Class Exercises for Chapter 6

1. Determine the discrete-time Fourier transforms (DTFTs) for $x[n] = (0.5)^n u[n]$ and $y[n] = 2^n u[n]$.

2. Let $h_c(t)$ be the impulse response of a linear time-invariant (LTI) continuous-time system and it has the form of:

$$h_c(t) = \begin{cases} e^{-at}, & t \geq 0, \quad a > 0 \\ 0, & t < 0 \end{cases}$$

   (a) Determine the Fourier transform of $h_c(t)$, $H_c(j\Omega)$.

   (b) The $h_c(t)$ is sampled with a sampling period of T to produce a sequence $h[n]$. Determine the DTFT of $h[n]$, $H(e^{j\omega})$.

   (c) Find the maximum values for $|H_c(j\Omega)|$ and $|H(e^{j\omega})|$.

3. Consider a LTI system with input $x[n]$, output $y[n]$ and impulse response $h[n]$. Let the DTFT of $h[n]$ be $H(e^{j\omega})$.

   (a) If $x[n] = e^{j\omega_1 n}$, determine $y[n]$ in terms of $H(e^{j\omega})$.

   (b) Extend the result of (a) when

$$x[n] = \sum_{k=1}^{K} \alpha_k e^{j\omega_k n}$$

4. Let $X(e^{j\omega})$ denote the DTFT of $x[n]$. Prove the following two properties:

   (a) The DTFT of $x^*[n]$ is $X^*(e^{-j\omega})$.

   (b) The DTFT of $x^*[-n]$ is $X^*(e^{j\omega})$.