable (large amplitude) limit cycle in the Sibirskii example. The two interior cycles of Fig. 4 have disappeared under a saddle-node bifurcation of cycles ($-0.5 \le x_1 \le 0.5$; $-0.5 \le x_2 \le 0.5$).



X1	=	-.2585438E-01
X2	=	.1670138
LAMBDA	=	.8538568E-03
A	=	2.000000
W	=	1.000000
THETA	=	-1.000000
NU	=	-1.000000
ETA	=	1.000000
MU	=	1.000000
XI	=	-.5000000
TO	=	6.265448
time	=	.000000
Md (1)	=	.000000
Md (2)	=	.000000
Ar (1)	=	.000000
Ar (2)	=	.000000

Figure 7. The continuation of periodic solutions varying the bifurcation parameter λ. Stable limit cycles are marked in yellow while unstable ones are depicted in red. The collisions of yellow and red curves denote saddle-node bifurcations of cycles ($-0.007 \le \lambda \le 0.002$; $0 \le x_2 \le 0.45$).



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and still rapidly growing array of information on potential applications of bifurcation control in engineering systems, it is literally impossible to give an all-rounded and comprehensive coverage of these materials in one single section of this article. Therefore, only a few selected topics are presented here.

Application in power network control and stabilization

Nonlinearity is an inherent and essential characteristic of electric power systems, especially in heavily loaded operation. Historically, power systems were designed and operated conservatively and, as a result, systems were normally operated within a region where system dynamical behaviors were fairly linear. Only occasionally would systems be forced to the limits where nonlinearities could begin to have significant impacts on the system behaviors.

Notably, the recent trend shows different promises. Economic and environmental factors, along with the current trend toward an open access market, have strongly demanded that power systems be operated much closer to their limits as they become more heavily loaded. Ultimately, there will be greater dependence on control methods that can enable the system capability rather than on expensive physical system expansion. It is therefore vital to gain greater understanding of the nonlinear phenomena of an operational power system.

In studying the electric power system shown in Fig. 2, voltage collapse refers to an event in which the voltage magnitudes

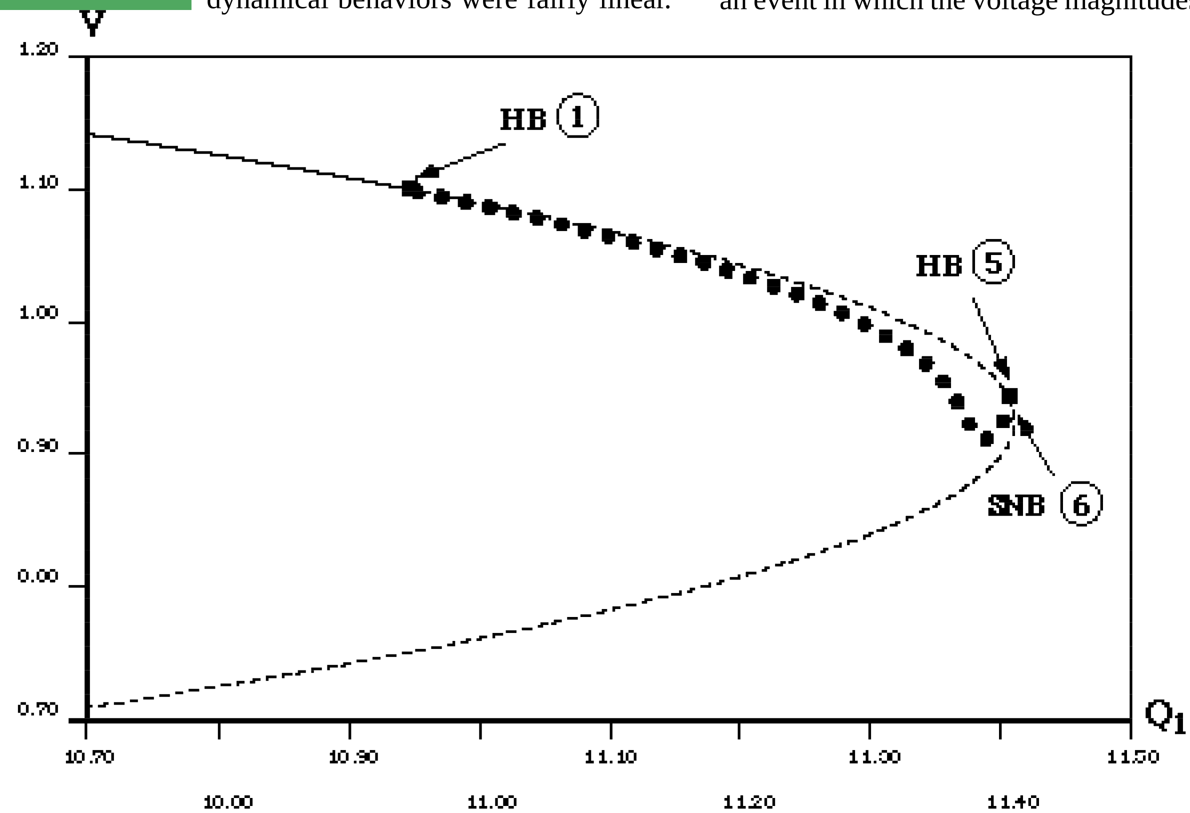
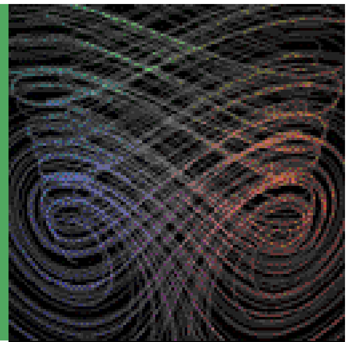


