********** **Mathematical Weekly** **********
(week fifteen)

Math Quiz

1. Find 3 natural numbers, $x > 1$, $y > 1$, and $z > 1$, such that

$$(x+1)^y - x^z = 1$$

I found one solution: $x = 2$, $y = 2$, and $z = 3$. And you?

2. Find 3 natural numbers, x, y, z, such that

$$a^x + b^y = c^z \quad \text{subject to} \quad \frac{1}{x}+\frac{1}{y}+\frac{1}{z}<1$$

where a, b, c are also natural numbers without prime factors in common.

I found 10 solutions:

$1^n + 2^3 = 3^2$ (for any $n > 6$)
$2^5 + 7^2 = 3^4$
$7^3 + 13^2 = 2^9$
$2^7 + 17^3 = 71^2$
$3^5 + 11^4 = 122^2$
$17^7 + 76271^3 = 21063928^2$
$1414^3 + 2213459^2 = 65^7$
$43^8 + 96222^3 = 30042907^2$
$33^8 + 1549034^2 = 15613^3$
$9262^3 + 15312283^2 = 113^7$

And you?

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