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*Invited Paper*

# Pinning Control and Synchronization on Complex Dynamical Networks

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**Abstract:** This article offers a survey of the recent research advances in pinning control and pinning synchronization on complex dynamical networks. The emphasis is on research ideas and theoretical developments. Some technical details, if deemed necessary for clarity, will be outlined as well.

**Keywords:** Complex network, controllability, pinning control, synchronizability, synchronization.

## 1. INTRODUCTION

The study of modern network science can be traced back to 1736, when Euler solved the interesting Königsberg seven bridge problem thereby laying down the foundation of graph theory [1]. A unified framework for in-depth studying the subject is the random graph theory, established by Erdös-Rényi in the late 1950s [2]. More recently, Watts-Strogatz described their small-world network model in *Nature* in 1998 [3], and Barabási-Albert formulated their scale-free network model in *Science* in 1999 [4], which together had marked a new milestone in the network science development, stimulating a great deal of interests and efforts in pursuing networks theory and its applications. All this is particularly significant in the present big-data era. In fact, network science and engineering has become a self-contained and standalone research paradigm in the realm of science and technology today.

Control theory, on the other hand, is a relatively well-established subject on systems science and engineering, with a rapid development since the 1960s under the unified state-space framework attributed to Kalman (see, e.g., [5]). Controllability, in particular, is the core notion of the whole theory because it determines if a system is controllable and, if not, under what conditions one can make it controllable. It has a dual concept, the observability, and a core component of optimal control [5], as well as many other important relevant issues such as stability and synchronization.

Network science and control theory are gradually merged through their individual and interactive developments. Today, control theory concerns more and more with controlling networks (e.g., power grids, robot teams, traffic networks) and networked control namely executing control tasks through wired and wireless networks (e.g., communication networks, ethernets, sensor networks). Meanwhile, more and more large-scale networks are embedded with various controllers. However, classical control theory typically focuses on control problems and methods for a single albeit higher-dimensional dynamical system, paying little attention to directed and irregularly-connected networks of many of such dynamical systems. Although a dynamical system as the underlying platform for control can be very complex (higher-dimensional, stochastic and nonlinear), it was not being investigated in a networking framework, especially not emphasizing on its internal topological connectivity and directionality. This was due mainly to historical especially technical reasons, because half a century ago there were no technical supports and demands from things like today's Internet, wireless communication networks, power grids, global transportation systems, biological gene regulation networks, and so on; and there were no facilities and resources like today's supercomputers, huge databases, GPS services and cloud computing environments. As a result, there were very few research activities and achievements in control theory formulated in the complex dynamical network setting in the past until very recently.

The current rapid development of network science and engineering has created a corpus of new opportunities as well as challenges to classical control systems theory. Usually, a dynamical network is considered complex if it is large-sized (with many nodes and edges), higher-dimensional (every node is a higher-dimensional dynamical system) and connected in an indefinite or irregular manner (such as random, small-world or scale-free structures), especially with nonlinearity in a time-varying (growing, evolving, impulsive, time-delayed) form, and even in multiple spatiotemporal scales. For such a complex network of nonlinear dynamical systems, it is obviously expensive and practically impossible to control all its nodes (dynamical systems) to achieve a certain desired objective. Hinted by this observation, the concept and notion of pinning control were introduced in [6,7], as an effective control strategy that takes into account both the node dynamics and the network

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topology in control systems design and implementation.

The idea of pinning control was motivated particularly by biology. One example is about the worm C. elegans, which has a simple but rather complete neural network in its body with statistically some 300 neurons and some 5000 synapse connections depending on the age and size of the warm in consideration. For such a worm, the question of stimulating (controlling) how many neurons one can expect to provoke its whole body has a rather surprising answer - 49 on average [8] - less than 17% of the total. Another example is about fish schools and bee hives who migrate to forage [9]: "Relatively few informed individuals within fish schools are known to be able to influence the foraging behaviour of the group and the ability of a school to navigate towards a target. Similarly, very few individuals (approximately 5%) within honeybee swarms can guide the group to a new nest site."

From a control theoretic point of view, these approximately 17% of neurons and 5% of bees can be seen as controlled individuals through which the entire network will be manageable. Such a control strategy is obviously very efficient and economical. Inspired by these examples, a sensible question arises naturally: to achieve some objective on a given and fixed network of dynamical systems, how many nodes one needs to control and at which nodes to apply the controllers can achieve a pre-designated objective most effectively? To answer this sort of questions, the so-called pinning control strategy was introduced in [6,7], aiming at developing an effective control approach that can "pull one hair to move the whole body."