Figure 8. Bifurcation control of voltage collapse.



in AC power systems decline to some unacceptably low levels that can lead to system blackout. The power system model exhibits rich nonlinear phenomena, including bifurcations and chaos.

One bifurcation control approach to the problem of controlling voltage collapse in this power system model is to add a control u to the system, where the control occurs in the excitation system and involves a purely electrical controller [8, 9]. Feedback signals, which are some dynamic functions of the angular velocity $\dot{\theta}$, are widely used in power system stabilizers (PSS). A nonlinear bifurcation control law of the form $u = k_n\dot{\theta}^3$ transforms the subcritical Hopf bifurcation to a supercritical bifurcation. It also ensures a sufficient degree of stability of the bifurcated periodic solutions, so that chaos and crises are eliminated. This control law allows stable operation very close to the point of impending collapse (saddle-node bifurcation). Figure 8 shows a bifurcation diagram for the closed-loop system with control gain $k_n = 0.5$, where HB indicates Hopf bifurcation and dark circles mean stable oscillations.

Another linear bifurcation control law, $u = k_l\dot{\theta}$ involves changing the critical parameter value, at which the Hopf bifurcations occur, by a linear feedback control. This linear feedback law eliminates the Hopf bifurcations and the resulting chaos and crises [8, 9]. Therefore, the linearly controlled system can operate at a stable equilibrium up to the saddle node bifurcation.

In summary, although the relative importance of the effects of the nonlinear phenomena in general power systems under stressed conditions is still a topic for further research, the bifurcation control approach appears to be a viable tech-

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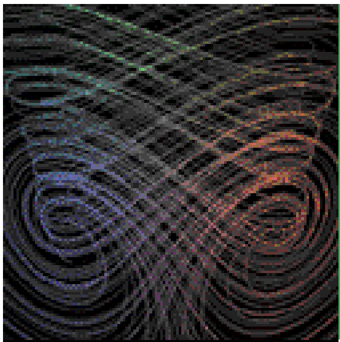
Guanrong (Ron) Chen received the M.Sc. degree in computer science from the Sun Yatsen (Zhongshan) University, China, in 1981, and the Ph.D. degree in applied mathematics from Texas A&M University in 1987. He is a fellow of the IEEE, since 1997, for his fundamental contributions to the theory and applications of chaos control and bifurcation analysis. He served as associate editor for the *IEEE Transactions on Circuits and Systems—I* (1993–1995; 1999–2001) and for the Chinese Academy of Sciences *Journal in Control Theory and Applications* since 1995. He also served as guest editor for the *International Journal of Bifurcation and Chaos* and several other journals. Recently, he is the chair-elect of the Nonlinear Circuits and Systems Technical Committee of the IEEE Circuits and Systems Society, 1999–2000.

