

Supplementary Information for paper “Local communities obstruct global consensus: Naming game on multi-local-world networks”

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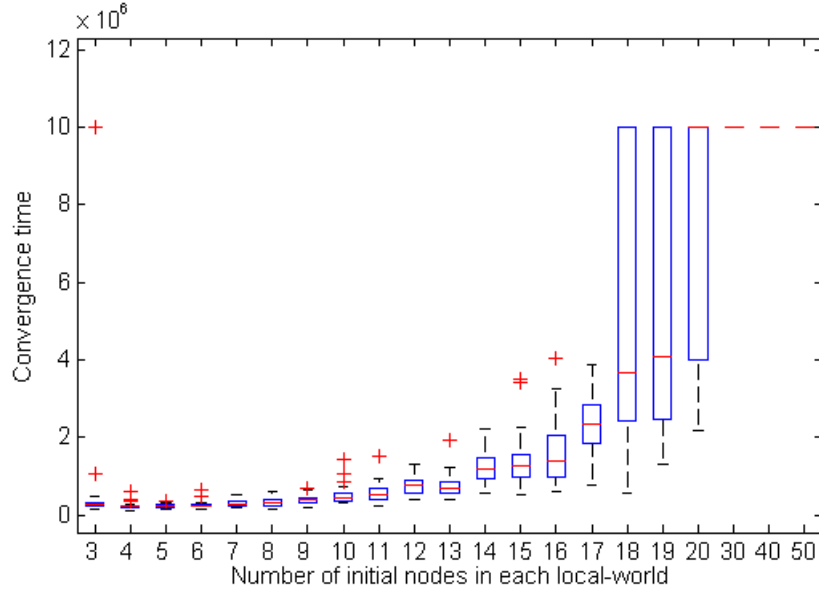
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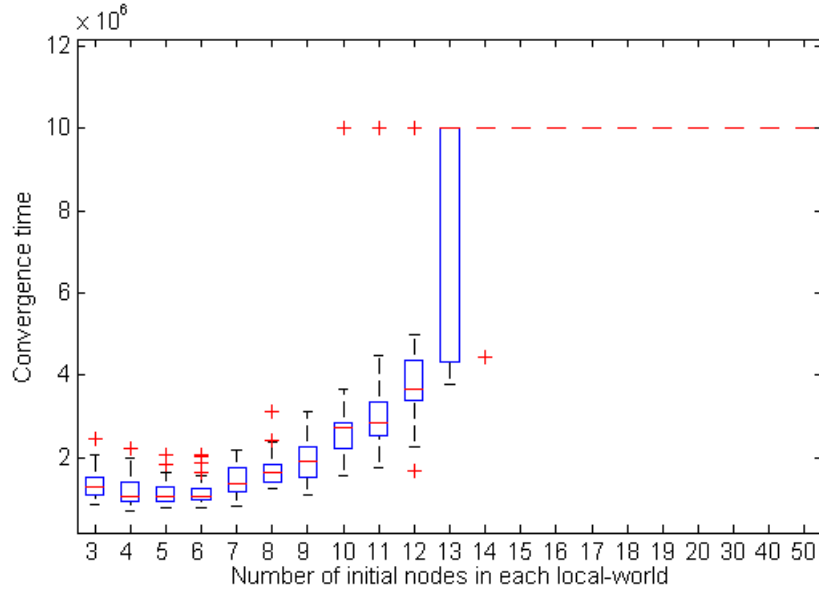
Abstract

This supplementary document further examines the scaling property of the population size (denoted by N) of 500 and 1500, respectively. *Convergence time* is used as the measure, which means the time steps needed to reach global convergence. The comparing simulation is carried out by varying parameters ρ , m_0 and N_{LW} . In the following comparison: 1) $\rho = 0.7$ is fixed and the *convergence time* affected by the dynamics of the number and size of the local-worlds is investigated; 2) the *convergence time* is studied when the rate ρ of the initial assigned nodes is varied, while m_0 and N_{LW} are all fixed; and 3) the convergence progresses of the multi-local-world (MLW) networks with the three typical models, namely random-graph (RG), small-world (SW) and scale-free (SF) networks, are compared.