To be more precise, suppose that a directed network has been given, which has a certain structure (e.g., in a scale-free topology) with nodes being some higher-dimensional nonlinear dynamical systems. Suppose also that an objective has been assigned (e.g., to achieve synchronization) with some optimality requirements (e.g., using shortest time, consuming minimum energy, having smallest oscillations) and, moreover, assume that the type of controllers has been determined (e.g., linear state-feedback controllers). Under this well-defined framework, a typical control theoretic problem is: how many controllers are needed and where to locate them can achieve the control objective with an optimal performance? This problem is referred to as *pinning control*, which means to find how many controllers to pin and where to pin them in the given network of dynamical systems can achieve the control objective most effectively.

It is clear that any answer to the above question depends upon the structure (e.g., regular, random-graph, small-world, scale-free topology) of the given network and its node systems (e.g., nonlinear, impulsive, hybrid dynamics). It can be easily imagined that there are some other questions of similar nature that can be formulated for complex dynamical networks, while these kinds of questions would not be asked by, or did not even exist in classical control theory. It is also clear that to answer such questions is by no means easy, for which the classical control theory is likely insufficient. This motivates the current attempts to extend the control theory and practice from complex systems to complex networks of such systems.

This article will not survey on complex network synchronization alone, which is a huge subject in its own right (see, e.g., [1,10]), but will use synchronization as the control objective to illustrate the notion of pinning control of complex dynamical networks, thereby presenting the state-of-the-art development of pinning-controlled synchronization on such networks. For this purpose, the concept and model of network synchronization are first briefly introduced in the next section.

## 2. A GENERAL MODEL FOR NETWORK SYNCHRONIZATION

A general undirected and unweighted continuous-time dynamical network of $N$ identical nodes can be described by

$$\dot{x}_i = f(x_i) + c\sum_{j=1}^{N} a_{ij} H(x_j), \quad i = 1,2,...,N, \tag{1}$$

where $f(\cdot)$ is a nonlinear function typically satisfying a local Lipschitz condition namely $\| f(x) - f(y) \| \le \rho \| x - y \|$ for some constant $\rho > 0$ and for all $x, y$ in their confined domain in $R^n$ (which is global if for all $x, y \in R^n$), $x_i = [x_i^{(1)}, x_i^{(2)}, \cdots, x_i^{(n)}]^T \in R^n$ is the state vector, constant $c > 0$ is the coupling strength, $H : R^n \to R^n$ is the inner coupling matrix (here, for simplicity, it is assumed to be a constant matrix, i.e., $H(x) = Hx$), and $A = [a_{ij}] \in R^{N \times N}$ is the outer coupling matrix defined as follows: If there is a connection between node $i$ and node $j$ then $a_{ij} = a_{ji} = 1$ otherwise $a_{ij} = a_{ji} = 0$ for all $i \neq j$; the diagonal elements $a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}$ for all $i = 1, 2, ..., N$, which is referred to as a diffusive condition. The Laplacian matrix of the network is $L = -A$, which for a connected network is irreducible with eigenvalues $0 = \lambda_1 < \lambda_2 \le \cdots \le \lambda_N$. Here, $\lambda_2 > 0$ can be used as the algebraic connectivity index of the network. For directed networks, however, $L$ is generally asymmetrical, so its eigenvalues are usually not real but complex values.

Note that the general network model (1) can describe all kinds of complex networks, including such as various regular networks, random-graph networks, small-world networks, and scale-free networks [1].

Network (1) is said to achieve complete (asymptotic) synchronization, if (see, e.g., [11])

$$\lim_{t\to\infty} \left\| x_i(t) - x_j(t) \right\| = 0 \quad \text{for all } i, j = 1,2,...,N, \tag{2}$$

where $\| \cdot \|$ is the Euclidean norm.

Physically, throughout the synchronization process all node states will be continuously governed by the differential equations of the node dynamical systems, therefore if all node states finally reach synchrony then the synchronized state must be one of the evolving states of the node system, namely one solution orbit of the node dynamical system. For this reason, as suggested in [10] (see also [1]), synchronization may also be defined as

$$\lim_{t\to\infty} \| x_i(t)-s(t) \| = 0 \quad \text{for all} \quad i=1,2,...,N \tag{3}$$

for some $s(t)$ satisfying $\dot{s}(t)=f(s(t)),\ s(t)\in R^n$. It is noted that for synchronization, this $s(t)$ is not specified, since (3) always leads to (2); but if this $s(t)$ is specified then (3) becomes a typical "tracking" problem in classical control systems theory.

