

***** Mathematical Weekly *****

Week 32

Let us start with the number 88209. You can split it into two parts: 88 and 209, and then add them together so as to get $88 + 209 = 297$. Now, if you square the result then you will get back to the beginning number 88209, namely:

$$88209 \rightarrow 88 + 209 = 297 \rightarrow 297^2 = 88209$$

This number is not unique. For example, you can easily verify the following:

$$494209 \rightarrow 494 + 209 = 703 \rightarrow 703^2 = 494209$$

Actually, there are more. But the most interesting one might be 1234567900987654321 (almost but not perfectly symmetrical!), which yields

$$123456790 + 0987654321 = 1111111111$$

→

$$1111111111^2 = 1234567900987654321$$

Yet, the largest of such amazing numbers known to us today is the following 100-digit number:

$$\begin{array}{r} 66942\ 14876\ 03305\ 78512\ 39669\ 42148\ 76033\ 05785\ 12396\ 69420 \\ 14876\ 03305\ 78512\ 39669\ 42148\ 76033\ 05785\ 12396\ 69421\ 48761 \end{array}$$

This large number, after the above “split and add” operations, is equal to $\underbrace{8181 \cdots 8181}_{25 \text{ pieces of } 81}^2$

Furthermore, you can find numbers that can be split into 3 parts:

$$1212 + 1388 + 2349 = 4949^3 = 121213882349$$

Or, into 4 parts:

$$20 + 15 + 11 + 21 = 67 \rightarrow 67^4 = 20151121$$

If you are smart enough, or very good in computer programming, you may be able to move forward this way to find other similar amazing numbers ... Have fun!