1 Convergence time vs the number and size of local-worlds



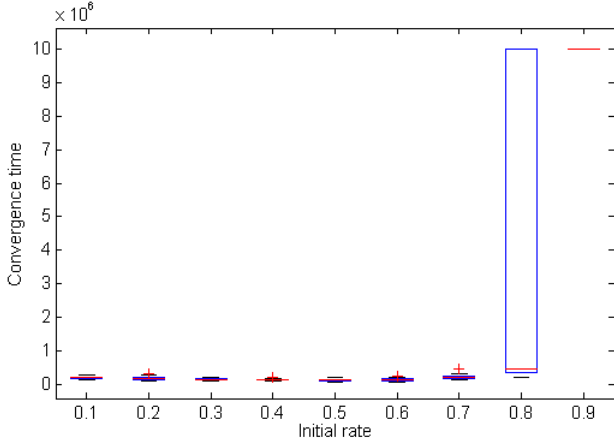
(a) $N = 500$



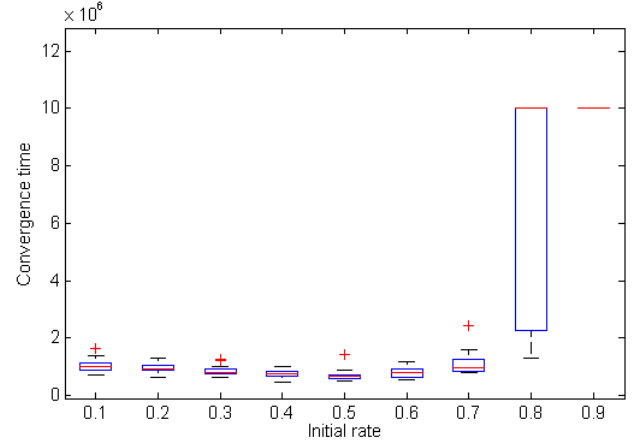
(b) $N = 1500$

Figure 1 The box plot of the convergence time vs the number m_0 of initial nodes in each local-world, with (a) $N = 500$ and (b) $N = 1500$. In both cases, $\rho = 0.7$. The number of local-worlds is calculated by Eq. (4) in the paper, i.e., $N_{LW} = \lfloor \rho N / m_0 \rfloor$, where $\lfloor x \rfloor$ is to the largest integer less than or equal to x . In (a), $N = 500$, the mean value of the convergence time seemingly does not change as m_0 increases from 3 to 5, but it can be clearly observed that the maximum number of outliers (red plus) becomes smaller when m_0 increases from 3 to 5. In (b), $N = 1500$, both the mean value (red bar) and the maximum number of outliers (red plus) decrease when m_0 increases from 3 to 5, and then increases as m_0 continues to increase. Both box plots show a similar feature as that in the paper, where $N = 1000$: the overall convergence time firstly decreases as m_0 increases from 3 to 5, and then increases when $m_0 > 6$.

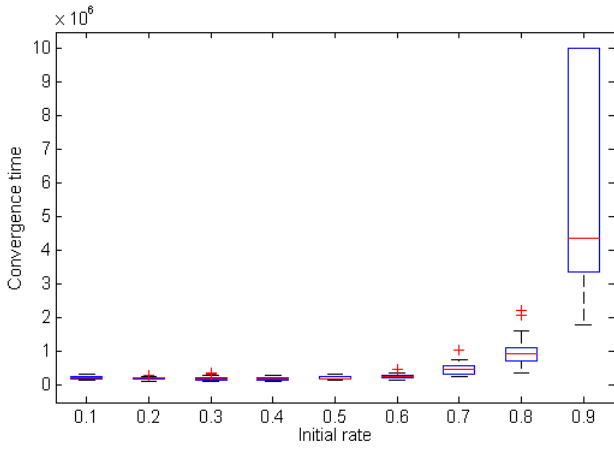
2 Convergence time vs the rate of initially assigned nodes



