Class Exercises for Chapter 3

1. Consider a discrete-time system S with input $x[n]$ and output $y[n]$. This system is obtained through a series interconnection of a system S_1 followed by another system S_2. The input and output relationships for S_1 and S_2 are:

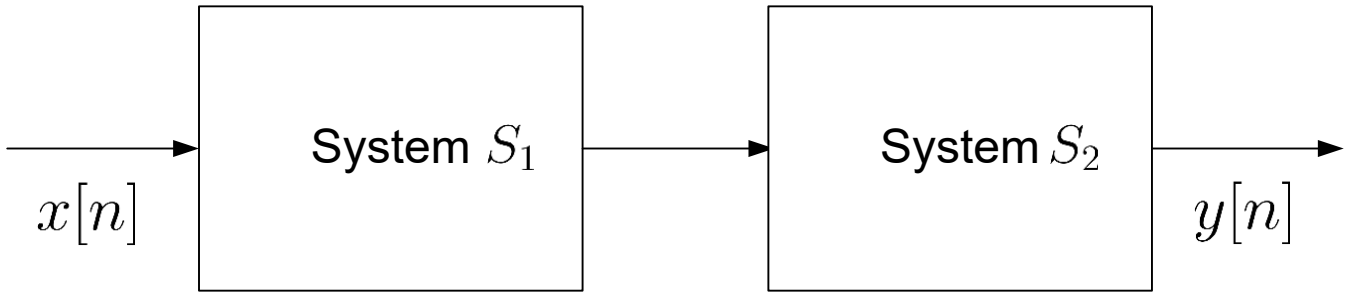
$$S_1 : y_1[n] = 2x_1[n] + 4x_1[n-1]$$

$$S_2 : y_2[n] = x_2[n-2] + \frac{1}{2}x_2[n-3]$$

where $x_1[n]$ and $x_2[n]$ denote inputs and $y_1[n]$ and $y_2[n]$ denote outputs.

(a) Determine the input-output relationship for system S, i.e., find the equation that relates $x[n]$ and $y[n]$.

(b) Does the input-output relationship of system S change if we first pass $x[n]$ through S_2 and then S_1?



2. Sketch $u[n] - u[2n]$.

3. Determine whether the following discrete-time system, with input signal $x[n]$ and output signal $y[n]$ is stable, causal, linear, and/or time-invariant:

$$y[n] = ax[n+1] + b, \qquad 0 < |\, a \,| < \infty, \quad 0 < |\, b \,| < \infty$$

4. Consider a linear time-invariant (LTI) discrete-time system with input $x[n]$, output $y[n]$ and impulse response $h[n]$. With the use of the convolution formula, show that the following two statements regarding causality are identical – (i) Output at time n depends on input up to time n, and (ii) $h[n] = 0$ for $n < 0$.

5. Describe the operation of a LTI discrete-time system whose impulse response is $h[n] = 0.5\delta[n] + 0.5\delta[n-1]$ by relating the input and output. Is the system stable? Why? Is it causal? Why?

6. Compute $y[n]$ for all values of n:

$$y[n] = a^n u[n] \otimes a^n u[n]$$

7. Determine the convolution of the following two discrete-time signals:

$$x[n] = \begin{cases} n^2, & -2 \le n \le -1 \\ 0, & \text{otherwise} \end{cases}$$

and

$$h[n] = \begin{cases} n-1, & 2 \le n \le 3 \\ 0, & \text{otherwise} \end{cases}$$