Mathematically, definition (2) is equivalent to the following definition:

$$\lim_{t\to\infty} \|x_i(t)-\bar{x}(t)\| = 0 \quad \text{for all} \quad i=1,2,...,N, \tag{4}$$

where $\bar{x}(t)=\sum_{i=1}^N \beta_i x_i(t)$, which was first introduced in [12-14] and used by many others later on (see, e.g., [15]), with $[\beta_1,\beta_2,\cdots,\beta_N]^T$ being the left eigenvector of the zero eigenvalue of the network Laplacian matrix $L$. In fact,

$$0 \le \| x_i - \bar{x} \| = \left\| x_i - \sum_{j=1}^N \beta_j x_j \right\| = \left\| \sum_{j=1}^N \beta_j x_i - \sum_{j=1}^N \beta_j x_j \right\|$$

$$= \left\| \sum_{j=1}^N \beta_j (x_i - x_j) \right\| \le \sum_{j=1}^N \beta_j \| x_i - x_j \| \to 0 \quad (t\to\infty)$$

for all $i=1,2,...,N$. Therefore, (2) implies (4). On the other hand,

$$\| x_i - x_j \| = \| x_i - \bar{x} + \bar{x} - x_j \|$$
$$\le \| x_i - \bar{x} \| + \| \bar{x} - x_j \| \le 2\max_{1\le i\le N} \| x_i - \bar{x} \|,$$

which, by taking limits on both sides, shows that (4) implies (2).

Now, one can also easily verify that definition (3) is equivalent to (2) at least for two general cases: (i) $f(\cdot)$ is linear homogeneous in the sense that $f(\sum_{j=1}^N \alpha_j x_j) = \sum_{j=1}^N \alpha_j f(x_j)$ for arbitrary constants $\alpha_i$, $i=1,2,...,N$, which includes linear systems as a special case; (ii) $f(\cdot)$ satisfies a local (or global) Lipschitz condition in the sense that $\| f(x)-f(y) \| \le \rho \| x-y \|$ for some constant $\rho > 0$ and for all $x$, $y$ in their confined domain in $R^n$, as indicated in the network model (1). For these two cases, it can be shown that the two definitions (3) and (4) are indeed equivalent [16]:

(i) If $f(\cdot)$ is linear homogeneous then it can be shown that $\bar{x}(t)=s(t)$, namely, $\dot{\bar{x}}(t)=f(\bar{x}(t))$. Indeed, one has

$$\dot{\bar{x}} = \sum_{j=1}^N \beta_j \dot{x}_j = \sum_{j=1}^N \beta_j \left( f(x_j) + c\sum_{k=1}^N a_{jk} H(x_k) \right)$$

$$= \sum_{j=1}^N \beta_j f(x_j) + c\sum_{j=1}^N\sum_{k=1}^N \beta_j a_{jk} H(x_k)$$

$$= f\left( \sum_{j=1}^N \beta_j x_j \right) + c\,[\beta_1,\beta_2,\cdots,\beta_N]\,A \begin{bmatrix} H(x_1) \\ H(x_2) \\ \vdots \\ H(x_N) \end{bmatrix}.$$

Since $[\beta_1,\beta_2,\cdots,\beta_N]^T$ is the left eigenvector of the zero eigenvalue of matrix $L$, one has $[\beta_1,\beta_2,\cdots,\beta_N]A = 0$, so that $\dot{\bar{x}} = f\left( \sum_{j=1}^N \beta_j x_j \right) = f(\bar{x})$.

(ii) If $f(\cdot)$ satisfies the Lipschitz condition, then

$$\left\| \dot{\bar{x}} - f(\bar{x}) \right\| = \left\| \sum_{j=1}^N \beta_j \dot{x}_j - f(\bar{x}) \right\|$$

$$= \left\| \sum_{j=1}^N \beta_j \left( f(x_j) + c\sum_{k=1}^N a_{jk} H(x_k) \right) - f(\bar{x}) \right\|$$

$$= \left\| \sum_{j=1}^N \beta_j f(x_j) + c\sum_{j=1}^N\sum_{k=1}^N \beta_j a_{jk} H(x_k) - f(\bar{x}) \right\|$$

$$= \left\| \sum_{j=1}^N \beta_j f(x_j) + c[\beta_1,\beta_2,\cdots,\beta_N]\,A \begin{bmatrix} H(x_1) \\ H(x_2) \\ \vdots \\ H(x_N) \end{bmatrix} - f(\bar{x}) \right\|$$

$$= \left\| \sum_{j=1}^N \beta_j \left( f(x_j) - f(\bar{x}) \right) \right\| \le \sum_{j=1}^N \beta_j \left\| f(x_j) - f(\bar{x}) \right\|$$

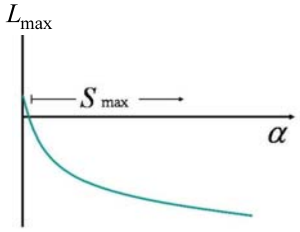
$$\le \rho \sum_{j=1}^N \beta_j \| x_j - \bar{x} \|.$$

Therefore, by taking limits on both sides, one has $\lim_{t\to\infty} \| \dot{\bar{x}}(t) - f(\bar{x}(t)) \| = 0$.