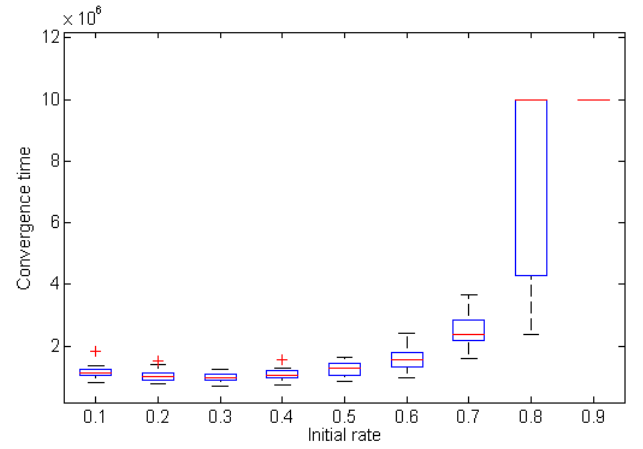
(a) $N = 500, m_0 = 4$



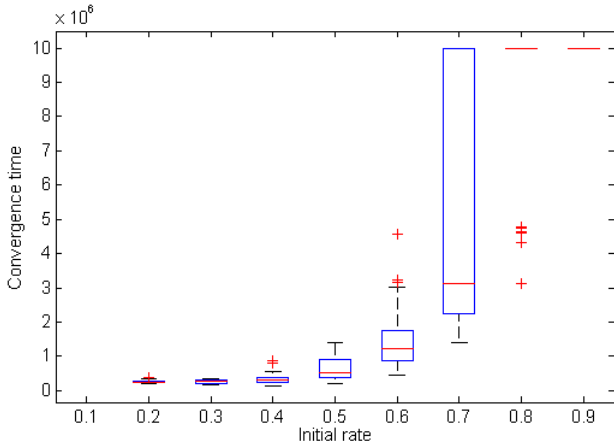
(d) $N = 1500, m_0 = 4$



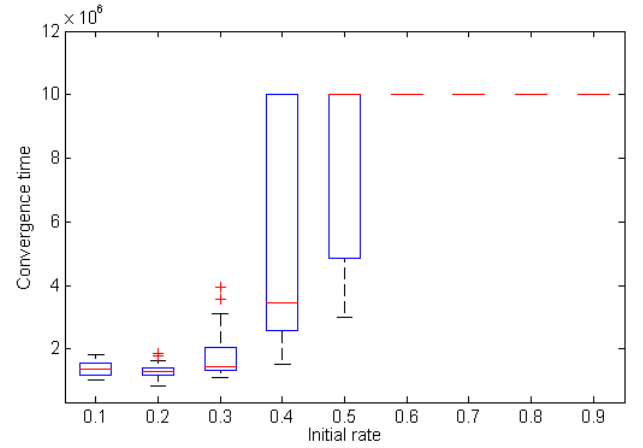
(b) $N = 500, m_0 = 10$



(e) $N = 1500, m_0 = 10$



(c) $N = 500, m_0 = 18$



(f) $N = 1500, m_0 = 18$

Figure 2 The box plot of the convergence time vs the rate ρ of the initially assigned nodes. The number m_0 of initial nodes in each local-world is set to 4, 10, and 18, respectively; the number of local-worlds is calculated by $N_{LW} = \lfloor \rho N / m_0 \rfloor$. In sub-figures (a), (b), (d), (e) and (f), ρ is varied from 0.1 to 0.9, while in sub-figures (c), ρ is varied from 0.2 to 0.9, to show different scenarios. The data for the case of $\rho = 0.1$ is missing because when $\rho = 0.1$, $N_{LW} = \lfloor \rho N / m_0 \rfloor = \lfloor 0.7 * 500 / 18 \rfloor = 2$, but both N_{LW} and m_0 are supposed to be greater than or equal to 3. A common feature is that, when ρ is small enough (i.e., $\rho \leq 0.7$ in sub-figures (a) and (d); $\rho \leq 0.6$ in (b); $\rho \leq 0.3$ in (c); $\rho \leq 0.5$ in (e); and $\rho \leq 0.2$ in (f)), the different values of ρ do not affect the convergence time at all. However, when ρ becomes greater than these values, the convergence time increases substantially.

3 Convergence processes

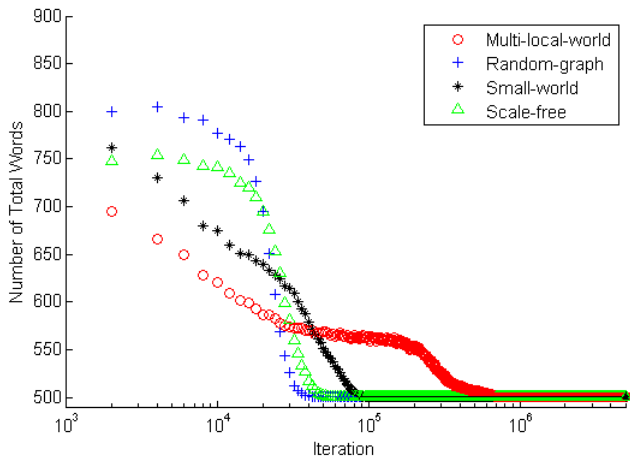
Table 1 The feature statistics of the four networks, when $N = 500$. Here, $\langle k \rangle$ is the average degree, $\langle pl \rangle$ is the average path length and $\langle cc \rangle$ is the average clustering coefficient. The data for MLW is collected from experiments in Section 1, while those for RG, SW and SF networks are generated using the $\langle k \rangle$ values of MLW for reference. As a result, the four types of networks have very similar $\langle k \rangle$ values.

Reference	$\langle k \rangle$				$\langle pl \rangle$				$\langle cc \rangle$			
	MLW	RG	SW	SF	MLW	RG	SW	SF	MLW	RG	SW	SF
$m_0 = 10$	9.40	9.40	10.00	9.92	3.52	3.01	3.43	2.76	0.63	0.02	0.34	0.08
$m_0 = 20$	15.85	15.84	16.00	15.83	3.19	2.56	2.84	2.46	0.78	0.03	0.36	0.10
$m_0 = 30$	22.30	22.33	22.00	21.72	3.00	2.31	2.59	2.28	0.82	0.04	0.38	0.12
$m_0 = 50$	37.43	37.42	38.00	37.19	2.70	1.98	2.20	2.01	0.88	0.08	0.40	0.17

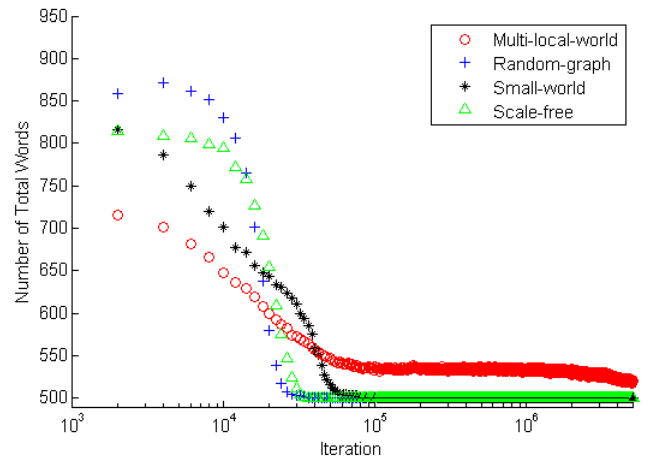
Table 2 The feature statistics of the four networks when $N = 1500$.

