

***** **Mathematical Weekly** *****
(week twelve)

Interesting Powers

Something simple:

$$1^3 = 1^2$$

$$1^3 + 2^3 = (1+2)^2$$

$$1^3 + 2^3 + 3^3 = (1+2+3)^2$$

.....

$$1^3 + 2^3 + 3^3 + \dots + n^3 = (1+2+3+\dots+n)^2$$

Then, something more complicated:

$$10^2 + 11^2 + 12^2 = 13^2 + 14^2 [= 365]$$

$$21^2 + 22^2 + 23^2 + 24^2 = 25^2 + 26^2 + 27^2$$

$$36^2 + 37^2 + 38^2 + 39^2 + 40^2 = 41^2 + 42^2 + 43^2 + 44^2$$

$$55^2 + 56^2 + 57^2 + 58^2 + 59^2 + 60^2 = 61^2 + 62^2 + 63^2 + 64^2 + 65^2$$

.....

Now, you need a big screen to continue:

$$3^5 + 2^5 + 1^5 = 5 \times 3^4 - 129$$

$$5^5 + 3^5 + 2^5 + 1^5 = 8 \times 5^4 - 129 - 2 \times 3 \times 5(19 + 2 \times 3 \times 5)$$

$$8^5 + 5^5 + 3^5 + 2^5 + 1^5 = 13 \times 8^4 - 129 - 2 \times 3 \times 5(19 + 2 \times 3 \times 5)$$

$$\quad \quad \quad - 3 \times 5 \times 8(19 + 2 \times 3 \times 5 + 2 \times 5 \times 8)$$

$$13^5 + 8^5 + 5^5 + 3^5 + 2^5 + 1^5 = 21 \times 13^4 - 129 - 2 \times 3 \times 5(19 + 2 \times 3 \times 5)$$

$$\quad \quad \quad - 3 \times 5 \times 8(19 + 2 \times 3 \times 5 + 2 \times 5 \times 8)$$

$$\quad \quad \quad - 5 \times 8 \times 13(19 + 2 \times 3 \times 5 + 2 \times 5 \times 8 + 2 \times 8 \times 13)$$

$$21^5 + 13^5 + 8^5 + 5^5 + 3^5 + 2^5 + 1^5 = 34 \times 21^4 - 129 - 2 \times 3 \times 5(19 + 2 \times 3 \times 5)$$

$$\quad \quad \quad - 3 \times 5 \times 8(19 + 2 \times 3 \times 5 + 2 \times 5 \times 8)$$

$$\quad \quad \quad - 5 \times 8 \times 13(19 + 2 \times 3 \times 5 + 2 \times 5 \times 8 + 2 \times 8 \times 13)$$

$$\quad \quad \quad - 8 \times 13 \times 21(19 + 2 \times 3 \times 5 + 2 \times 5 \times 8 + 2 \times 8 \times 13 + 2 \times 13 \times 21)$$

.....

See the patterns?