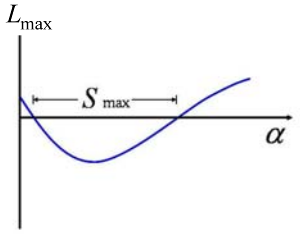
It is further remarked that for a very special case of network (1), if the network is synchronized to a constant state vector $s$, then by taking the time limit on both sides of (1), due to the diffusive coupling of the network, one has $\dot{s} = f(s)$. Here, since $s$ is a constant vector, one has $\dot{s}=0$, so $f(s)=0$, namely this constant state vector $s$ is an equilibrium of the node dynamical system.

In performing pinning control to achieve synchronization of network (1), for instance when the network is scale-free [17], if the controllers are pinned at the network nodes according to the descending order of the node degrees, the needed number of controllers will be much smaller than that if they are randomly placed in, just to gain the same effects. For small-world networks [18], however, even if the controllers are randomly applied to some nodes, as the coupling probability increases, which creases more long-range edges, the number of needed controllers will decrease.

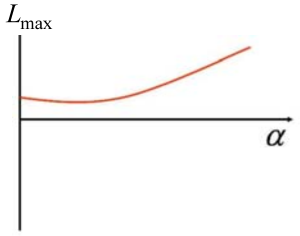
Next, consider the local synchronization problem for the dynamical network (1) with a constant matrix $H$,




(a) Type 1.




(b) Type 2.




(c) Type 3.

Fig. 1. Synchronized regions.

namely $H(x) = Hx$. Linearizing the node dynamical system at its solution (e.g., equilibrium) $s$, which satisfies $\dot{s}(t) = f(s(t))$ as mentioned above, yields a master stability equation [19],

$$\dot{y}_k = [Df(s) - c\lambda_k H]y_k, \quad k = 2,3,...,N,$$

where $\lambda_1 = 0$. The maximum Lyapunov exponent $L_{max}$ of these equations is called the master stability function. A common criterion for determining the stability of the above network synchronization manifold is that its $L_{max}$ is negative [19,20]. The common region $S = S_{max}$ for all $c\lambda_k$ values in which $L_{max}$ is negative is called the synchronized region of the network, which is obviously determined by both $f(\cdot)$ and $H$. Thus, if

$$c\lambda_k \subseteq S, \quad k = 2,3,...,N,$$

then the largest (hence, every) Lyapunov exponent will be negative, $L_{max} < 0$, implying that the synchronization manifold is stable, so the network synchronizes.

According to different situations, synchronized regions can be classified into the following types (Fig. 1) [1,10]:

**Type 1** [17,18]**:** The corresponding synchronized region is $S_1 = (\alpha_1, \infty)$, where $0 \le \alpha_1$ is determined by $L_{max}(\alpha) = 0$. For this type of networks, if the product of the coupling strength $c > 0$ and the smallest nonzero eigenvalue $\lambda_2$ of the Laplacian matrix satisfies

$$\alpha_1 < c\lambda_2,$$

then the network synchronizes. Therefore, the synchronizability of this kind of networks can be characterized by the eigenvalue $\lambda_2$ of the Laplacian matrix $L$: the larger the $\lambda_2$, the smaller the coupling strength $c > 0$ is needed, hence the stronger the network synchronizability.

**Type 2** [20]**:** Its corresponding synchronized region is $S_2 = (\alpha_2, \alpha_3)$, where $0 \le \alpha_2 < \alpha_3 < \infty$ is determined by the equation $L_{max}(\alpha) = 0$. If

$$\alpha_2 < c\lambda_2 \le c\lambda_N < \alpha_3,$$

then the network synchronizes. Therefore, the synchronizability of this kind of networks is characterized by the ratio $\lambda_2 / \lambda_N$ of the eigenvalues of the Laplacian matrix $L$: the more closer to each other between $\lambda_2$ and $\lambda_N$, the easier this inequality is satisfied, so the larger the ratio $\lambda_2 / \lambda_N > \alpha_2 / \alpha_3$, the stronger the network synchronizability.

**Type 3:** The corresponding synchronized region is an empty set, $S_3 = \varphi$. In this case, no matter how strong the coupling strength $c > 0$ is, the network will not synchronize by itself (unless external control is applied, as further discussed below).

**Type 4** (see, e.g., [21])**:** The corresponding synchronized region is a union of several regions of the forms $S_1 = (\alpha_1, \infty)$ and $S_2 = (\alpha_2, \alpha_3)$. In this case, only if all $c\lambda_i$ fall into these sub-regions the network will synchronize. This situation is fairly complicated, but also rather rare.

It is worth noting that graph theory, especially digraph theory, is a powerful mathematical tool which classical control theory did not intend to take advantage of. Studying control theory under complex dynamical network environments, for example studying the synchronizability and stability of various dynamical networks, these tools are particularly important and useful - oftentimes they could provide new results and criteria that classical control theory did not or could not offer [21]. This is especially true for controlling directed networks, a topic to be further discussed next.