Reference	$\langle k \rangle$				$\langle pl \rangle$				$\langle cc \rangle$			
	MLW	RG	SW	SF	MLW	RG	SW	SF	MLW	RG	SW	SF
$m_0 = 10$	9.22	9.05	10.00	9.98	4.25	3.57	4.07	3.12	0.64	0.01	0.33	0.03
$m_0 = 20$	16.10	15.75	16.00	15.94	3.69	2.92	3.40	2.77	0.78	0.01	0.37	0.05
$m_0 = 30$	23.36	23.39	22.00	21.91	3.38	2.67	3.02	2.61	0.84	0.02	0.39	0.05
$m_0 = 50$	37.35	37.23	38.00	37.73	3.22	2.36	2.64	2.30	0.90	0.02	0.38	0.08

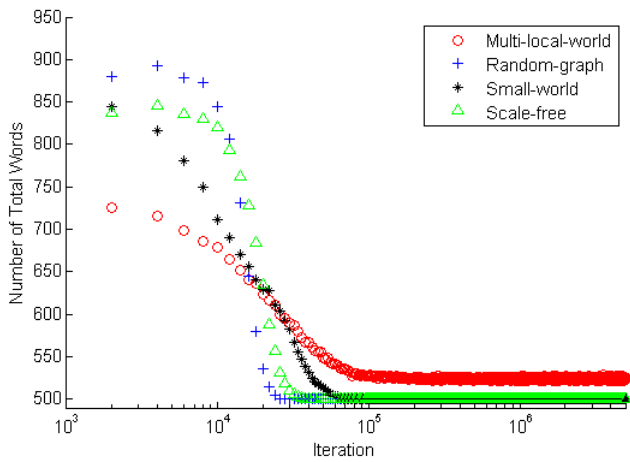
In the following, Figures 3 and 6 show the convergence processes in terms of *the number of total words*, Figures 4 and 7 show the convergence processes in terms of *the number of different words*, and Figures 5 and 8 show the convergence processes of *the success rate*, with $N = 500$ and $N = 1500$, respectively. There are totally 24 sub-figures, and all these sub-figures show the same phenomenon that the blue plus (RG) converges the fastest, followed by the green triangles (SW), and the black stars (SF) ranks the third, and the red circles (MLW) always converge the slowest. This result supports the conclusions summarized in the paper.



