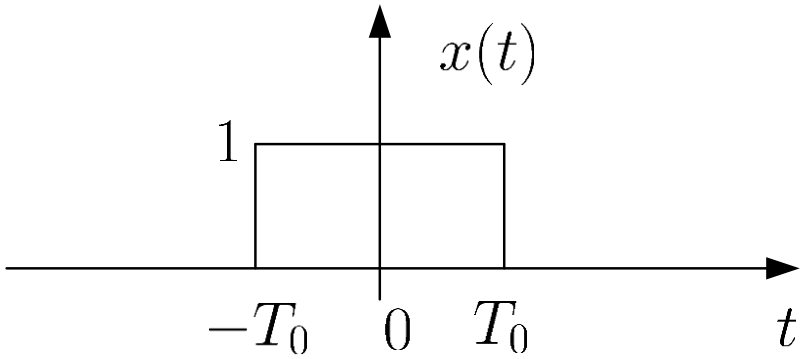
Class Exercises for Chapter 2

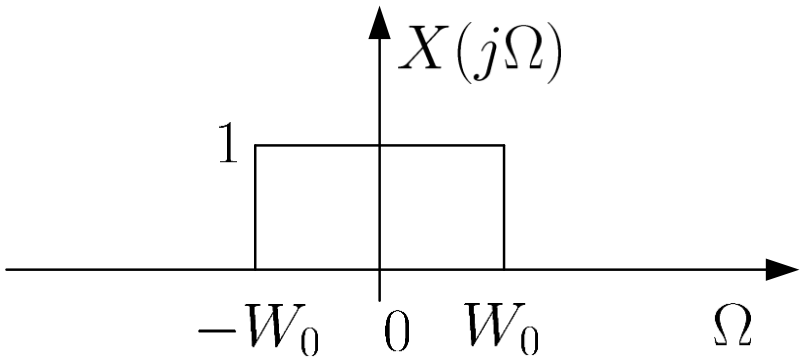
1. Find the Fourier series coefficients for the following continuous-time periodic signal $x(t)$:

$$x(t) = \begin{cases} 1.5, & 0 \le t < 1 \\ -1.5, & 1 \le t < 2 \end{cases}$$

where the fundamental period is $T_p = 2$ and fundamental frequency is $\Omega_0 = \pi$.

2. Represent $x(t)$ and $X(j\Omega)$ using unit step function:





3. By using Example 2.4 or otherwise, prove

$$\frac{2\sin(\Omega T)}{\Omega} = 2T$$

when $\Omega = 0$.

4. Find the real part, imaginary part, magnitude and phase of

$$\frac{a + jb}{b - jc}$$

5. Compute the Fourier transform of

$$x(t) = e^{-\alpha|t|}, \quad \alpha > 0$$

Then find the magnitude and phase of $X(j\Omega)$.

6. Compute the Fourier transform of $x(t) = \cos(100t)$.

7. Compute the Fourier transform of $x(t) = 1$.

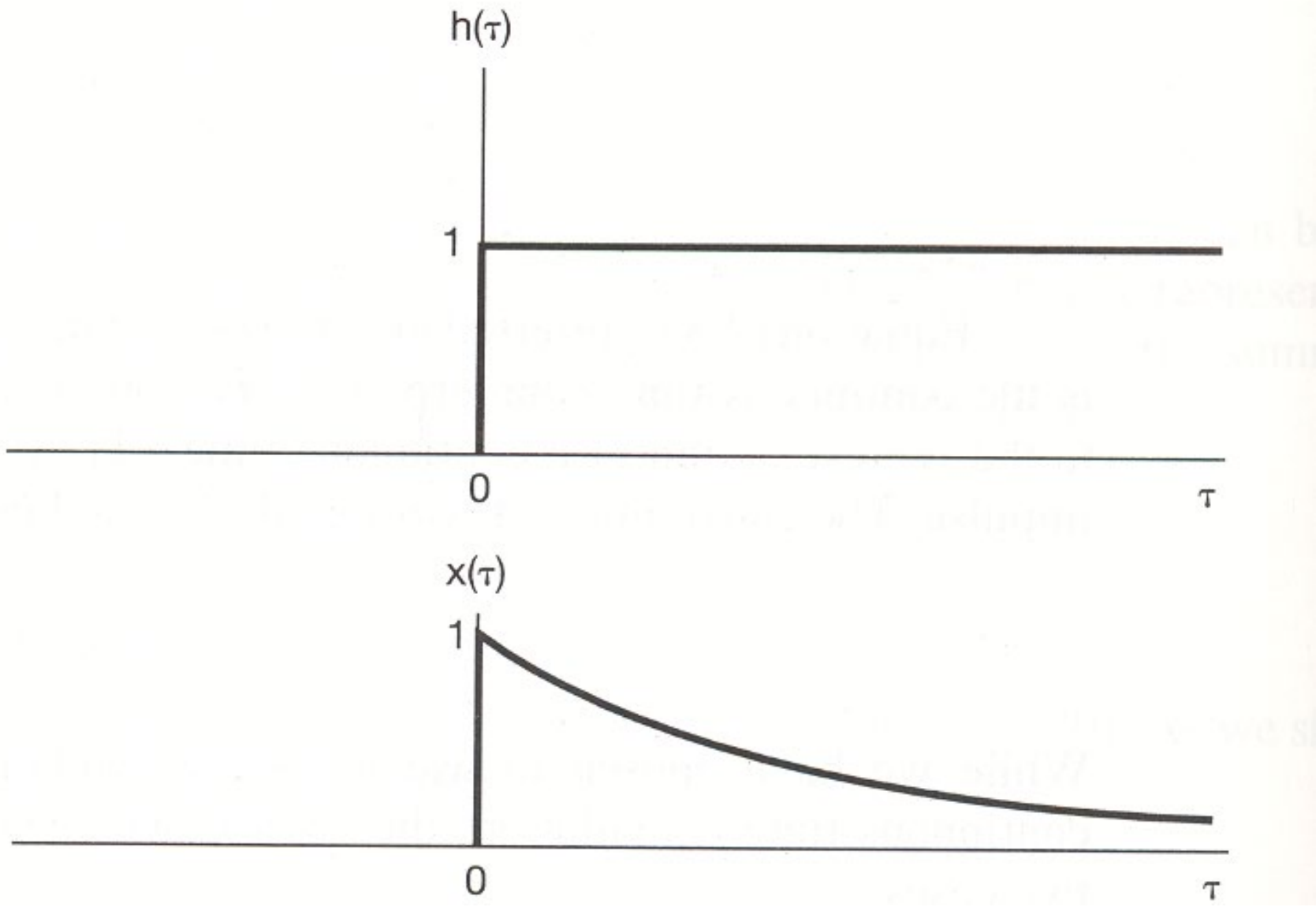
8. Prove the conjugation property of Fourier transform:

$$x(t) \leftrightarrow X(j\Omega) \Rightarrow x^*(t) \leftrightarrow X^*(-j\Omega)$$

   Then show that if $x(t)$ is real-valued, then the magnitude of Fourier transform is symmetric around $\Omega = 0$:

$$|X(j\Omega)| = |X(-j\Omega)|$$

9. Compute the output $y(t)$ if the input is $x(t) = e^{-at}u(t)$ with $a > 0$ and the linear time-invariant system impulse response is $h(t) = u(t)$.


h(τ)
1
0
τ
x(τ)
1
0
τ