## 3. CONTROLLABILITY OF DIRECTED DYNAMICAL NETWORKS

The (complete) controllability is a fundamental concept and basic notion in classical control theory, which is attributed to Kalman and has been well documented (see, e.g., [5]). For a linear system, there is an elegant necessary and sufficient condition for controllability; that is, the system controllability matrix has a full rank. For linear time-invariant systems, $\dot{x} = Ax + Bu$ with $x \in R^n$ and $u \in R^m$, $1 \le m \le n$, denoted as $(A, B)$, the corresponding controllability matrix is $[B \quad AB \quad \cdots \quad A^{n-1}B]$. For linear time-varying systems it has an integral form; for nonlinear systems, the situation is much more complicated where usually only case studies can provide some useful criteria, which are both beyond the scope of this survey therefore will not be further discussed below.

In the investigation of controllability for linear time-invariant dynamical systems under the framework of directed networks, there were already some encouraging successes in the 1970-80s, referred to as the "structural controllability" [22,23]. Yet, not too much progress was made thereafter, and unfortunately not much attention was received from the control theory community for some historical reasons. Today, spurred by the demands from network science and engineering, the same question was revisited from a network perspective and it brought up more interesting problems and also new challenges to both the networks and the control communities. First, the networks under consideration are directed; hence, one cannot simply consider a directed network of multiple dynamical systems as one single huge-dimensional overall system and then tackle it by using the conventional system-decoupling techniques. Second, complex networks have various models and forms, such as random-graph, small-world, scale-free, weighted, evolutionary, impulsive and hybrid networks; therefore, the descriptions and conditions on their controllability would be fairly different from one to another, even in the case of linear node systems, for which existing results and tools from the classical control theory are insufficient or even inapplicable, at least not in a straightforward manner.

A distinct feature of directed networks is that it is very important to decide where to locate the controllers. A trivial example is a two-node network with a directed edge pointing from one node to another. Obviously, if a controller is put at the latter then the former will not be affected by any control input. Usually, a node with only one input is called an (independent) driver node. Clearly, if every node is a driver then the network can be (completely) controllable. However, a practical question will be: Is there a minimum number $N_D$ of driver nodes using which the whole network is controllable? Moreover, choosing which $N_D$ nodes as drivers can achieve a control objective most effectively? This is a typical pinning control problem discussed above.

To be more precise, consider a directed network of identical linear time-invariant dynamical systems, with every node being described by the same $(A, B)$. It is said to be structurally controllable if there exist a set of elements in matrices $A$ and $B$ such that the networked system $(A, B)$ is controllable in the classical sense. Furthermore, if for all nonzero elements of $(A, B)$ the networked system is always controllable, then it is said to be strongly structurally controllable [22,23].

For illustration, consider the simple example shown in Fig. 2, which has four 3-dimensional systems with $X_1$, $X_2$, $X_3$ representing their state variables [22-24]. Their controllability matrices are, respectively, given by

$$b_1\begin{bmatrix}1 & 0 & 0\\ 0 & a_{21} & 0\\ 0 & 0 & a_{32}a_{21}\end{bmatrix},\ b_1\begin{bmatrix}1 & 0 & 0\\ 0 & a_{21} & 0\\ 0 & a_{31} & 0\end{bmatrix},$$

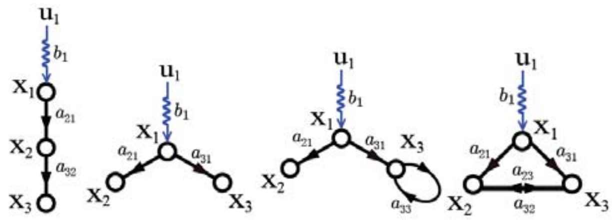



Fig. 2. An example illustrating the structural controllability [22-24].

$$b_1\begin{bmatrix}1 & 0 & 0\\ 0 & a_{21} & 0\\ 0 & a_{31} & a_{33}a_{31}\end{bmatrix},\ b_1\begin{bmatrix}1 & 0 & 0\\ 0 & a_{21} & a_{23}a_{31}\\ 0 & a_{31} & a_{32}a_{21}\end{bmatrix}.$$

In Fig. 2, networked systems **a** and **c** are strongly structurally controllable, since for any choice of nonzero elements their corresponding controllability matrices have a full rank. System **d** is structurally controllable but not strongly, because for elements satisfying $a_{32}a_{21}^2 = a_{23}a_{31}^2$, its controllability matrix is not of full rank, yet for other choices of elements it is so. This case also explains the importance of the weights in determining the network controllability. System **b** is simply uncontrollable, since for whatever choice of nonzero elements, its controllability matrix is not of full rank. Notice that, compared to system **b**, system **c** has an extra self-loop in state $X_3$, but this small difference in structure leads to an essential change in the nature of the controllability. Thus, to some extent, this case reflects the complexity and difficulty in the study of controllability for directed networks.