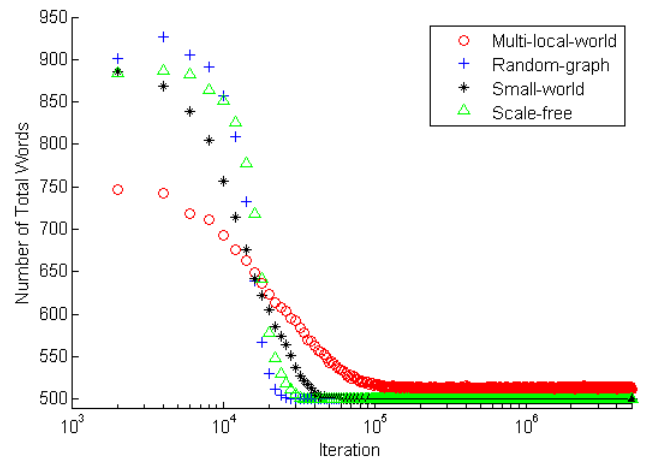
(a) $\langle k \rangle \approx 9.40$



(b) $\langle k \rangle \approx 15.85$

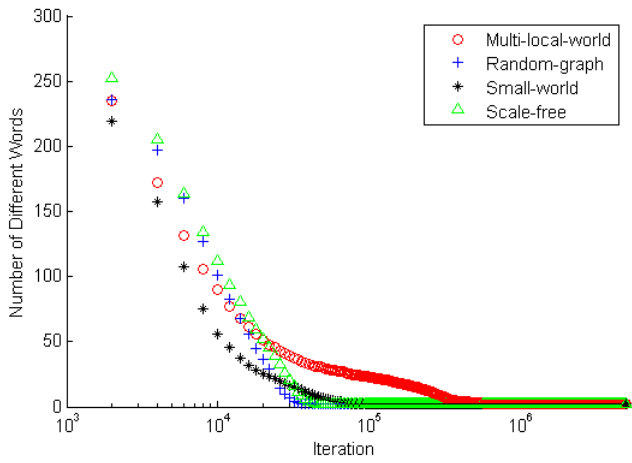


(c) $\langle k \rangle \approx 22.30$

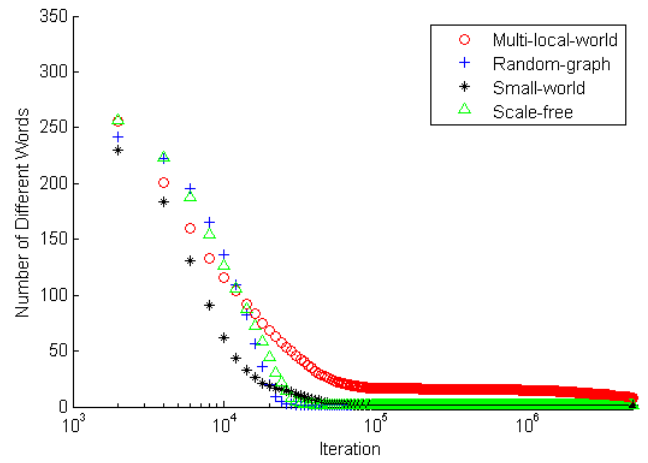


(d) $\langle k \rangle \approx 37.43$

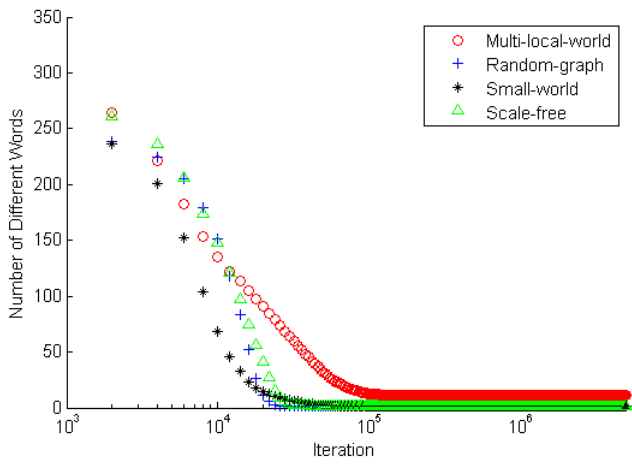
Figure 3 Comparison of the convergence processes in terms of the number of total words ($N = 500$).