Jorge L. Moiola received the M.Sc. in electrical engineering from the University of Houston, in 1991, and the Ph.D. from the Universidad Nacional del Sur (UNS), Bahia Blanca, in 1992. Since then he has been with the Department of Electrical Engineering at UNS, where he is currently assistant professor. In 1995 he was with the Department of Electrical and Computer Engineering, University of Houston, as a visiting scholar. In 1998 he was a Fulbright scholar at the University of California at Berkeley. His primary research interests are in the field of nonlinear oscillations and bifurcations related to control theory and applications. He currently is the chief editor of the *Latin American Applied Research Journal*.

Hua Wang received the Ph.D. degree from the University of Maryland at College Park in 1993. From 1993 to 1996, he was an associate research engineer/scientist at the United Technologies Research Center (UTRC), East Hartford, Connecticut. Since September 1996, he has been with Duke University, where he is an assistant professor in the Department of Electrical and Computer Engineering. Dr. Wang is a recipient of the 1994 O. Hugo Schuck Best Paper Award of the American Automatic Control Council, the 1999 IFAC Congress Poster Paper Prize of the Fourteenth Triennial World Congress of the International Federation of Automatic Control (IFAC), several publication awards, and the High Impact Performer recognition from the UTRC. His research interests include nonlinear dynamics and control, intelligent control, multisensor fusion, and biomedical control applications.



Bifurcations: Control and Anti-Control

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nique for controlling these systems.

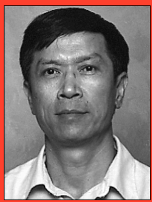
Applications in axial flow compressors and jet engine control

Another application of bifurcation control is in the hearts of aeroengines: the axial flow compressors. Recent years have witnessed a flurry of research activities in axial flow compressor dynamics, both in terms of analysis of stall phenomena and their control. This interest is due to the increased performance that is potentially achievable in modern gas turbine jet engines by operating near the maximum pressure rise. The increased performance comes at the price of a significantly reduced stability margin. Specifically, axial flow compressors are subject to two distinct aerodynamic instabilities, rotating stall and surge, which are associated with bifurcations. Both of these instabilities are disruptions of the normal operating condition that is designed for steady and axisymmetric flow, and both can bring catastrophic consequences to jet airplanes. Because these instabilities occur at the critical operating point of the highest pressure rise, the compressors are forced to operate at a much lower pressure rise in order to provide adequate stability margin, which limits greatly the performance of axial flow compressors.

Due to the design constraint, there has been much work on enhancing compression system stability using active control. Many of the early control strategies were designed to extend the stable axisymmetric operating range by delaying the onset of stall. The application of bifurcation control to compression systems has initiated a promising paradigm aiming at solving this challenging problem. These bifurcation control approaches look for controllers to enhance the operability of the compression system by modifying the nonlinear stability characteristics of the compression system. Using the popular third-order Moore-Greitzer model, it was found that the first stalled flow solution is born through a subcritical bifurcation. The practical importance of the subcritical stall bifurcation is that when the axisymmetric flow operating point becomes subject to perturbations, the system will jump to a large-amplitude, fully developed stall cell. Subcritical bifurcations also imply hysteresis, and so returning the throttle to its original position may not bring the system out of stall.