The minimum number of driver nodes needed to successfully control a given directed network can be determined by the maximum matching set [1,10]. A (sub)set of directed edges, $E^*$, is called a matching set if every pair of edges in $E^*$ do not have common starting nodes nor common ending nodes. A node is called a matched node if it is the end point of an edge in $E^*$; otherwise, it is unmatched. In a directed network, a matching set with the largest number of matched nodes is called a maximum matching set. Further, a matching set is said to be perfect if all its nodes are matched nodes; thus, the largest possible perfect matching set is the network itself.

In investigating the controllability of directed networks, there was an attempt [24] that received a lot of attention. In its supplementary materials, there is a "minimum input theorem" saying that for a directed network to be controllable, the needed minimum number of driver nodes is $N_D = \max\{N - |E^*|, 1\}$, where $|E^*|$ is the number of elements in $E^*$. In particular, if a network has a perfect matching set, then $N_D = 1$, and in this case every node can be chosen to be the driver. Yet, if a network does not have a perfect matching set, then $N_D = N - |E^*|$; that is, $N_D$ is the number of unmatched nodes in any maximum matching set of the network. In this case, one should choose unmatched nodes as drivers for pinning control of the network. Of course, this is quite intuitive since otherwise the unmatched nodes will never be controlled. If a network does not have perfect matched sets, one may apply some control means to create a perfect matching set in it [24].

In [24], it was pointed out, that "Here we develop analytical tools to study the controllability of an arbitrary complex directed network, identifying the set of driver nodes with time-dependent control that can guide the system's entire dynamics. We apply these tools to several real networks, finding that the number of driver nodes is determined mainly by the network's degree distribution. We show that sparse inhomogeneous networks, which emerge in many real complex systems, are the most difficult to control, but that dense and homogeneous networks can be controlled using a few driver nodes. Counterintuitively, we find that in both model and real systems the driver nodes tend to avoid the high-degree nodes." Note that, here, "using a few driver nodes" is in fact a pinning control strategy discussed above. Of course, the next step is to answer questions like "how many (to pin)? " and "which ones (to pin)? "

Before the paper [24] appeared in *Nature*, a staff writer of *Science*, Mr Adrian Cho, sent me the galley and then called me for a discussion. Afterwards, he wrote a commentary [8] and said: "The work is both more general and more practical than earlier efforts to apply control theory to networks, says Guanrong Chen, an electrical engineer at City University of Hong Kong. … The new work treats the more common case of directed networks. … Also, Chen says, the algorithm for finding a set of control nodes is very important because it's useful."

It is fair to say that the publication of the timely article [24] has in effect provoked the current active research on the controllability of directed dynamical networks. Therein, it shows some insightful observations on 37 empirical examples from 12 different types of real-world networks, such as the aforementioned observation "the driver nodes tend to avoid the high-degree nodes". These deserve special attention from classical control theorists. It is also worth mentioning that, from a control-cost perspective, similar conclusions could be made [25]: better driver nodes typically are not hub nodes with large degrees. Moreover, there are some recent reports on this commonly-concerned subject. For example, in [26] it is pointed out that scale-free directed networks are easier to control than those directed networks with low degree-degree correlations; in [27] an analytical framework is developed to identify critical intermittent or redundant nodes, leading to the discovery of two distinct control modes in complex systems: centralized versus distributed controls; in [28] the concept of control capacity is introduced to quantify the likelihood that a node is a driver, and it demonstrates that the possibility of being a driver node decreases with its in-degree, which however is independent of its out-degree. Along the same line, in [29] it studies minimizing the number of controllers towards the network controllability by optimizing structural perturbations, and in [30] it investigates the controllability of directed and weighted networks which in a way generalizes some results of [24].

In retrospect, as earlier as in 1974, Lin [22] was the first to consider the controllability of networked linear time-invariant dynamical systems from a graph-theoretic approach, deriving some necessary and sufficient conditions in terms of graph theory. In 1976, the notion was extended to multi-input systems [31]. In 1977, Lin [32] further introduced the concept of minimal structural controllability, giving a necessary and sufficient condition in both graphic and algebraic terms. These stimulated a great deal of interest in the important subject of system structural controllability for a while [33].

After a silent period of about two decades, in 2007 Lombardi and Hörnquist [34] revisited the controllability of digraphs, and Liu *et al.* [35] studied the controllability of a leader-follower dynamic network with switching topology, prior to which there were already some studies on the controllability of leader-follower networks [36-39]. However, these studies were essentially based on the Kalman criterion in terms of the controllability matrix rank. When the size of a network is large, it becomes extremely difficult if not impossible to verify, so graph-theoretic criteria become more desirable.