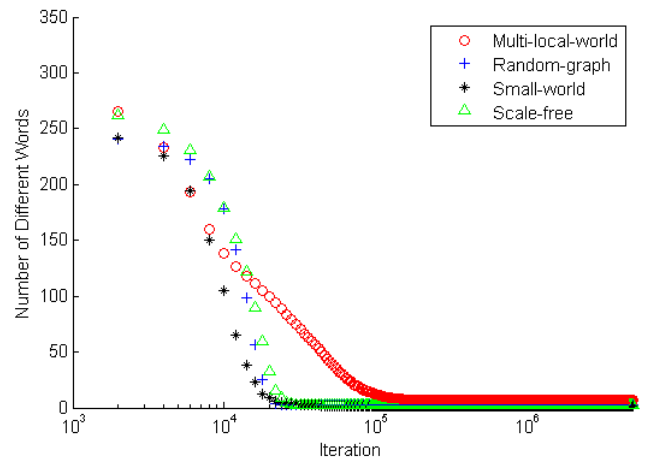
(a) $\langle k \rangle \approx 9.40$



(b) $\langle k \rangle \approx 15.85$

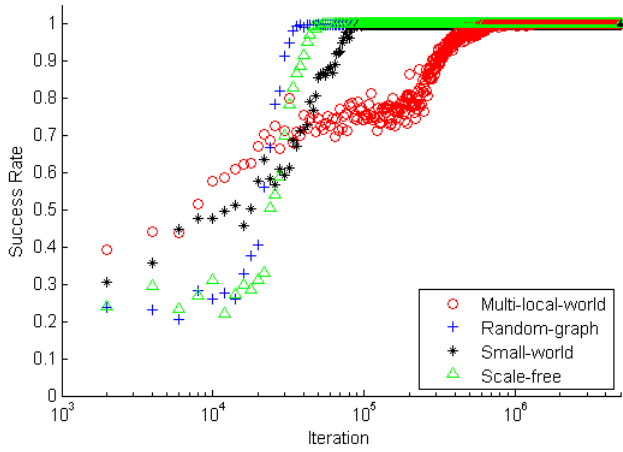


(c) $\langle k \rangle \approx 22.30$

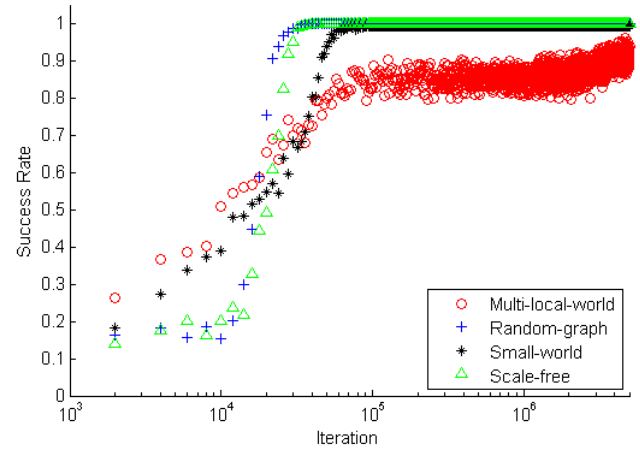


(d) $\langle k \rangle \approx 37.43$

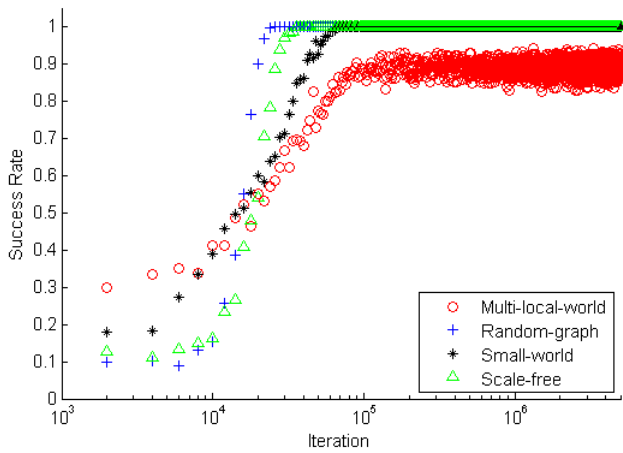
Figure 4 Comparison of the convergence processes in terms of the number of different words ($N = 500$).



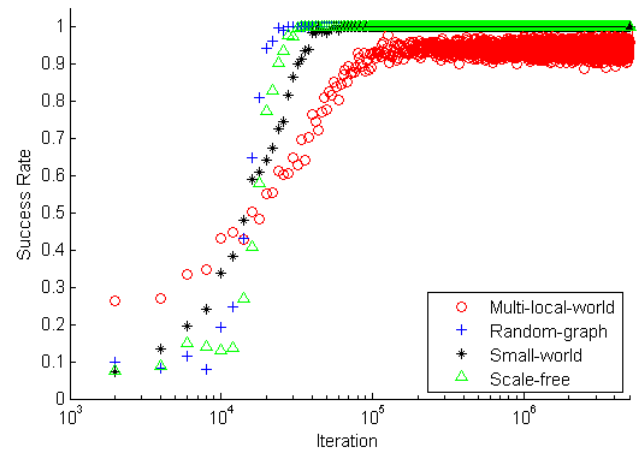
(a) $\langle k \rangle \approx 9.40$



(b) $\langle k \rangle \approx 15.85$

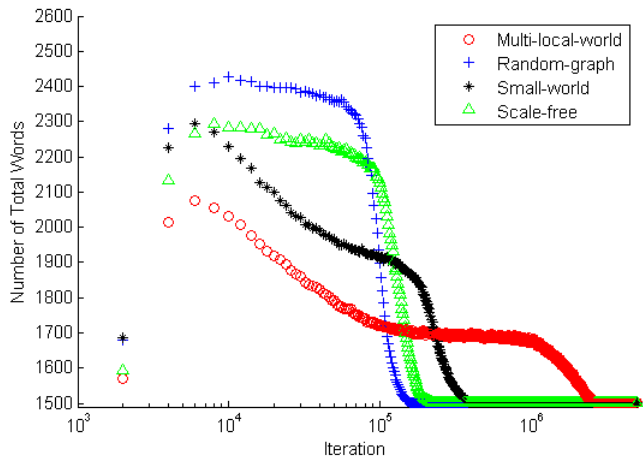


(c) $\langle k \rangle \approx 22.30$

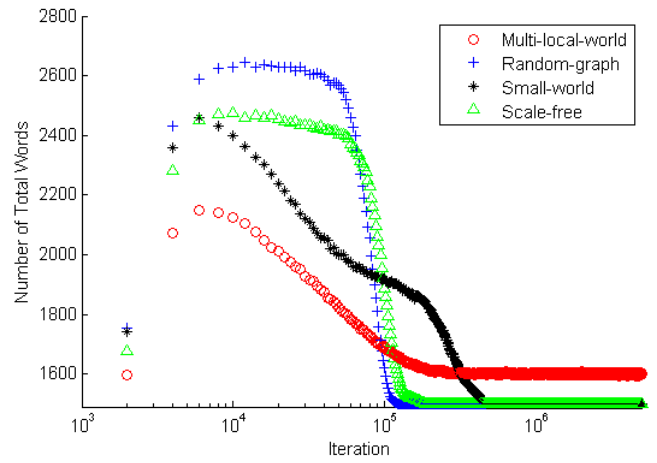


(d) $\langle k \rangle \approx 37.43$

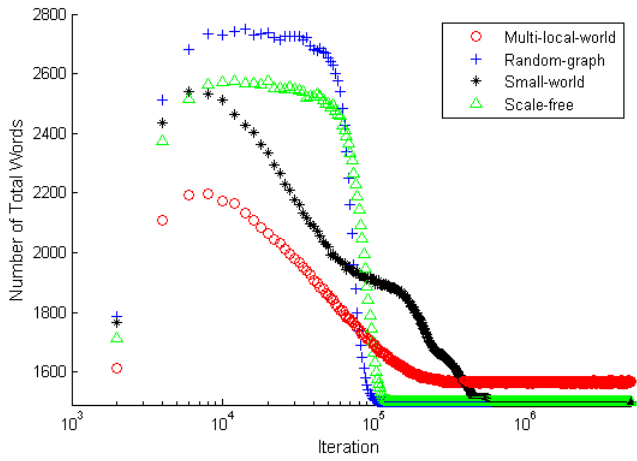
Figure 5 Comparison of the convergence processes in terms of the success rate ($N = 500$).



