Class Exercises for Chapter 7

1. Prove the following orthogonality identity:

$$\frac{1}{N}\sum_{n=0}^{N-1} e^{j(2\pi/N)(k-r)n} = \begin{cases} 1, & k = r \\ 0, & k = 0,1,\cdots,N-1, \quad k \neq r \end{cases}$$

2. Determine the discrete Fourier transform (DFT) of

$$x[n] = \begin{cases} a^n, & 0 \leq n \leq N-1 \\ 0, & \text{otherwise} \end{cases}$$

3. Consider sampling $x(t) = \cos(\Omega_0 t)$ with a sampling period T to produce $x[n]$. Determine the condition of Ω_0 and T if $x[n]$ is periodic.

4. Let $\tilde{x}[n] \leftrightarrow \tilde{X}[k]$ be a discrete Fourier series (DFS) pair. Prove the symmetric properties:

$$\tilde{x}^*[n] \leftrightarrow \tilde{X}^*[-k]$$

and

$$\tilde{x}^*[-n] \leftrightarrow \tilde{X}^*[k]$$

Then show that if $\tilde{x}[n]$ is real, we have:

- $\text{Re}\{\tilde{X}[k]\} = \text{Re}\{\tilde{X}[-k]\}$ and $\text{Im}\{\tilde{X}[k]\} = -\text{Im}\{\tilde{X}[-k]\}$
- $|\tilde{X}[k]| = |\tilde{X}[-k]|$ and $\angle\tilde{X}[k] = -\angle\tilde{X}[-k]$