Also noticeably, most of the aforementioned research efforts essentially ignored the node dynamics while concentrating on the effects of the network topologies on the networks controllability. An early study to introduce a structural measure for the controllability of undirected networks by taking node dynamics into consideration was reported in [40]. The main idea is to introduce a virtual node to the network, related to control, thereby augmenting the system to have one higher dimension, so that the powerful master stability function method [19,20], mentioned above regarding network synchronizability, can be applied for controllability analysis. It was pointed out that this structural measure depends not only on the information about the network topology but also on the choice of the controlled nodes and their control gains.

Moreover, in [41] the controllability problem for non-symmetrical weighted scale-free networks was investigated, revealing a threshold for pinning control: when the ratio of pinning-controlled nodes increases to be over this threshold, the pinning controllability will be achieved and also the control performance will be improved.

Very recently, the pinning control problem for non-diagonalized directed networks with non-identical nodes was studied in [42], using the network algebraic connectivity (defined in network model (1) above) as the controllability index. It was shown that the controllability is closely related to both the node dynamical functions and the control gains, revealing a key issue and also some essential difficulty in controllability analysis of directed dynamical networks. More precisely, a general linear leader-followers network is considered, which can be represented in a compact form by

$$\dot{X}(t) = [(C-L)\otimes H]X(t) + (\Delta \otimes B)U(t),$$

where $X(t)$ is the state vector composing of all node-state vectors, $U(t)$ is the pinning control input vector composing of all pinning-controller input vectors, $\Delta$ is a 0-1 diagonal matrix determining which node to pin, $C$ is a diagonal matrix of all the coupling constants, $L$ is the overall Laplacian matrix, $H$ is the inner coupling matrix, $B$ is the control gain matrix, and $\otimes$ is the Kronecker product. It was proved in [42] that this network is controllable if and only if the following two conditions are satisfied simultaneously: (i) $(H, B)$ is a controllable matrix-pair (in the classical sense); (ii) there exists no left-eigenvector of matrix $[L - C]$ with the first $q$ entries being zeros, where $q$ is the number of pinning-controlled followers. As consequences, this theorem implies several known and unknown results, such as: (i) a directed path is controllable if the beginning node is selected to be the only leader; (ii) a directed cycle with a single leader is always controllable; (iii) a complete digraph (of size > 2) with a single leader is uncontrollable; (iv) a star digraph (of size > 2) is uncontrollable even with the center node being the leader (consistent with Fig. 2(b) discussed above).

In summary, the subject of directed and weighted network controllability is extremely important and there are many fundamental theoretic and applied research problems awaiting for further exploration under a uniform framework of complex dynamical networks. Nevertheless, some research progress has been made recently, as reported in, e.g., [43-48].

As a side note, the (complete) observability is a dual concept to (complete) controllability [5], which means the ability to determine a system's initial state from its outputs thereby enabling reconstruction of all the states of the system. The observability has a simple and elegant necessary and sufficient condition for linear systems; that is, the system observability matrix has a full rank. There are already some attempts to extend this classic notion to directed networks with a graph-theoretic approach to determining the necessary number of observers needed to reconstruct all full internal states of a networked system, verified by biochemical reaction systems in [49].

To this end, the focal issue of pinning controlled synchronization of complex dynamical networks is further discussed, which will be detailed in the next section.

## 4. PINNING-CONTROLLED NETWORK SYNCHRONIZATION

Once again, consider network model (1) for simplicity of presentation and discussion.

The objective here is to let the network synchronize to some desired state of the node system, for example an equilibrium state $s \in R^n$ satisfying $\dot{s} = f(s) = 0$. If, under certain conditions as reviewed above, the network can self-synchronize to $s \in R^n$, then no external control input is needed. However, if the network is unable to self-synchronize to $s \in R^n$, then one needs to apply external control such as pinning control to force the network to achieve the objective.

Without loss of generality in applying the pinning control strategy, suppose that the first $l$ nodes are selected to pin. Then, referring to network (1), the controlled network is described by

$$\dot{x}_i = f(x_i) + c\sum_{j=1}^{N} a_{ij} H x_j + u_i\,, \quad i = 1, 2, ..., l, \tag{5a}$$

$$\dot{x}_i = f(x_i) + c\sum_{j=1}^{N} a_{ij} H x_j\,, \quad i = l+1, l+2, ..., N. \tag{5b}$$

For simplicity here, the following linear state-feedback controllers are used:

$$u_k = -c\kappa_k H(x_k - s)\,, \quad k = 1, 2, ..., l, \tag{5c}$$

where $\{\kappa_k\}$ are positive constant feedback gains to be determined. Let $D = diag\{\kappa_1, \kappa_2, ..., \kappa_l, 0, ..., 0\}$. Then, some local and global criteria for the above pinning controllability of network synchronization can be established.