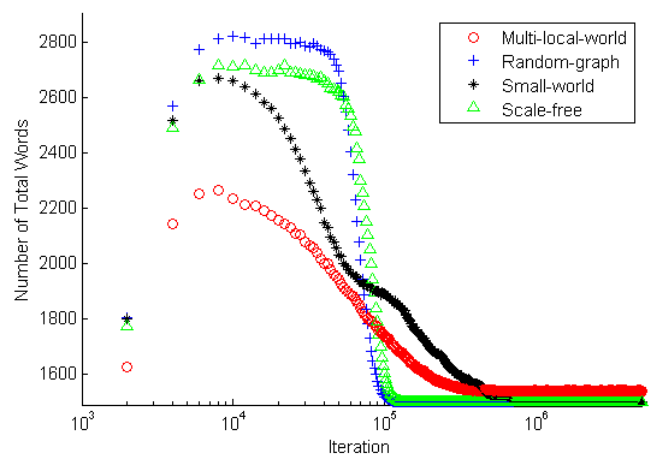
(a) $\langle k \rangle \approx 9.22$



(b) $\langle k \rangle \approx 16.10$

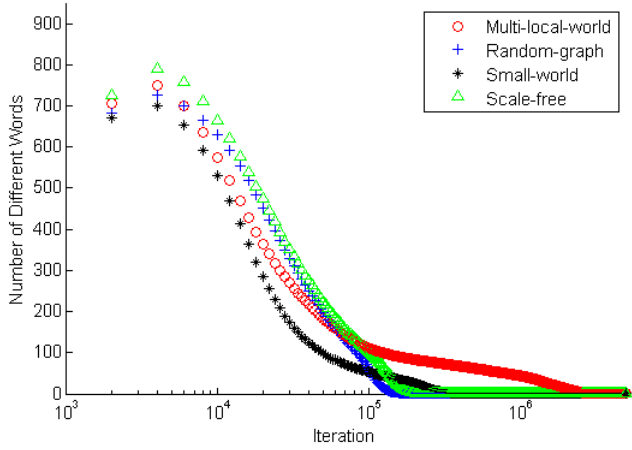


(c) $\langle k \rangle \approx 23.36$

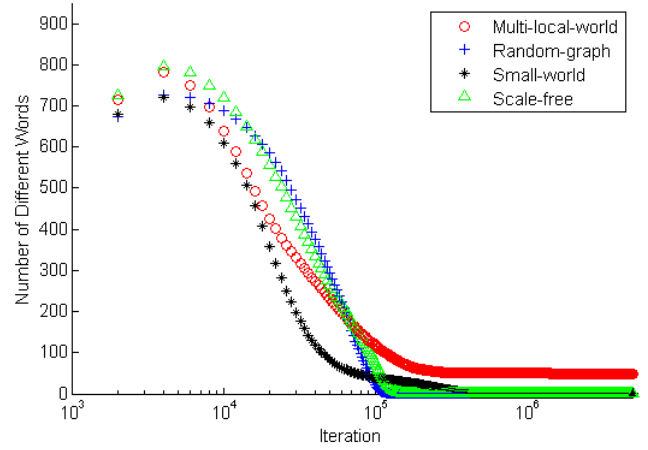


(d) $\langle k \rangle \approx 37.35$

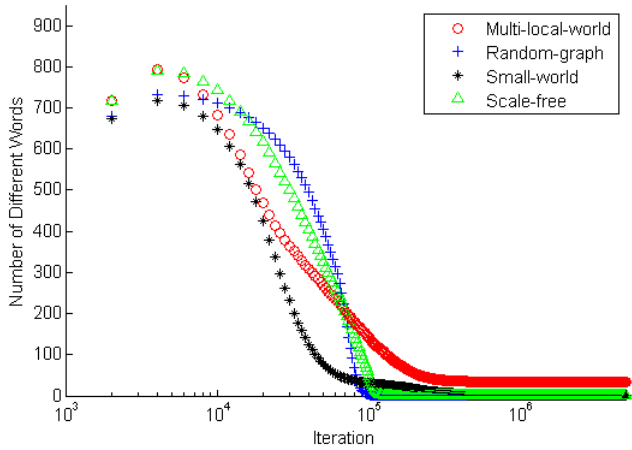
Figure 6 Comparison of the convergence processes in terms of the number of total words ($N = 1500$).