One control strategy seeks to transform the hard subcritical bifurcation at the onset of stall into a soft supercritical bifurcation, thereby eliminating the hysteresis associated with rotating stall. The compressor stall application is an excellent example for illustration (both theory and experimental validation) of a guiding philosophy in bifurcation control. It relates to stabilization, or “softening”, of bifurcations, with implications to improving system performance and robustness. Other approaches employ more conventional control approaches such as the backstepping technique to arrive at control laws for surge and rotating stall.

Some other techniques for bifurcation control of compression systems involve output feedback, under the assumption that the unstable modes corresponding to the critical eigenvalue of the linearized system are not linearly controllable. Some stabilizability conditions can be derived for the situation where the critical mode is lin-



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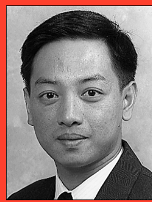

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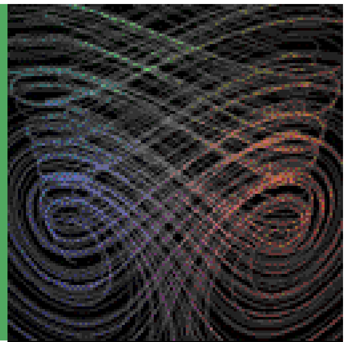

early observable through output measurement that includes state-feedback as a special case. It is shown that linear controllers are adequate for stabilization of transcritical bifurcation, and quadratic controllers are adequate for stabilization of pitchfork and Hopf bifurcations, respectively.

Application in cardiac alternans and rhythms control

One interesting application of bifurcation control is the control of pathological heart rhythms. The rhythm of the heart is determined by a wave of electrical impulses (in the form of action potential), which travels in the heart condition pathway. Arrhythmias in the heart such as fibrillation and ectopic foci are life threatening. Understanding the mechanism leading to arrhythmias is an important medical problem with enormous impact. Within this context, an even more challenging problem is the control and curing of such abnormal biological disorders. For a control engineer, a natural question is concerned with the role of feedback in such situations. From a bifurcation control point of view, what is interesting about arrhythmias is that they have been closely linked to a variety of bifurcations, both static and dynamic, and chaos. This connection enables bifurcation control methods to be used for controlling heart rhythms.

As an application, dynamic bifurcation control has been applied to suppression of pathological rhythm (cardiac alternans) in an atrioventricular modal conduction model. It has been shown that this theoretical model, which incorporates physiological concepts of recovery, facilitation and fatigue, can accurately predict a variety of experimentally observed complex rhythms of nodal conduction.

Other examples of bifurcation control applications

A list of potential applications of bifurcation control can be continued. In some physical systems, such as the stressed system, delay of bifurcations offers an opportunity to obtain stable operating conditions for the machine beyond the margin of operability at the normal situation. Sometimes, it is desirable that the stability of bifurcated limit cycles can be modified, with application to some conventional control problems such as thermal convection experiments. Other examples include stabilization via bifurcation control in tethered satellites and magnetic bearing systems; delay of bifurcation in rotating chains via external periodic forcing, and in various mechanical systems such as robotics and electronic systems such as laser machines and nonlinear circuits.

To Probe Further

When leaving the idealized mathematical domain and looking around the natural world, one certainly finds a very interesting and realistic phenomenon — there is almost nothing that is linear but is not man-made out there, is there? The nonlinear nature of the real world, and of the real life, have brought up a great number of technological challenges to scientists and engineers—the most difficult yet also most exciting complexities in dynamics, for which bifurcations, chaos, and fractals alike all get to interplay within a common ground of the mathematical as well as physical wonderland.

The field of bifurcation control is still very much in a rapidly evolving phase. This is the case not only in deeper

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and wider theoretical studies but also in many newly found real-world applications. It calls for further efforts and endeavors from the communities of engineering, physics, applied mathematics, and biological as well as medical sciences. New results and new publications on the subject of bifurcation control continue to appear, leaving a door wide open to every individual who has the desire and courage to pursue further in this stimulating and promising direction of new research.

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…Shlomo Karni

Ohm is where the heart is,
and therefore you are …
(to be continued).

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