# Class Exercises for Chapter 5

1. Determine the z transform of

$$x[n] = (0.5)^n (u[n+5] - u[n-5])$$

Specify its region of convergence (ROC).

2. Determine the $z$ transform of

$$x[n] = \begin{cases} \left(\frac{1}{3}\right)^n \cos\left(\frac{\pi}{4}n\right), & n \leq 0 \\ 0, & n > 0 \end{cases}$$

Specify its ROC.

3. A causal linear time-invariant (LTI) system has impulse response $h[n]$ and its z transform is

$$H(z) = \frac{1+z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)}$$

(a) What is the ROC of $H(z)$?
(b) Is the system stable? Why?
(c) Find the impulse response $h[n]$ of the system.

4. Consider the $z$ transform of a discrete-time signal $h[n]$:

$$H(z) = \frac{1-2z^{-1}}{(1+0.3z^{-1})(1-0.5z^{-1})(1-0.7z^{-1})(1+0.9z^{-1})}$$

Determine the zero(s) and pole(s) of $H(z)$. Determine the all the possible ROCs for $H(z)$.

5. If the input $x[n]$ to a LTI system is $x[n] = u[n]$, the system output $y[n]$ is

$$y[n] = \left(\frac{1}{2}\right)^{n-1} u[n+1]$$

(a) Find $H(z)$, the z transform of the system impulse response, and plot its pole-zero diagram.
(b) Find the system impulse response $h[n]$.
(c) Is the system stable?
(d) Is the system causal?

Hint: Consider the z transform convolution property:

$$x_1[n] \otimes x_2[n] \overset{Z}{\leftrightarrow} X_1(z) \cdot X_2(z), \qquad \text{ROC} \quad \textit{includes} \quad R_{x1} \cap R_{x2}$$