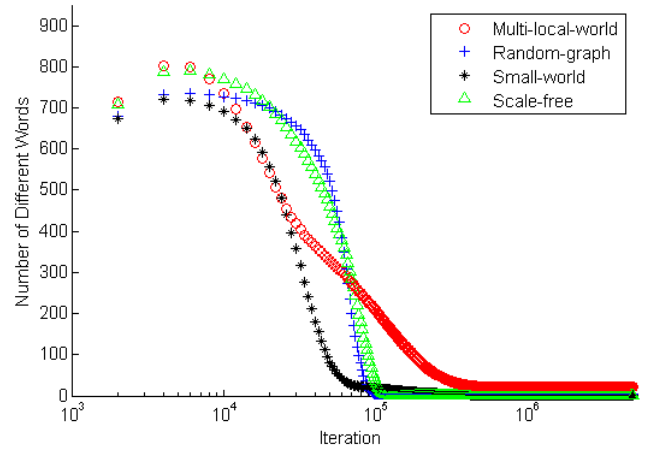
(a) $\langle k \rangle \approx 9.22$



(b) $\langle k \rangle \approx 16.10$

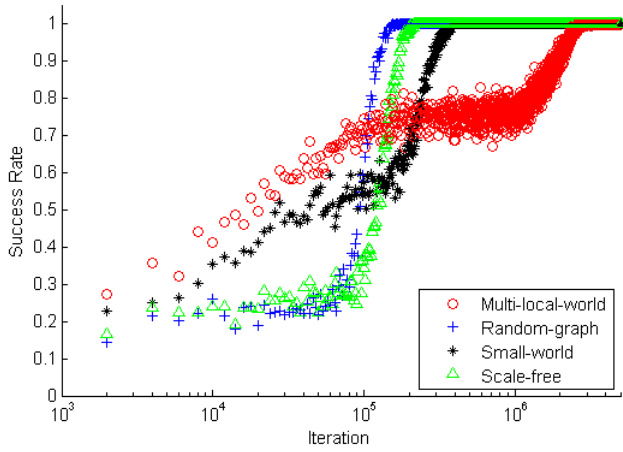


(c) $\langle k \rangle \approx 23.36$

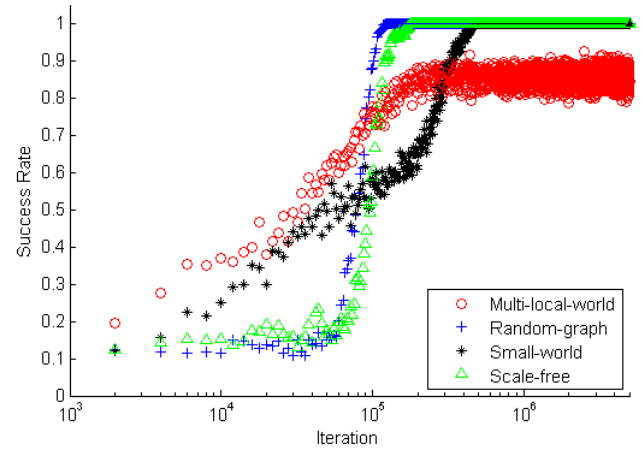


(d) $\langle k \rangle \approx 37.35$

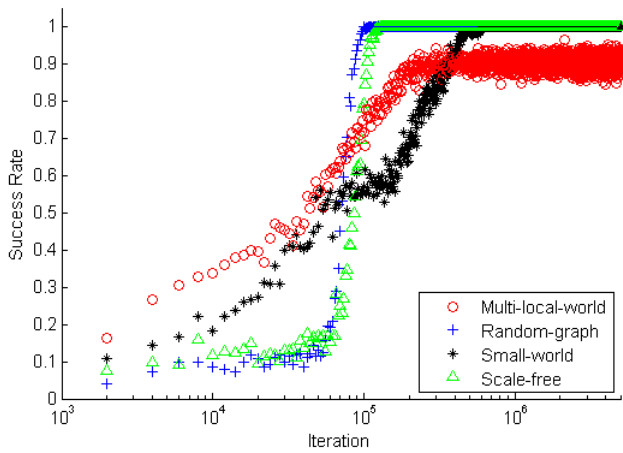
Figure 7 Comparison of the convergence processes in terms of the number of different words ($N = 1500$).



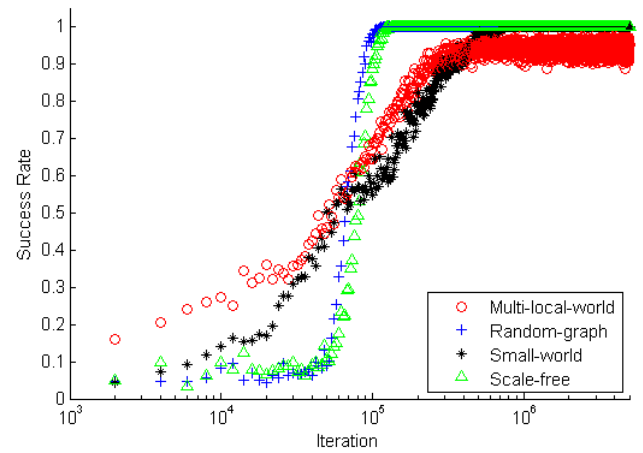
(a) $\langle k \rangle \approx 9.22$



(b) $\langle k \rangle \approx 16.10$



(c) $\langle k \rangle \approx 23.36$



(d) $\langle k \rangle \approx 37.35$

Figure 8 Comparison of the convergence processes in terms of the success rate ($N = 1500$).