First, the simplest possible pinning control scheme is to use only one single controller (with $l = 1$), namely, in (5) with $u_1 = -c\kappa_1 H(x_1 - s)$ and all the other $u_i = 0$, $i = 2, 3, ..., N$. This is possible under some local conditions (e.g., using linearization and large coupling strength $c$), as shown in [50].

Then, to derive a global criterion, let us introduce the concept of $V$-uniformly decreasing functions (see, e.g., [7]): a function $\phi : R^n \times R \to R^n$ is $V$-uniformly decreasing if there exists a square matrix $V$ and a constant $\rho > 0$ such that for all $V$ and all $t \geq 0$,

$$(z - y)V[\phi(z,t) - \phi(y,t)] \leq -\rho\|z - y\|^2. \tag{6}$$

Now, consider the controlled network (5). Suppose that matrix $(U \otimes V)[\rho(A + D) \otimes H + I \otimes T] > 0$ is symmetrical and positive semi-definite, and let $T$ be a square matrix such that $f(x_k) + Tx_k$ is $V$-uniformly decreasing for some symmetrical and positive definite matrix $V$ and for all $x_k \in R^n$, $k = 1, 2, ..., N$. It can be shown that if there exists a positive-definite diagonal matrix $U$ such that

$$(U \otimes V)[\rho(A + D) \otimes H + I \otimes T] > 0, \tag{7}$$

then the controlled network (5) is globally stable about a state $\bar{x}$ which satisfies $\dot{\bar{x}} = f(\bar{x})$.

To verify the above global criterion, construct a Lyapunov function

$$W(\tilde{x}) = \frac{1}{2}\tilde{x}^T (U \otimes V)\tilde{x},$$

where $\tilde{x} = [\tilde{x}_1^T, \tilde{x}_2^T, \ldots, \tilde{x}_N^T]^T \in R^{Nn}$ with $\tilde{x}_k = x_k - \bar{x}$, $k = 1, 2, ..., N$. The derivative of $W(\tilde{x})$ along the state trajectories of the network is

$$\begin{aligned}\dot{W}(\tilde{x}) &= \tilde{x}^T (U \otimes V)\dot{\tilde{x}} \\ &= \tilde{x}^T (U \otimes V)[f(x) - f(\theta \otimes \bar{x}) + (I \otimes T)\tilde{x}\end{aligned}$$

$$
\begin{aligned}
&-[\rho(A+D)\otimes H + I\otimes T]\tilde{x}] \\
&\le \tilde{x}^T(U\otimes V)[f(x)-f(\theta\otimes\bar{x})+(I\otimes T)\tilde{x}],
\end{aligned} \tag{8}
$$

where $\theta=[1,1,\ldots,1]^T \in R^N$ and $x=[x_1^T,x_2^T,\ldots,x_N^T]^T \in R^{Nn}$, and the last inequality follows from (7). Because $f(x_k)+Tx_k$ is $V$-uniformly decreasing and $U = diag\{u_1,u_2,...,u_N\}$ is positive definite, it follows from (8) that

$$
\begin{aligned}
\dot{W}(\tilde{x}) &= \tilde{x}(U\otimes V)[f(x)-f(\theta\otimes\bar{x})+(I\otimes T)\tilde{x}] \\
&= (x-\theta\otimes\bar{x})(U\otimes V)[f(x)+(I\otimes T)x \\
&\quad -f(\theta\otimes\bar{x})-(I\otimes T)(\theta\otimes\bar{x})] \\
&= \sum_{k=1}^{N} u_k(x_k-\bar{x})V[f(x_k)+Tx_k-f(\bar{x})-T\bar{x})] \\
&\le -\sum_{k=1}^{N} u_k\rho\|x_k-\bar{x}\|^2.
\end{aligned}
$$

Hence, by the Lyapunov direct method, the criterion is proved.

It is remarked that the criterion presented and verified here has corrected some minor errors in Theorem 1 of [7], by strengthening the requirement on matrix $U$ as stated above and by simply changing (7) from originally "positive semi-definite" to presently "positive definite" and replacing $D$ with $\rho D$ therein (see also [51]).

A broad spectrum of research on pinning control of network synchronization has been carried out in the last few years [52,53], for example with adaptive pinning control [48,54-56] and on time-varying networks and networks with time-delayed couplings [57,58], for which the network may even be pinned to a rather arbitrary trajectory (such as chaotic orbits [59,60]), and equipped with digital controllers [61], towards some real-world engineering applications [62].

## 5. CONCLUSIONS

This article has surveyed the recent research progress in pinning control and pinning synchronization on complex dynamical networks. The aim is to present the background and some new developments in the fields. Wherever deemed necessary, some technical details were provided along with key references for the readers' convenience. Due to the space limitation, this article of modest size cannot cover all important topics; for example, network synchronization as a standalone subject itself is a huge research area which was not covered herein. Even in regard to the topics under review, this short overview article is by no means comprehensive or exhaustive, regrettably leaving out many relevant publications without referencing and discussing.

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